Abstract—Based on the study of cultural algorithms and particle swarm optimization, a new method of FIR digital filters design, called cultural particle swarm optimization (CPSO) is presented in this paper. CPSO is a global convergent algorithm that can find out the global optima of the problem. The algorithm concept and implementing procedure of CPSO are described in detail. The simulation results show that CPSO is better than the original PSO and QPSO with more rapidly convergence speed and better performance of the designed filter.

Index Terms—FIR digital filters, filter design, cultural algorithms, particle swarm optimization

I. INTRODUCTION

FIR digital filters are widely used in the field of signal processing due to its distinguishing features such as: the stability, linear phase and easiness for realization [1]. Traditionally, there exist some methods for FIR digital filters design, such as window method, frequency sampling method and best uniform approximation. Unfortunately, each of them is only suitable for a particular application. In recent years, many evolutionary computation techniques, such as simulated annealing approach (SA) [2], genetic algorithms (GA) [3]-[5], particle swarm optimization (PSO) [6], have been employed to design FIR digital filters. GA is a good global searching method, but it is difficult to realization because of the complexity of coding. PSO is a recently proposed random search algorithm and has been applied to many real-world problems, but not a good global searching algorithm [7]. W. Fang proposed quantum particle swarm optimization (QPSO) for the design of FIR digital filters and the performance is prior to the original PSO [8].

Initially, cultural algorithms (CAs) were applied with population spaces based on evolutionary programming [9]-[10] approaches to real parameter optimization. Recently, particle swarm optimization has also been proposed as a population space [11]. In literature [9]-[10], the belief space is divided into four knowledge sources: situational, normative, topographical, and historical knowledge. New concept of optimization called cultural particle swarm optimization (CPSO) based on normative knowledge and situational knowledge of CAs to be used for the optimization of a FIR digital filters design problem is presented in this paper.

This paper will focus on the implementation of FIR digital filters based on CPSO algorithm. The rest of the paper is organized as follows. In section 2, model of FIR digital filters is presented. In section 3, particle swarm optimization is introduced and CPSO is discussed in detail, then the application of CPSO to FIR digital filters design is presented. The experiment results and the conclusions can be seen in Section 4 and Section 5.

II. FIR DIGITAL FILTERS

FIR filter has a finite number of nonzero entries of its impulse such as $h[n]$, $n=0,1,\ldots,N-1$. Generally assume that $h[n] \neq 0$. The transfer function of the FIR filter is

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n} \quad (1)$$

and the frequency response of the form is

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-in\omega}. \quad (2)$$

Consider the ideal frequency response $H_J(e^{j\omega})$ with the samples divided into equal frequency interval, thus we can get

$$H_J(k) = H_J(e^{j\omega}) \bigg|_{\omega = \frac{2\pi k}{M}} \quad (3)$$

where $H_J(k)$ is regarded as the frequency response of the filter designed. Equation (3) can be re-written as

$$H_J(k) = H_J(e^{j\omega}) \bigg|_{\omega = \frac{2\pi k}{M}} \quad (4)$$

To design linear-phase FIR, we must minimize the error between actual and ideal output. There are two forms of error function for the filter design [6]. One is the least-squares sense, which is employed in this paper and the other is the largest absolute error. We define the error function as the error between the desired magnitude and the actual amplitude at a certain frequency, that is

$$E(e^{j\omega}) = H_J(e^{j\omega}) - H(e^{j\omega}) \quad (5)$$

Thus we can adopt the objective function for the minimization as total squared error across frequency domain as follows

$$E = \sum_{k=1}^{M} \left| H_J(e^{j\omega}) - H(e^{j\omega}) \right|^2 \quad (6)$$

where $M$ is the number of frequency interval. From (2), we can write the above equation as

$$E = \sum_{k=1}^{M} \left| \sum_{n=0}^{N-1} h(n)e^{j\omega n} - H_J(e^{j\omega}) \right|^2. \quad (7)$$

The problem is reduced to find out $h(n)$ by minimizing the squared error $E$. 

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III. FIR DIGITAL FILTER USING CULTURAL PARTICLE SWARM OPTIMIZATION

A. Particle Swarm Optimization Algorithm

Particle swarm optimization is a novel multi-agent optimization system inspired by social behavior metaphor [12]. Each agent called particle flies in a D-dimensional space \( S \) according to the historical experiences of its own and its colleagues. There are \( m \) particles in a swarm that is in a space of \( D \) dimensions, the \( i \)th particle’s position in the space is \( x_i = (x_{i1}, x_{i2}, \ldots, x_{id}) \), \( i = 1,2,\ldots,m \), which is a latent solution. The \( i \)th particle’s \( \text{fit} \) speed is \( v_i = (v_{i1}, v_{i2}, \ldots, v_{id}) \) and until now the best position of the \( i \)th particle is \( p_i = (p_{i1}, p_{i2}, \ldots, p_{id}) \), \( i = 1,2,\ldots,m \). \( p_s = (p_{s1}, p_{s2}, \ldots, p_{sd}) \) is the best position discovered by the whole population. At each generation, the \( i \)th particle is updated according to the following move equations:

\[
v_{i+1} = w v_i + c_1 r_1 (p_i - x_i) + c_2 r_2 (p_s - x_i)
\]
\[
x_{i+1} = x_i + v_{i+1}
\]

where \( r_1 \) and \( r_2 \) are uniform random values between 0 and 1; \( w \) is called inertia weight and \( c_1 \) and \( c_2 \) are usually set to 2. The inertial weight \( w \) represents the degree of the momentum of the particles. The use of the variable \( w \), inertia weight is responsible for dynamically adjusting the speed of the particles [13].

B. A Novel Cultural Particle Swarm Optimization Algorithm

The key idea behind cultural particle swarm optimization algorithms is to acquire problem-solving knowledge (beliefs) from the evolving in return making use of that knowledge to population and guiding the search [11]. CA models have two levels of evolution: the population and the belief space. Besides a population space, CA has a belief in which the problem-solving knowledge can be acquired space from the evolving population can be stored and integrated. Cultural particle algorithms for optimization problems above framework can be represented as:

\[
P[m,D]^{t+1} = \text{select}(\text{generate}(P[m,D]^t, Beliefs'))
\]

(10)

Here, Beliefs can be used to influence generate() operator and/or select() operator. The beliefs can be adjusted as the following:

\[
\text{Beliefs}'^{t+1} = \text{update}(\text{Beliefs'}, \text{accept}(P[m,D]^t))
\]

(11)

The formal syntax of beliefs structure defined in this paper is \( \{N[D], s\} \), where \( s \) is situational knowledge component, and \( N \) is the normative knowledge component which include the intervals information for each \( D \) parameter, \( N[i] \) is represented as \( N[i] = (I_i, L_i, U_i) \). \( I_i \) is the closed interval of parameter \( i \) and \( I_i = [l_i, u_i] = \{x | l_i \leq x \leq u_i, x \in R\} \), where lower bound \( l \) and upper bound \( u \) are initialized as the given domain values and can be changed later. \( L_i \) shows the performance score of the lower bound of parameter \( i \) and \( U_i \) denotes the performance score of the upper bound \( u \) for parameter \( i \).

The accept() function selects individuals who can directly impact the formation of current belief space. Just like that different knowledge can be contributed by different people in social society, the normative knowledge can be impacted by different individuals in cultural algorithms. The acceptance function selects the top 20% individuals in this paper.

The component \( s' \) will be updated by update() function as follows.

\[
s_{t+1}^{i} = \begin{cases} s_{i}^{\text{best}}, & \text{if } f(x_{i}^{\text{best}}) < f(s'); \\ s', & \text{otherwise}. \end{cases}
\]

(12)

where \( s' \) is on behalf of the best individual of cultural population at the \( t \)th generation.

The component of \( N \) will be updated by update() function as follows.

Assume now it is the \( j \)th individual that affects the lower bound for parameter \( i \), the lower boundary and its score are given below.

\[
L'_{i}^{j+1} = \begin{cases} p_{i}^{j}, & \text{if } p_{i}^{j} \leq L_i \text{ or } f(p_{i}^{j}) < L_i; \\ L_i, & \text{otherwise}. \end{cases}
\]

(13)

\[
U'_{i}^{j+1} = \begin{cases} f(p_{i}^{j}), & \text{if } p_{i}^{j} \leq U_i \text{ or } f(p_{i}^{j}) < U_i; \\ U_i, & \text{otherwise}. \end{cases}
\]

(14)

where \( L_i \) represents lower bound for parameter \( i \) at generation \( t \) and \( L_i \) denotes the performance score for it.

Assume now it is the \( k \)th individual that impacts the upper bound for parameter \( i \).

\[
u'_{i}^{t+1} = \begin{cases} p_{i}^{k}, & \text{if } p_{i}^{k} \geq U_i \text{ or } f(p_{i}^{k}) < U_i; \\ U_i, & \text{otherwise}. \end{cases}
\]

(15)

\[
u'_{i}^{t+1} = \begin{cases} f(p_{i}^{k}), & \text{if } p_{i}^{k} \geq U_i \text{ or } f(p_{i}^{k}) < U_i; \\ U_i, & \text{otherwise}. \end{cases}
\]

(16)

where \( U_i \) represents the upper bound for parameter \( i \) at generation \( t \) and \( U_i \) denotes the performance score for it.

By changing the search size and direction of the variation with belief space, cultural individuals are updated by normalized knowledge.

\[
x_{i}^{j+1} = \begin{cases} x_{i}^{j} + \text{size}(I_i) \cdot N(0,1), & \text{if } x_{i}^{j} < l_i; \\ x_{i}^{j} - \text{size}(I_i) \cdot N(0,1), & \text{if } x_{i}^{j} > u_i; \\ x_{i}^{j} + \eta \cdot \text{size}(I_i) \cdot N(0,1), & \text{otherwise}. \end{cases}
\]

(17)
Where $N(0,1)$ is the random number of standard normal distribution, $\text{size}(I_i)$ is adjustable variable length of the belief space range and $\eta$ (0.01–0.6) is a constant.

By changing the search size and direction of the variation with belief space, cultural individuals are updated by situational knowledge.

$$
x_{ji}^{t+1} = \begin{cases} 
    x_{ji}^t + c_i \cdot v_{ji}^t \cdot N(0,1), & \text{if } x_{ji}^t < x_{ji}^t; \\
    x_{ji}^t - c_i \cdot v_{ji}^t \cdot N(0,1), & \text{if } x_{ji}^t > x_{ji}^t; \\
    x_{ji}^t + \eta \cdot v_{ji}^t \cdot N(0,1), & \text{otherwise.}
\end{cases} 
$$

where $c$ (1 ~ 0.001) is a regressive descending with iteration increase. The purpose is to focus on global search at the beginning of the algorithm, and with the increase of iteration numbers tends to improve accuracy with local search.

C. FIR Digital Filter Based on Cultural Particle Swarm Optimization

The initial population is created randomly from the solution space. The goal of the fitness function is to evaluate the status of each individual. In the FIR digital filter based on cultural particle swarm optimization, the target is the minimization of the following objection function.

$$
f = \sum_{i=1}^{N} \left( \sum_{n=0}^{N-1} h(n)e^{j\omega n} - |H_j(e^{j\omega})| \right)^2. 
$$

The procedure for implementing the global version of CPSO is given by the following steps:

Step1: Initialize a population of particles with random positions and velocities in the $N$ dimensional problem space using uniform probability distribution function, and then initialize belief space $B$;

Step2: Evaluate the performance scores of population space $P$;

Step3: According to influence function $\text{Influence}()$, effect on each parent individual of population space, and each individual select normative knowledge and situational knowledge in certain probability, normative knowledge is selected in the probability of linear reduction with generation increase (in order to maintain its global search characteristics, the selection probability of normative knowledge is not less than 0.8 and the initial selection probability is 0.9) to generate a corresponding son-individual;

Step4: Calculate fitness of the son individual;

Step5: Comparison to pbest (personal best): Compare each particle’s fitness with the particle’s pbest. If the current value is better than pbest, then set the pbest value equal to the current value and the pbest location equal to the current location in D-dimensional space;

Step6: For each particle, the pbest itself compared with the whole best position $p_g$, if pbest is better than $p_g$, then reset $p_g$.

Step7: According to $\text{Accept}()$ function, belief space is updated by equation (12)-equation(16) to update the belief space;

Step8: If haven’t got the stop condition (the termination condition is setting as maximum iteration times $N_g$ usually), then back to step3, else the algorithm stops.

IV.EXPERIMENT AND SIMULATION RESULTS

The example filters are a low-pass filter and a band-pass filter. Their frequency response is as follows

$$
H_j(e^{j\omega}) = \begin{cases} 
    1, & 0 \leq \omega \leq \omega_p \\
    0, & \omega_p \leq \omega \leq \pi
\end{cases} 
$$

where $\omega_p = 0.2\pi \cdot \omega_s = 0.3\pi$.

$$
H_j(e^{j\omega}) = \begin{cases} 
    1, & 0 \leq \omega \leq \omega_{p1} \\
    0, & \omega_{p1} \leq \omega \leq \omega_{p2} \\
    0, & \omega_{p2} \leq \omega \leq \omega_s \\
    1, & \omega_s \leq \omega \leq \pi
\end{cases} 
$$

where $\omega_{p1} = 0.33\pi$ , $\omega_{p2} = 0.64\pi$ , $\omega_s = 0.23\pi$ , $\omega_s = 0.74\pi$.

We employ QPSO and PSO to design the filter for the performance comparison with CPSO. In each running of CPSO, QPSO and PSO, population size is set to 100. PSO experiment settings are as follows: PSO: Maximum velocity=0.1, $w_{max} = 0.9$ , $w_{min} = 0.4$ . The search scope of the coefficient of the filter is set as [-1, 1]. The number of frequency interval is $N=30$.

Figure 1 shows the comparison of convergence behavior of FIR low-pass filter designed by QPSO, PSO and CPSO with 100 trial runs. It can be seen the convergence speed of CPSO is significantly improved under the same iterations.

Figure 1. Convergence behaviors of three algorithms in design low-pass FIR filter.

Figure 2 shows the comparison of magnitude response of FIR low-pass filter designed by three different algorithms with iteration =300. It can be concluded that CPSO works better than the others with much smaller stop-band attenuation.
Figure 2. Magnitude response of low-pass FIR filters designed with three algorithms.

Figure 3 shows the comparison of magnitude response of FIR band-pass filter designed by three different algorithms with iteration =300. It can be concluded that CPSO works better than the other two algorithms with the same population size.

Figure 3. Magnitude response of band-pass FIR filters designed with three algorithms.

V. Conclusions

In this paper, CPSO, a novel population-based search technique is proposed for the design of FIR digital filters. Compared to QPSO and the original PSO algorithm, CPSO is able to converge to the global optima and the fitness is greatly improved. The simulation results of low-pass and band-pass filters demonstrate that CPSO is an efficient and alternative approach for FIR digital filters design and has higher optimizing precision and strong global search capability.

References