The Multi-user Detection for the MIMO-OFDM System Based on the Genetic Simulated Annealing Algorithm
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Abstract—In the MIMO-OFDM systems, the traditional methods of multi-user detection have some limitations in the rank-deficient scenarios, where the number of supported users exceeds the number of receiver antennas. The multi-user detection (MUD) based on the genetic simulated annealing algorithm (GSAA) is proposed for the rank-deficient problems in the multi-user MIMO-OFDM system. The simulation results show that the GSAA-MUD algorithm can achieve higher performance than the Genetic algorithm (GA) MUD in the rank-deficient scenarios.

Index Terms—MUD, Rank-deficient, GSAA, GA, MIMO-OFDM

I. INTRODUCTION
In the MIMO-OFDM system, the number of users always exceeds the number of receivers, which is defined as "rank-deficient" [1]. Owing to the number of users in the coverage area of a base station is beyond control, the performance of traditional detection methods reduces greatly. The GSAA-MUD algorithm was proposed to solve the rank-deficient problem. The genetic algorithm (GA) has strong global search ability. The simulated annealing algorithm (SAA) has strong local search ability and also has the ability to avoid falling into the local optimal solution [2]. GA and SAA are combined together to form a new global search algorithm GSAA, which was introduced into the multi-user MIMO-OFDM system to solve the rank-deficient problem. The simulation results in the rank-deficient scenarios show that the GSAA-MUD algorithm can achieve higher performance than the GA-MUD algorithm.

II. THE MUD OF THE MIMO-OFDM SYSTEM
The process of the multi-user detection in the MIMO-OFDM system is introduced as following. The coding signals $s(l) (l=1,\ldots,L)$ of $L$ users is firstly modulated at the transmit station, then transmitted through the MIMO channel. We demodulate the received signals and output $x_p (p=1,\ldots,P)$ in each element of the receiving antenna array at the base station, then separated into signals of different users through the multi-user detector [3]. In the MIMO-OFDM system, the signal vector $x[n,k]$ is received by the receiving antenna array, and recorded as:

$$x = Hs + n$$  \hspace{1cm} (1)

Where $[n,k]$ is omitted for expressing simply. And $x$ is the received signal, $H$ is the frequency domain channel transfer function, $s$ is the output signal, $n$ is the AWGN signal.

III. THE GENETIC SIMULATED ANNEALING ALGORITHM
The optimization problem of seeking the maximum value can be formulated as:

$$\max_{X \in \{0,1\}^l} f(X), \text{s.t. } X \in R$$  \hspace{1cm} (2)

The individual $X$ is the decision-making variable in the GA, $f(X)$ is the objective function, $l$ is the chain length of the individual, and $R$ is the feasible solution which meets the restriction conditions. The GSAA generated a new set of individuals through some genetic operations, then simulates annealing to every individual independently, and makes these results as the next generation of population. The operations iterate repeatedly until meeting the termination condition.

The GSAA algorithm can be described as following steps [2]:

1. Setting the relevant parameters of the GSAA, generating the initial population $P_0(t)$ and evaluating the fitness functions of the individuals.
2. Operating with the genetic operations of the crossover and mutation, then generating a new population $P_1(t)$ with the simulated annealing, and evaluating the fitness functions of the population.
3. Generating a new population through the selection and reproduction operations: $P(t + 1) \subseteq \{P_0(t) \cup P_1(t)\}$.
4. If the result does not meet the termination conditions, let $t \leftarrow t + 1$, and turn to the second step. If the result meets the termination conditions, then output the best individual.

IV. THE GSAA-MUD ALGORITHM
In the MIMO-OFDM system, the MUD algorithm based on the GSAA can be analyzed as the follow steps:

A. The first genetic generation
In the GSAA-MUD, the symbols which are resulted from the MMSE algorithm are used as the first generation of genetic population [1]. The estimated signal vector
is obtained by combining the signals linearly, and by $P$ different receiver antennas with the aid of the MMSE array weight matrix as follow:

$$
\begin{align*}
\hat{s}_{\text{MMSE}} &= W^H_{\text{MMSE}} x \\
W_{\text{MMSE}} &= \left(HH^H + \sigma_n^2 I\right)^{-1} H
\end{align*}
$$

Where, $H$ denotes the Hermitian transpose, $W_{\text{MMSE}}$ is the MMSE-based weight matrix, $I$ is the identity matrix and $\sigma_n^2$ is the AWGN noise variance. When $i_{\text{MMSE}}$ becomes available, the first genetic generation can be created. The $m$-th individual can be formulated as:

$$
\hat{s}_{(n,m)} = \left[\hat{s}_{1,(n,m)}, \hat{s}_{2,(n,m)}, \ldots, \hat{s}_{L,(n,m)}\right],
$$

$m = 1, \ldots, M$; $n = 1, \ldots, N$

Where, $N$ is the largest genetic generation, $i_{(n,m)}$ is a representative of the valuable symbols in all individuals.

### B. Fitness Function

The best decision metric based on the maximum likelihood algorithm can be used to estimate the transmission symbol vector $\hat{s}_{\text{GA}}$ [1][8]. The estimated transmitted symbol vector of $L$ users based on the received signal at the $p$-th receiver antenna is given by:

$$
\hat{s}_{\text{GA}}_p = \arg\left\{\min_{\hat{s}} w_p(\hat{s})\right\}
$$

Since there are $P$ receiver antennas, the combined object function can be formulated as:

$$
w(\hat{s}) = \sum_{p=1}^{P} w_p(\hat{s}) = \|x - HH^H\|^2
$$

Hence, the decision rule is to find the specific transmission symbol vector that minimizes $w(\hat{s})$. Record the fitness function $f(n,m)$ of the $m$-th individual in the $n$-th generation as:

$$
f(n,m) = \max_{k \in [1,K]} \left\{w\left(\hat{s}_{n,k}\right) - w\left(\hat{s}_{n,m}\right) + c\right\}
$$

Where, $c$ is a small positive constant, which is used to ensure that the fitness degree $f(n,m)$ is non-negative.

### C. Genetic Operator

Selection operator is used to select a valuable individual in the population. And the individuals whose fitness values are poor can be deleted from the population by the deletion operator. The selection and the deletion reflect the principle of “survival of the fittest”. Hybrid plays a central role in the genetic algorithm, which can be broadened the search capability of the genetic algorithm. Mutation operator can broaden the local searching capabilities and increase the population’s diversity of the genetic algorithm.

Genetic manipulation searches the global optimal solution through the iterative population, and the populations achieve its iterative through some significant genetic operators, such as the selection operator, the hybridization operator and the mutation operator. Selecting operation is divided into two steps. First, we can choose a part of individuals from the population to establish a hybrid collection based on the fitness functions. Second, we can select some pairs of crossover parents from the hybrid collection based on the roulette wheel selective method. Then, the hybridization and mutation operations are introduced. By the Two-point crossover operation, we can select two encoding bit points randomly as the cross-points for each pair of the crossover parents and exchange the intermediate part of the two points. Finally, selecting an individual and its two encoding bit points randomly, reversing the intermediate part of the two points as the mutate manipulation.

### D. The Simulated Annealing Algorithm

The energy function $E$ of the GSAA is the objective function which needs to be optimized, and the minimum is the optimal solution. In the state $z_{\text{old}}$, the system can change to $z_{\text{new}}$ through a random disturbance operator, and the corresponding energy value will change from $E(z_{\text{old}})$ to $E(z_{\text{new}})$. Where, the acceptable probability $p$ is determined by the rules of the Metropolis [2]. If the increment of the objective function is smaller than zero, the new value can be received as the optimal solution. If it is not smaller than zero, the new value can be received as the optimal solution by the probability:

$$
p = \exp\left\{-\frac{E(z_{\text{new}}) - E(z_{\text{old}})}{T_e}\right\}
$$

Where, $T_e$ is the temperature which can fall to zero.

### E. Optimal Solution

The highest fitness value of the individuals in the specific OFDM sub-carriers is considered to be the transmitted symbol vector, which is detected in the $L$ users. The signals of different users are detected synchronously in the MUD of the MIMO-OFDM system, and the estimated transmission signals of each user can eliminate the imposed interference.

### V. Simulations And Results

In the “rank-deficient” MIMO-OFDM system, we set parameters of the GSAA-MUD in table 1. The number of the receiving antennas is less than the number of users in the rank-deficient scenarios. And forty objective function values can be gained as the first genetic generation by the MMSE algorithm. The simulation is shown as figure 1.

The first generation of the GSAA population can be expressed to be forty discrete points, which correspond to forty objective functions values. Also, these discrete points can be fitted to be a curve by the least square method, and the fitting curve has an increasing trend. Based on the first genetic generation of the GSAA, twenty individuals who
have better performance were selected to be a hybrid assembly. Then, we choose two individuals from the assembly as hybrid parents each time in accordance with the principles of the roulette wheel. Repeat this operation for ten times, we can gain twenty genetic offspring individuals through the hybridization operator and the mutation operator. All the twenty new individuals and the twenty initiatory better ones compose a new generation of population.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Sizes</td>
<td>40</td>
</tr>
<tr>
<td>Crossover Probability</td>
<td>0.8</td>
</tr>
<tr>
<td>Mutation Probability</td>
<td>0.05</td>
</tr>
<tr>
<td>Genetic Generations</td>
<td>5</td>
</tr>
<tr>
<td>The Number of Users</td>
<td>4</td>
</tr>
<tr>
<td>Receiving Antennas</td>
<td>3</td>
</tr>
</tbody>
</table>

Simulating the GA-MUD in the rank-deficient scenarios, the curves of the object functions in five generations are shown in figure 2. With the increasing of the genetic times, the object function values in five generations decline overall, which proves that the object function values of individuals are closer and closer to each other, and become equal eventually. If the values keep stable for several generations, the stable value will be output as the final optimal solution. The minimum object function values in each population were recorded in the second column of table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data</th>
</tr>
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<tbody>
<tr>
<td>Ng</td>
<td>GA</td>
</tr>
<tr>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

Based on the GSAA-MUD in the rank-deficient scenarios, each generation of population conducts the simulated annealing operation for three times based on the genetic operations, and we can get the minimum objective function values in table 2, the simulation is shown in figure 3.

When the energy function of the system is increasing, the system will accept the new state with a probability based on the rules of the Metropolis, and the minimum value after the simulated annealing operation will be bigger than the initial minimum value in the first generation of population. Comparing the initial value and the final value of the first generation of population, the GSAA can reduce the minimum objective function value of each generation significantly, and the minimum value remain unchanged after the second simulated annealing.
operation of the second generation. Finally, the stable minimum objective function value is the optimal solution. The minimum objective function values in each generation of the GA and the GSAA are shown in figure 4. The GA-MUD and the GSAA-MUD have the same trend, that is, with the increasing of the genetic generation, the objective function values in five generations of populations reduced globally. Furthermore, the result of the first GSAA generation can achieve the result of the third GA generation, and the second GSAA generation can achieve the optimal solution, which is smaller than the fifth GA generation for 1. All the above show that, the GSAA-MUD of the MIMO-OFDM system can achieve better performance than the GA-MUD in the rank-deficient scenarios.

VI. THE CONCLUSION

In this paper, we introduce the genetic simulated annealing algorithm (GSAA) into the multi-user detection (MUD) of the MIMO-OFDM system to solve the rank-deficient problem. The GSAA-MUD uses the objective function values which are gained from the MMSE algorithm as the first genetic generation of population, and gets the minimum value of the objective function through the operation of the genetic simulated annealing algorithm. The simulations and the results show that the GSAA-MUD of the MIMO-OFDM system can achieve much higher performance than the GA-MUD in the rank-deficient scenarios.

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REFERENCES


