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A Novel Track-Before-Detect Algorithm for Weak Target

Hui Guo, Hong-bing Ji, and Bin Wu
School of Electronic Engineering, Xidian University, Xi'an, P. R. China
Email: ghui_xd@yahoo.cn

***Abstract*—In this paper, a novel track-before-detect algorithm is proposed for weak target with low signal to noise ratio. Multi-sensor information combining with track-before-detect technology is used to improve the signal to noise ratio. According to frame sequences of each sensor, Rao-Blackwellized particle filter, which is a clever combination of the Kalman filter and the standard particle filter, is employed to obtain state estimation and likelihood ratio of the target. In the detection process, the proposed cusum joint detection is adopted to realize detection of target appearance and disappearance. Calculation of the disappearance statistic is started the moment a target is detected, which ensures the continuous detection of target appearance and shorter delay for the detection of target disappearance. Simulation results show the efficiency of the proposed algorithm.**

***Index Terms*—Target appearance and disappearance, Rao-Blackwellized particle filter, Cusum joint detection**

I. INTRODUCTION

The problem of weak target detection and track has always been a challenge in signal processing. Recently, a class of algorithms called track-before-detect (TBD) [1-3] has been developed to solve the problem with increased attention due to its effective detection performance in adverse environment.

Dynamic programming [3] and multiple hypotheses testing [4] are the representatives of TBD algorithms, whose essence is to get high signal-to-noise ratio (SNR) at the expense of computation and storage. Nevertheless, the performance of these algorithms decline seriously when dealing with non-linear and non-gaussian system. Particle filter, as an effective method to handle this problem, has made particle filter based on recursive TBD method become the research hotspot currently. Salmond [5] first put forward the PF algorithm based on TBD, in whose detection processing, the statistic is taken as existing probability lacking theoretical foundation. The idea of cumulative likelihood ratio test based on multi-observations has been introduced in references [6] and [7] respectively, however, the concrete detection steps are unknown. Gong yaxin [8] puts forward a recursive TBD algorithm based on PF, combined sequential probability ratio test and fixed sample size likelihood ratio test, where the likelihood ratio test statistics is derived and specific detection steps are given, but its large computation with high dimensional space-state limits the practical applications.

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Rao-Blackwellized particle filter (RBPF) [9], which can get better estimation performance than PF, has been applied in a number of state estimation problems. According to the above problems, a novel track-before-detect algorithm based on RBPF is proposed combined with Multi-sensor information. In the algorithm, Multi-sensor information of current time is used to improve the current SNR, through which the detection of target appearance and disappearance can been implemented rapidly with shorter delay. According to frame sequences of each sensor, Rao-Blackwellized particle filter is employed to obtain state estimation and likelihood ratio of the target. During the detection processing, firstly, the cusum statistic is adopted to determine target appearance. Subsequently, calculation of the disappearance statistic is started the moment a target is detected.

The remainder of the paper is organized as follows. Section II introduces the target and sensor models. The implementation of the TBD algorithm is presented in Section III. In Section IV, simulation results with corresponding analyses are given. Finally, conclusions are given in Section V.

II. TARGET AND SENSOR MODELS

A. Target Model

Assuming the target in uniform motion, the discrete time evolution of state is defined as a Markov Process, The target is represented by a five-dimensional state vector $\mathbf{x}_k = [x_k \;\; \dot{x}_k \;\; y_k \;\; \dot{y}_k \;\; I_k]^T$, where (x_k, y_k), $(\dot{x}_k, \dot{y}_k)$ and I_k denote the position, velocity, and the intensity of the target at time *k*, respectively.

$$\mathbf{x}_{k+1} = \begin{bmatrix} 1 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k + \mathbf{w}_k \qquad (1)$$

where T is time sample interval and $\mathbf{w}_k$ is a Gaussian noise with zero mean and covariance $\mathbf{Q}$, which is determined by q_1 and q_2 [6].

Target existence is modeled by a state Markov Chain $E_k \in \{0,1\}$, where 0 denotes the event that the target is absent while 1 denotes the opposite. The transition probabilities of the Markov Chain are defined as

$$P_b \triangleq P\{E_k = 1 \mid E_{k-1} = 0\}$$
$$P_d \triangleq P\{E_k = 0 \mid E_{k-1} = 1\} \quad (2)$$

The other two transition probabilities of the Markov Chain, the probability of staying alive and of staying dead is given by $1-P_b$ and $1-P_d$, respectively. Hence the transitional probability matrix of the Markov Chain is,

$$\Pi = \begin{bmatrix} 1-P_b & P_b \\ P_d & 1-P_d \end{bmatrix} \quad (3)$$

Furthermore, the initial probability of target existence $P\{E_0 = 1\}$ is assumed to be known.

B. Sensor Model

The sensor gives a sequence of images of the surveillance region. Each image consists of $n \times m$ resolution cells. The centre of each cell (i, j) is defined to be at $(i\Delta_x, j\Delta_y)$. The measurements of the *r-th* sensor are recorded at discrete time k with sampling interval *T*. The measured intensity at each resolution cell (i, j) is given by

$$z_{k,r}^{(i,j)} = \begin{cases} h_k^{(i,j)}(\mathbf{x}_k) + \mathbf{v}_{k,r}^{(i,j)}, & \mathrm{E}_k = 1 \\ \mathbf{v}_{k,r}^{(i,j)}, & \mathrm{E}_k = 0 \end{cases} \quad (4)$$

where $\mathbf{v}_{k,r}^{(i,j)}$ is measurement noise of the *r-th* sensor in cell (i, j), assumed to be independent from pixel to pixel and frame to frame. For simplicity, we assume $\mathbf{v}_{k,r}^{(i,j)}$ to be Gaussian, i.e. $\mathbf{v}_{k,r}^{(i,j)} \propto \mathrm{N}(0, \sigma^2)$. $h_k^{(i,j)}(\mathbf{x}_k)$ is the target contribution to cell (i, j) approximated by a point spread function [6],

$$h_k^{(i,j)}(\mathbf{x}_k) \approx \frac{\Delta_x \Delta_y I_k}{2\pi\Omega^2} \exp\{-\frac{(i\Delta_x - x_k)^2 + (j\Delta_y - y_k)^2}{2\Omega^2}\} \quad (5)$$

where Ω is a known parameter that represents the extent of blurring of the sensor.

Assuming $\mathbf{z}_{k,r} = \{z_{k,r}^{(i,j)} : i = 1,...,n, j = 1,...m\}$ denotes the complete measurement recorded of the *r-th* sensor at time *k*. While the complete set of measurements of the *r-th* sensor from time 1 to k is denoted as $\mathbf{Z}_k = \{\mathbf{z}_{i,r}, i = 1,...,k, r = 1,...,R\}$. Based on the above model, the likelihood function is given below,

$$p(\mathbf{z}_{k,r} \mid \mathbf{x}_k, E_k) = \begin{cases} \prod_{i=1}^{n}\prod_{j=1}^{m} p_1(z_{k,r}^{(i,j)} \mid \mathbf{x}_k), & E_k = 1 \\ \prod_{i=1}^{n}\prod_{j=1}^{m} p_0(z_{k,r}^{(i,j)}), & E_k = 0 \end{cases} \quad (6)$$

where $p_0(z_{k,r}^{(i,j)})$ and $p_1(z_{k,r}^{(i,j)} \mid \mathbf{x}_k)$ denote the PDF of the background noise in pixel (i, j) and likelihood of the target signal plus noise in pixel (i, j).

III. ALGORITHM ANALYSIS

A. State Estimation

According to the Target and Sensor Model described in part II, the state vector can be partitioned as $\mathbf{x}_k = [\mathbf{x}_k^n \ \mathbf{x}_k^l]^{\mathrm{T}}$, where $\mathbf{x}_k^n = [x_k, y_k, I_k]^{\mathrm{T}}$ denotes the nonlinear state variable and $\mathbf{x}_k^l = [\dot{x}_k, \dot{y}_k]^{\mathrm{T}}$ denotes the linear state variable. Then the target and sensor model can be rewritten as below,

$$\mathbf{x}_{k+1}^l = \mathbf{A}^l \mathbf{x}_k^l + \mathbf{w}_k^l$$
$$\mathbf{x}_{k+1}^n = \mathbf{F}\mathbf{x}_k^n + \mathbf{A}^n \mathbf{x}_k^l + \mathbf{w}_k^n \quad (7)$$
$$z_{k,r}^{(i,j)} = \begin{cases} h_k^{(i,j)}(\mathbf{x}_k^n) + \mathbf{v}_{k,r}^{(i,j)}, & \mathrm{E}_k = 1 \\ \mathbf{v}_{k,r}^{(i,j)}, & \mathrm{E}_k = 0 \end{cases}$$

where, $\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{A}^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{A}^l = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{w}_k^n \sim \mathrm{N}\left(0, \mathbf{Q}^n\right)$, $\mathbf{w}_k^l \sim \mathrm{N}\left(0, \mathbf{Q}^l\right)$.

Assuming the number of particles is N, filtering process of the *r-th* sensor based on RBPF can been summarized as following according to the reference [6].

Step1: Set $k = 1$, draw N samples denoted as $\{\mathbf{x}_{0|0}^{n\ (i)}, E_{0|0}^{(i)}, \mathbf{x}_{0|0}^{l(i)}, \mathbf{P}_{0|0}^{(i)}\}_{i=1}^{N}$ according to initial distribution $p_0(\mathbf{x}_0^n)$, $\{\mathbf{x}_0^l, \mathbf{P}_0\}$ and $P\{E_0 = 1\}$.

Step2: Draw $\{E_{k|k-1}^{(i)}\}_{i=1}^{N}$ according to $\{E_{k-1|k-1}^{(i)}\}_{i=1}^{N}$ and Π defined in A of II. If $E_{k-1|k-1}^{(i)} = 0$ and $E_{k|k-1}^{(i)} = 1$, draw $\mathbf{x}_{k|k-1}^{n(i)} \propto p(\mathbf{x}_0^n \mid \mathbf{z}_k)$ and set $\{\mathbf{x}_{k|k-1}^{l(i)}, \mathbf{P}_{k|k-1}^{(i)}\} = \{\mathbf{x}_0^l, \mathbf{P}_0\}$. If $E_{k-1|k-1}^{(i)} = 1$ and $E_{k|k-1}^{(i)} = 1$, draw $\mathbf{x}_{k|k-1}^{n(i)}$ from a Gaussian distribution with the mean $\boldsymbol{\mu} = \mathbf{F}\mathbf{x}_{k-1|k-1}^{n(i)} + \mathbf{A}^n \mathbf{x}_{k-1|k-1}^{l(i)}$ and covariance $\boldsymbol{\Sigma} = \mathbf{A}^n \mathbf{P}_{k-1|k-1}(\mathbf{A}^n)^{\mathrm{T}} + \mathbf{Q}^n$. $\{\mathbf{x}_{k|k-1}^{l(i)}, \mathbf{P}_{k|k-1}^{(i)}\}$ is updated as follows

$$\mathbf{x}_{k|k-1}^{l(i)} = \mathbf{x}_{k|k-1}^{l(i)*} + \mathbf{K}_k^{(i)}(\mathbf{y}_k^{(i)} - \mathbf{A}^n \mathbf{x}_{k|k-1}^{l(i)*})$$
$$\mathbf{P}_{k|k-1}^{(i)} = \mathbf{P}_{k|k-1}^{(i)*} - \mathbf{K}_k^{(i)} \mathbf{L}_k^{(i)} (\mathbf{K}_k^{(i)})^{\mathrm{T}}$$
$$\mathbf{K}_k^{(i)} = \mathbf{P}_{k|k-1}^{(i)*} (\mathbf{A}^n)^{\mathrm{T}} (\mathbf{L}_k^{(i)})^{\mathrm{T}} \quad (8)$$
$$\mathbf{L}_k^{(i)} = \mathbf{A}^n \mathbf{P}_{k|k-1}^{(i)*} (\mathbf{A}^n)^{\mathrm{T}} + \mathbf{Q}^n$$

where

$$\mathbf{x}_{k|k-1}^{l(i)*} = \mathbf{A}^l \mathbf{x}_{k-1|k-1}^{l(i)}$$
$$\mathbf{y}_k^{(i)} = \mathbf{x}_{k|k-1}^{n(i)} - \mathbf{F}\mathbf{x}_{k-1|k-1}^{n(i)} \quad (9)$$
$$\mathbf{P}_{k|k-1}^{(i)*} = \mathbf{A}^l \mathbf{P}_{k-1|k-1}^{(i)} (\mathbf{A}^l)^{\mathrm{T}} + \mathbf{Q}^l$$

Step3: Obtain the respective weights by

$$\tilde{w}_k^{(i)} = p(\mathbf{z}_{k,r} \mid \mathbf{x}_{k|k-1}^{n(i)}, E_{k|k-1}^{(i)})$$
$$w_k^{(i)} = \frac{\tilde{w}_k^{(i)}}{\sum_{i=1}^{N} \tilde{w}_k^{(i)}}, i = 1, \cdots, N \quad (10)$$

Step4: Resample $\{\mathbf{x}_{k|k}^{n(i)}, \mathbf{x}_{k|k}^{l(i)}, \mathbf{P}_{k|k}^{(i)}, E_{k|k}^{(i)}\}_{i=1}^{N}$ according to $\{\mathbf{x}_{k|k-1}^{n(i)}, \mathbf{x}_{k|k-1}^{l(i)}, \mathbf{P}_{k|k-1}^{(i)}, E_{k|k-1}^{(i)}, w_k^{(i)}\}_{i=1}^{N}$.

Step5: Increase time and repeat from step 2.

B. Cusum Joint Detection

The conventional cusum detection is proposed by

Page [10] in 1954 based on likelihood, which can been represented by a dual hypothesis as follow

$H_{\lambda,\gamma}$: target appear at time λ and disappear at time γ, $1 \le \lambda \le k, \gamma \ge \lambda + 1$

H_0 : target do not appear, $\lambda \notin [1,k]$

Supposing the complete set of state form 1 to k is denoted as $\mathbf{X}_k = \{\mathbf{x}_i, i = 1,...,k\}$, according to the above hypothesis, the log-likelihood ratio (LLR) for the observation $\mathbf{Z}_k$ is given below

$$\begin{aligned}\Lambda(\mathbf{X}_k) &= \log \frac{p(\mathbf{Z}_k \mid H_{\lambda,\gamma})}{p(\mathbf{Z}_k \mid H_0)} \\ &= \sum_{n=\lambda}^{\gamma} \sum_{r=1}^{R} \log \frac{p_1(\mathbf{z}_{n,r} \mid \mathbf{z}_{1,r}, \ldots, \mathbf{z}_{n-1,r}; \mathbf{x}_n)}{p_0(\mathbf{z}_{n,r} \mid \mathbf{z}_{1,r}, \ldots, \mathbf{z}_{n-1,r})}\end{aligned} \quad (11)$$

By L denote the generalized LLR [11],

$$L(\mathbf{X}_k) = \max_{\substack{1\le\lambda\le k \\ \gamma\ge\lambda+1}} \Lambda(\mathbf{X}_k) = \sum_{r=1}^{R} \max_{\substack{1\le\lambda\le k \\ \gamma\ge\lambda+1}} \sum_{n=\lambda}^{\gamma} \Lambda_r(\mathbf{x}_n) \quad (12)$$

where $\Lambda_r(\mathbf{x}_n) = \log \frac{p_1(\mathbf{z}_{n,r} \mid \mathbf{z}_{1,r}, \ldots, \mathbf{z}_{n-1,r}; \mathbf{x}_n)}{p_0(\mathbf{z}_{n,r} \mid \mathbf{z}_{1,r}, \ldots, \mathbf{z}_{n-1,r})}$ denote the LLR of the r-*th* sensor at time *n*. It is easily shown that the statistic $L(\mathbf{X}_k)$ satisfies the following system of recursive relations

$$\begin{aligned}L(\mathbf{X}_k) &= \max\{L(\mathbf{X}_{k-1}), U(\mathbf{X}_{k-1})\} \\ U(\mathbf{X}_k) &= \Lambda(\mathbf{x}_k) + \max\{0, U(\mathbf{X}_{k-1})\},\ k \ge 1\end{aligned} \quad (13)$$

where $\Lambda(\mathbf{x}_k) = \sum_{r=1}^{R} \Lambda_r(\mathbf{x}_k)$ denote the cumulative LLR at time *n*. This shows that the test is equivalent to the sequential test which is defined by the stopping time $\tau = \min\{k : U(\mathbf{X}_k) \ge T_U\}$, $\tau = \infty$ if no such *k*, where T_U is threshold that is defined on the give probability of false alarm. We have $T_U = \log(1/\bar{P}_{fa})$, here, $\bar{P}_{fa}$ is upper bound of the frequency of false alarms defined as $\bar{P}_{fa} = 1/M$ (M is the number of frames). According to the Bayesian approach, if $\tau < \infty$, then the hypothesis $H_{\lambda,\gamma}$ is accepted, while if $\tau = \infty$, the hypothesis H_0 is accepted. However, it will be failed when only using the statistic U, because of its cumulation after target appearance detected.

In this paper, we consider an extended decision making process assuming that each decision on target disappearance the precious data are discarded. Specifically, we accept the following hypothesis

$H_{1,\lambda,\gamma}$: target appear at time λ and do not disappear, $1 \le \lambda \le k$

$H_{2,\lambda,\gamma}$: target appear at time λ and disappear at time γ, $1 \le \lambda < k, \lambda + 1 \le \gamma < k$

H_0 : target do not appear, $\lambda \notin [1,k]$

Clearly, at time moment *k* the LLR of subalternative $H_{1,\lambda,\gamma}$ is determined by the statistic $U(\mathbf{X}_k)$, while the LLR of subalternative $H_{2,\lambda,\gamma}$ is determined by the statistic $L(\mathbf{X}_{k-1})$. Thus the structure of the decision making algorithm may be as follows.

Step1: When the *i-th* target disappeared, the statistic $U(\mathbf{X}_k)$ is formed with the mull initial condition. If $U(\mathbf{X}_k) \ge T_U$, target is detected. Otherwise, the observation is continued.

Step2: When the $i+1$-*th* target appeared, this statistic $U(\mathbf{X}_k)$ is computed further and the algorithm starts to compute the statistic $L(\mathbf{X}_k)$ with the initial condition $L_\tau = U_\tau$.

Step3: At each step $k = \tau+1, \tau+2, \cdots$, the statistic $\Delta(\mathbf{X}_k) = L(\mathbf{X}_{k-1}) - U(\mathbf{X}_k)$ is compared with another threshold T_Δ. If $\Delta(\mathbf{X}_k) < T_\Delta$, then target is also present and the next time step is analyzed. If $\Delta(\mathbf{X}_k) \ge T_\Delta$, then target disappearance is made, the statistic $L(\mathbf{X}_k)$ is not computed and the statistic $U(\mathbf{X}_k)$ is formed with the mull initial condition. Then the whole cycle is repeated.

C. Algorithm Process

Combined with Rao-Blackwellized particle filter (RBPF) and cusum joint detection, the algorithm can be summarized as follow.

(1) When target do not appear, the new observation is filtered according to the RBPF algorithm described in A of III, meanwhile, the statistic U is computed using the formula (13). Then the target is detected according to step1 in B of III.

(2) Supposing the target appear at time *k*, the filer processing is continued and the detection of target disappearance is made according to step2 and step3 in B of III.

(3) When target disappearance is made, another target is searched from the last target disappearing moment. Then the filer processing and detection processing are repeated.

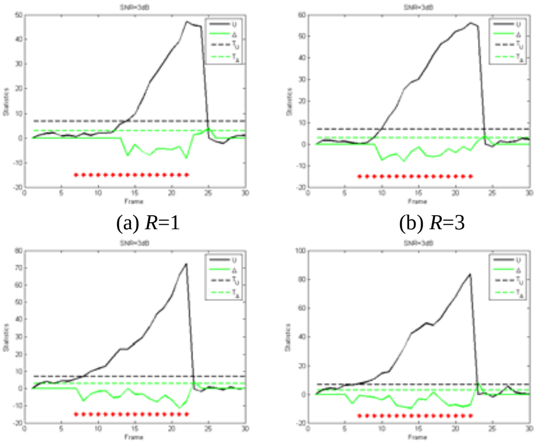
IV. Experriments

Consider the following condition as an experiment. For each sensor, sequence of simulated image contains 30 frames, and is generated according to the model in section II with the following parameters: T = 1, the level of process noise covariance is $q_1 = 0.001$ and $q_2 = 0.01$, n=m=20, $\Delta_x = \Delta_y = 1$, the blurring parameter $\Omega = 0.7$, the intensity $I_0 = 20$. The measurement SNR is defined as

$$SNR = 10 \log \left[\frac{I_0 \Delta_x \Delta_y}{2\pi\Omega^2\sigma} \right]^2 \quad (14)$$

Target is present from frame 7 to frame 22 with initial state $[4.2\ 0.45\ 7.2\ 0.25\ 20]^{\mathrm{T}}$. The algorithm parameters are selected as follows: target birth probability $P_b = 0.05$ and target death probability $P_d = 0.05$, initial existence probability $\mathrm{P}\{E_0 = 1\}$ = 0.05, $N = 6000$, target intensity range from $I_{\min} = I_0 - 10$ to $I_{\max} = I_0 + 10$.

Under the condition of $\sigma = 4.6$, that is SNR=3dB, with different sensor number *R*=1, 3, 5 and 7, Fig.1 shows the results of detection of target appearance and disappearance. Asterisk signs (*) at the bottom of the figure indicate the presence of the target.



(a) *R*=1 (b) *R*=3

(c)*R*=5 (d) *R*=7

Figure 1. Detection of target appearance and disappearance

The detection of target occurs when the statistic U exceeds the threshold T_U and target disappearance is detected when the statistic Δ exceeds the threshold T_Δ. When the decision on target disappearance is made, the statistic U is renewed from 0. It can be seen that the proposed algorithm is able to detect target with 3dB efficiently, especially with more sensors. To be precise, the algorithm detects the target with the delay about 6 frames for single sensor, 3 frames for *R*=3, 1 frames for *R*=5 and no delay for *R*=7. Then the fact of target's disappearance is detected with very small delay, with the delay about 2 frames for single sensor, 1 frames for *R*=3, and no delay for *R*=5 and *R*=7.

Table I shows the statistic analysis of the proposed algorithm with different SNB and different sensor number in 100 independent simulations. It is seen that when the SNR is high, the algorithm shows a good detection performance with no delay, even for single sensor. As a whole, the delay of target appearance and disappearance reduce as the sensor increase. In addition, the average of detection delay is defined as the average of total delay in 100 independent simulations.

TABLE I.
STATISTIC ANALYSIS

	Average delay of appearance			Average delay of disappearance		
SNR (dB)	12	6	3	12	6	3
R=1	0	1.8	5.4	0	0.8	1.2
R=3	0	0.5	3.2	0	0.2	0.6
R=5	0	0	1.1	0	0	0.4
R=7	0	0	0.3	0	0	0.1

V. CONCLUSIONS

In this paper, a novel track-before-detect algorithm is proposed for weak target with low signal to noise ratio. Multi-sensor information of current time is used to improve the current SNR, through which the detection of target appearance and disappearance can been implemented rapidly with shorter delay. Rao-Blackwellized particle filter, which is a clever combination of the Kalman filter and the standard particle filter, is employed in the algorithm to obtain better estimations and reduce the computational demand. The proposed cusum joint detection is adopted to realize detection of target appearance and disappearance. The simulation results show the algorithm based on multi-sensor has a good detection performance.

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