Frequency Domain Constant Modulus Algorithm Based on Fractionally Spaced Blind Equalizer

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Abstract—Aiming at the disadvantages of large mean square error(MSE) and slow convergence rate of Constant Modulus Algorithm(CMA), a frequency domain constant modulus algorithm based on fractionally spaced blind equalizer(FS-FCMA) is proposed. In this proposed algorithm, fractionally spaced blind equalizer is used to avoid the spectrum aliasing caused by the sub-sampling and decrease the MSE effectively, and FFT technique and overlapping retention algorithm is employed for realizing the frequency domain constant modulus algorithm to improve the convergence performance of equalizer and to reduce the computational complexity greatly. The FS-FCMA has smaller MSE and faster convergence rate comparison with the constant modulus algorithm in time domain(TCMA) and constant modulus algorithm in time domain based on fractionally spaced blind equalizer(FS-TCMA). The good performance was simulated using underwater acoustic channel.

Index Terms—constant modulus algorithm; fractionally spaced equalizer; FFT overlapping retention algorithm

I. INTRODUCTION

In underwater communication systems, multipath fading and channel distortion may create serious inter-symbol interference (ISI) at the receiver, it decreases the communication efficiency. To overcome ISI, the most effective method is to use blind equalization technology. The conventional approach to channel estimation and channel equalization requires an initial training sequence, and waste the finite bandwidth. The blind equalization technology without initial training sequence can save the bandwidth and improve the communication efficiency. Because of its superiority, it becomes the research focus at present.

Among the blind equalization algorithms, Constant Modulus Algorithm(CMA) is a classical algorithm because of its simple principle, but it has large mean square error(MSE) and slow convergence rate. Fractionally spaced blind equalizer was introduced in [4]-[5], and achieved good equalization results, but its computation complexity is very high. A frequency LMS equalization algorithm was carried out by FFT technique and overlapping retention algorithm in [6], obtained better equalization results than time domain equalization one. Furthermore, its computation complexity is decreased greatly.

A frequency domain constant modulus algorithm based on fractionally spaced blind equalizer (FS-FCMA) is proposed by introducing frequency domain algorithm to fractionally spaced constant modulus equalizer. This proposed algorithm makes use of the advantage of fractionally spaced equalizer, CMA and frequency domain algorithm, and has fast convergence speed and small MSE. The simulation results demonstrate the efficiency of the proposed algorithm.

II. TIME DOMAIN CONSTANT MODULUS ALGORITHM BASED ON FRACTIONALLY SPACED

Taking sampling to the input signal at \( \tau / p \) rate (\( \tau \) is sampling interval, \( p \) is positive integer), and the weight vector of equalizer updating with CMA in time domain, then A Time Domain Constant Modulus Algorithm Algorithm based On Fractionally Spaced (FS-TCMA) is obtained, its structure is shown in figure 1.

Where \( s(k) \) is the transmitted symbol sequence with independent and identically distributed; \( e^{j\phi}(k) \) is the impulse response of the \( p \)-th sub-channel fractionally spaced equalizer, \( y^{(p)}(k) \) is the input signal of the \( p \)-th sub-channel, \( f^{(p)}(k) \) is the input signal of the \( p \)-th sub-channel, \( z^{(p)}(k) \) is the output signal of the \( p \)-th sub-channel, \( z(k) \) is the signal after merger. By the theory of fractionally spaced equalizer and time domain constant modulus algorithm, the input of sub-channel is written as
\[ y^{(p)}(k) = \sum_{i=0}^{L-1} s(i)e^{(p)}(k-i) + n^{(p)}(k). \] (1)

The output of sub-channel equalizer is given by
\[ z^{(p)}(k) = \sum_{i=0}^{L-1} f^{(p)T}(k-i). \] (2)

The output of fractionally spaced equalizer is denoted as
\[ z(k) = \sum_{p=0}^{P-1} z^{(p)}(k). \] (3)

The updating formula of weight vector in sub-equalizer is denoted as by CMA
\[ f^{(p)}(k+1) = f^{(p)}(k) - \mu y^{(p)\ast}(k + L) \cdot (R - |z(k)|^2) z^{(p)}(k + L). \] (4)

Where \( R = E[s(k)^4]/E[s(k)^2] \) is the module of the transmitted signal sequence \( s(k) \), this algorithm is called as the Time Domain Constant Modulus Algorithm based On Fractionally Spaced (FS-TCMA).

The updating rule of weight vector in \( k + L \) is given by
\[ f^{(p)}(k + L) = f^{(p)}(k) - \mu \sum_{m=0}^{L-1} y^{(p)\ast}(k + m) \cdot (R - |z(k + m)|^2) z^{(p)}(k + m). \] (5)

And obtained by recursive iterative
\[ f^{(p)}(k + L) = f^{(p)}(k) - \mu \sum_{m=0}^{L-1} y^{(p)\ast}(k + m) \cdot (R - |z(k + m)|^2) z^{(p)}(k + m). \] (6)

The CMA is called as Block CMA (Block CMA,BCMA) when the weight vector update as (6) implement only one time from \( k \) to \( k + L \). Because of its weigh vector not update every time, thus, the error signals and output signals of BCMA are little different from classic CMA, they are calculated by the previous weight vector through (7) and (8):
\[ z^{(p)}(k + m) = y^{(p)\ast}(k + m) f^{(p)}(k) \] (7)
\[ e^{(p)}(k + m) = R - |z^{(p)\ast}(k + m)|^2 \] (8)

where, \( m = 1, 2, \ldots, L - 1 \). When the interval of weigh vector update is the same as filter order, the weight vector update rate of BCMA is slower than input signal speed, for convenience, redefine a new block time measure \( n \), make \( n \) increase a measure unit when the original time measure \( k \) increase \( L \), so (6) can express as
\[ f^{(p)}(k + 1) = f^{(p)}(k) - \mu \sum_{m=0}^{L-1} y^{(p)\ast}(k + L + m) \cdot (R - |z(k + L + m)|^2) z^{(p)}(k + L + m). \] (9)

As shown in(9), the very difference between CMA and CMA is the gradient estimate value of MSE surface. The former takes the average value of stochastic gradient from current time to first \( L - 1 \) time and smoothes the noise.

Compared with instantaneous stochastic gradient, its accuracy is increased and increased with \( L \).

III. FREQUENCY DOMAIN CONSTANT MODULUS ALGORITHM BASED ON FRACTIONALLY SPACED

Frequency equalization takes the frequency characteristic of tunable filter to compensate that of baseband system and to make the total characteristic of baseband system meet the conditions of undistorted transmission, the structure is shown in figure 2.

Where \( Y^{(p)}(k) \) is the input signals after FFT to \( y^{(p)}(k) \), \( F^{(p)}(k) \) is the weight vector of frequency equalizer in the \( p \)th sub-channel, \( Z^{(p)}(k) \) is the output signal of sub-equalizer. According to the principle of FFT and overlapping retention algorithm, dividing \( Y^{(p)}(k) \) into blocks of length \( L \), and the updating of weigh vector implement every \( L \) sampling. The process of FFT technique and overlapping retention algorithm is shown as follows:

\[ Y^{(p)}(k) = \text{FFT} \left[ y^{(p)}(k), y^{(p)}(k+1), \ldots, y^{(p)}(k+L-1) \right] \]
\[ y^{(p)}(kL), y^{(p)}(kL+1), \ldots, y^{(p)}(kL+L-1) \]
where \( 0 \leq k \leq 2L - 1 \)

The weight vector of equalizer is expressed as
\[ F_{j}(K) = \text{FFT} \left[ f^{(p)}(k), 0, 0, \ldots, 0 \right] \]
\[ \text{the } j \text{ block} \]
\[ \text{the } k \text{ block} \]
\[ \text{the } L \text{ block} \]

The output of this block equalizer is determined as
\[ Z^{(p)}(k) = F^{(p)}(K) \cdot Y^{(p)}(K) \]
\[ \text{the } \text{block} \]
\[ \text{the } \text{block} \]
\[ \text{the } \text{block} \]

The output of fractionally spaced equalizer is given by
\[ z(k) = \text{IFFT} \sum_{p=0}^{P-1} Z^{(p)}(K) \]
\[ \text{the } \text{block} \]
\[ \text{the } \text{block} \]
\[ \text{the } \text{block} \]

The updating formula of weight vector is written as
Where
\[ F^{(p)}(k+1) = F^{(p)}(k) + 2\mu \nabla^{(p)} \]

\[ \nabla^{(p)} = \text{IFFT}[E_0^{(p)}(K) \cdot \text{conj}(Y_0^{(p)}), E_1^{(p)}(K) \cdot \text{conj}(Y_1^{(p)}) \cdots E_{2L-1}^{(p)}(K) \cdot \text{conj}(Y_{2L-1}^{(p)})]. \]

IV. COMPUTATION COMPLEXITIES

Frequency domain CMA can decrease the computation complexity. In fact, time domain CMA equalizer of \( L \) node needs \( 2L^2 \) real multiplication to produce \( L \) output, and to the same output, frequency equalizer needs three FFT of \( L \) node and \( 2L \) complex multiplication, but there is the half of weight not need to calculate to real input. FFT of \( L \) node can be come out by FFT of \( L/2 \) node and \( L/2 \) complex multiplication. Thus, frequency domain needs \( 3\log_2(L/2) + 4L \) real multiplication. The computation complexities rate of frequency domain LMS to time domain CMA is

\[
\frac{\text{frequency domain CMA multiplication}}{\text{time domain CMA multiplication}} = \frac{3\log_2(L/2) + 4L}{2L^2} \tag{15}
\]

When \( L = 16 \), the rate is 0.41; \( L = 32 \), the rate is 0.25; when \( L = 256 \), \( N = 256 \), the rate is 0.049. Thus, the operate of save is large when \( N \) is big.

V. SIMULATION RESULTS

In order to prove the effectiveness of the proposed T/2FF-CMA algorithm, we compared it with FF-CMA and TF-CMA algorithms. The simulation makes use of channel

\[ c = [1, 0, 0.3e^{0.7j}, 0, 0, 0.2e^{0.8j}] \].

**Experiment 1** Transmitted signal is 4QAM; variance is 1, SNR=20dB; the length of equalizer is 12 and all take center taps are initialized; \( \mu_{\text{TF-CMA}} = 0.002 \), \( \mu_{\text{FF-CMA}} = 0.004 \), \( \mu_{\text{T/2FF-CMA}} = 0.004 \); Average of 5000 independent simulation results is shown in figure 3

**Experiment 2** Transmitted signal is 4PSK; variance is 1, SNR=20dB; the length of equalizer is 12 and all take center taps are initialized; \( \mu_{\text{TF-CMA}} = 0.006 \), \( \mu_{\text{FF-CMA}} = 0.006 \), \( \mu_{\text{T/2FF-CMA}} = 0.008 \); Average of 5000 independent simulation results is shown in figure 4
Figure 3(a) shows that the convergence rate of T/2FF-CMA is faster than TF-CMA about 450 steps and faster than FF-CMA about 300 steps; the mean square errors of T/2FF-CMA is less than TF-CMA about 5dB and less than FF-CMA about 3dB. Figure 3(b)-(e) show that the constellation of T/2FF-CMA is the clearest.

Figure 4(a) shows that the convergence rate of T/2FF-CMA is faster than TF-CMA about 450 steps and faster than FF-CMA about 200 steps; the mean square errors of T/2FF-CMA is less than TF-CMA and FF-CMA about 3dB. Figure 4(b)-(e) show that the constellation of T/2FF-CMA is the clearest.

VI. CONCLUSIONS

In this paper, a Frequency Domain CMA Based on Fractionally Spaced (FS-FCMA) is proposed. The important feature of FS-FCMA is to use fractionally spaced equalizer to reduce frequency aliasing caused by under-sampling to decrease MSE, use the frequency characteristic of tunable filter to compensate the frequency characteristic of baseband system and make the total characteristic of baseband system meet the conditions of undistorted transmission, and use FFT technique and overlapping retention algorithm calculate linear convolution to decrease the computation complexity. The simulation results of underwater acoustic channel demonstrate the efficiency of the proposed algorithm.

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REFERENCE