

Fractionally Spaced Mixed Blind Equalization Algorithm Based on Orthogonal Wavelet Transform

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Abstract—Aiming at huge computation of super-exponential iteration (SEI) algorithm, meanwhile the severe inter-symbol interference (ISI) caused by multi-path fading, in the fractionally spaced blind equalization algorithm which based on super-exponential iteration algorithm, fractionally spaced mixed blind equalization algorithm which based on orthogonal wavelet transform (FWSW) is proposed. The proposed algorithm uses the fast convergence speed characteristic of SEI algorithm to revise the iteration equation of feedforward weight vectors for decision feedback structure blind equalizer, uses the de-correlation ability of wavelet transform to improve the convergence rate, and the computation complexity can be reduced by rarefying the feedforward weight vectors, then uses the phase-locked loop (PLL) to correct phase rotation. The simulation results show that the proposed algorithm has fast convergence rate, low ISI and computational complexity, and can correct the channel-introduced carrier phase rotation.

Index Terms—blind equalization, orthogonal wavelet transform, fractionally spaced, decision feedback

I. INTRODUCTION

In the underwater acoustic communication, inter-symbol interference (ISI) caused by multi-path and bandwidth limited channel are an important factor in affecting on communication quality, and can be overcome via equalization technique. It is an important means to employ the blind equalization without the aid of any training sequence for greatly suppressing the ISI. Fractionally spaced constant modulus blind equalization algorithm[1](FSE-CMA) has slow convergence rate, large Mean Square Error (MSE) and can't correct the channel-introduced carrier phase rotation because of the characteristics of underwater acoustic channel are time-varying, multi-path, fading, and limited bandwidth. Thus, the orthogonal wavelet transform is introduced to FSE-CMA, because we use the de-correlation ability of orthogonal wavelet transform to speed up convergence[2]. We can also use SEI (super-exponential iteration) algorithm[3] to speed up convergence, but SEI algorithm is blind to phase and can't correct the carrier phase rotation. More and worse, the SEI algorithm's complexity increases with the square of the equalizer length, which makes it unsuitable for the project implementation. To enhance the project implementation, we can rarefy the weight vectors to reduce its computation load [4], which uses the sparse

characteristic of the underwater acoustic channel. The effective method of correct phase rotation is using PLL (phase-locked loop) to track the phase change[5]; nonlinear Decision Feedback Equalizer [6] (DFE) can remove ISI of feedforward filter, when the channel has severe frequency selective fading, it has excellent tracking performance.

According to those algorithm's characteristics, a new fractionally spaced mixed blind equalization algorithm based on orthogonal wavelet transform (FWSW) is proposed. The proposed algorithm uses orthogonal wavelet transform and SEI algorithm to speed up convergence, and decreases the algorithm's computation load through rarefying the weight vectors, uses Decision Feedback Equalizer (DFE) to remove ISI, uses phase-locked loop to correct phase rotation. The proposed algorithm simultaneously possess the excellent performance of the orthogonal wavelet transform, super exponential iterative algorithm, Decision Feedback Equalizer, and phase-locked loop.

II. THE PROPOSED ALGORITHM

A. *T/2 Fractionally Spaced Super Exponential Iteration Decision Feedback Equalizer Blind Equalization Algorithm Based on Orthogonal Wavelet Transform*

Based on wavelet theory, assume that the $f(n)$ denotes the weight vector of the finite impulse response filter or equalizer and can be expressed by the orthogonal wavelet functions and scale functions. In the T/2 fractionally spaced equalizer, we assume that the every subchannel equalizer tap number is $M_f = 2^J$, subchannel equalizer $f^{(m)}(n)$ may be expressed as

$$f^{(m)}(n) = \sum_{j=1}^J \sum_{k=0}^{k_j} d_{jk}^{(m)} \cdot \phi_{jk}(n) + \sum_{k=0}^{k_J} v_{Jk}^{(m)} \cdot \varphi_{Jk}(n). \quad (1)$$

where $n = 0, 1, \dots, M_f$, M_f is the length of subchannel equalizer, $\phi_{jk}(n)$ and $\varphi_{Jk}(n)$ denote wavelet functions and scale functions. k_j is the maximum shift of wavelet function at the scale j and given by $k_j = M_f / 2^j - 1$ ($j=1, 2, \dots, J$), J is maximum scale of wavelet decomposition, $d_{jk}^{(m)}$ and

$v_{jk}^{(m)}$ are subchannel equalizer weight coefficients. The subchannel equalizer input signal is given by

$$\mathbf{y}^{(m)}(n) = [r_{1,0}^{(m)}(n), r_{1,1}^{(m)}(n), \dots, r_{J,k_j}^{(m)}(n), s_{1,0}^{(m)}(n), \dots, s_{J,k_j}^{(m)}(n)]^T. \quad (2)$$

Similarly, the subchannel equalizer weight vector is given by

$$\mathbf{f}^{(m)}(n) = [d_{1,0}^{(m)}(n), d_{1,1}^{(m)}(n), \dots, d_{J,k_j}^{(m)}(n), v_{1,0}^{(m)}(n), \dots, v_{J,k_j}^{(m)}(n)]^T. \quad (3)$$

where

$$\begin{cases} r_{jk}^{(m)}(n) = \sum_{i=0}^{M_f-1} \mathbf{y}^{(m)}(n-i) \cdot \phi_{jk}(i) \\ s_{jk}^{(m)}(n) = \sum_{i=0}^{M_f-1} \mathbf{y}^{(m)}(n-i) \cdot \varphi_{jk}(i) \end{cases} \quad (m=0,1) \quad (4)$$

According to Mallat algorithm, orthogonal wavelet transform matrix \mathbf{V} [7] is given by

$$\mathbf{V} = [\mathbf{G}_0; \mathbf{G}_1 \mathbf{H}_1; \mathbf{G}_2 \mathbf{H}_2 \mathbf{H}_1; \dots; \mathbf{G}_{J-1} \mathbf{H}_{J-2} \dots \mathbf{H}_1 \mathbf{H}_0; \mathbf{H}_J \mathbf{H}_{J-2} \dots \mathbf{H}_1 \mathbf{H}_0]. \quad (5)$$

where \mathbf{H}_j and \mathbf{G}_j are matrix consisted of coefficients of wavelet filter $h(k)$ and scale filter $g(k)$, respectively, and the elements of \mathbf{H}_j and \mathbf{G}_j are given by $H_j(l, k) = h(k-2l)$, $G_j(l, k) = g(k-2l)$ ($l=1 \sim M_f/2^{j+1}$, $k=1 \sim M_f/2^j$).

For obtaining the possible fast convergence rate, correcting the channel-introduced carrier phase rotation, and removing ISI, T/2 fractionally spaced super exponential iteration Decision Feedback Equalizer blind equalization algorithm based on orthogonal wavelet transform (WFSD) is proposed and its principle structure is shown in Fig.1.

In Fig.1, $s(n)$ is transmission signal sequences; $\mathbf{c}^{(0)}(n)$ and $\mathbf{c}^{(1)}(n)$ are the impulse responses of the even and odd subchannel; $\mathbf{w}^{(0)}(n)$ and $\mathbf{w}^{(1)}(n)$ are the white Gaussian noises of the even and odd subchannel; the output of the even and odd subchannel are $\mathbf{x}^{(0)}(n)$ and $\mathbf{x}^{(1)}(n)$; WT represents

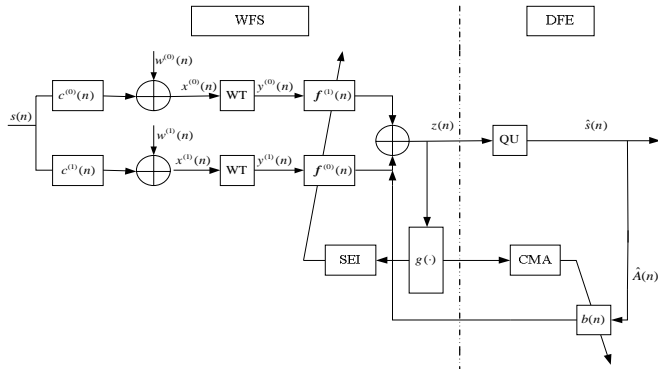


Fig 1. The principle structure of WFSD

orthogonal wavelet transform; $\mathbf{y}^{(0)}(n)$ and $\mathbf{y}^{(1)}(n)$ are the input signal of the even and odd feedforward subchannel; $\mathbf{f}^{(0)}(n)$ and $\mathbf{f}^{(1)}(n)$ are the weight vectors for the even and odd subchannel of length M_f ; $z(n)$ is output of the equalizer; QU is the Quantizer decision device; $\hat{s}(n)$ is the decision output of $z(n)$; the feedback filter weight vector $\mathbf{b}(n)$ is given by

$$\mathbf{b}(n) = [b(n), b(n-1), \dots, b(n-N_b+1)]^T. \quad (6)$$

where N_b is the length of feedback filter, which is positive constant, $(\cdot)^T$ denotes transpose of a vector; the feedback filter input regressor vector $\hat{\mathbf{A}}(n)$

$$\hat{\mathbf{A}}(n) = [\hat{\mathbf{A}}(n), \hat{\mathbf{A}}(n-1), \dots, \hat{\mathbf{A}}(n-N_b+1)]^T. \quad (7)$$

When the feedforward and feedback weight coefficients of equalizer are updated by the SEI algorithm and CMA, its recursively equations are given by

$$\mathbf{y}^{(m)}(n) = \mathbf{V} \mathbf{x}^{(m)}(n), \quad (8)$$

$$\mathbf{z}^{(m)}(n) = (\mathbf{f}^{(1-m)}(n))^T \mathbf{y}^{(m)}(n), \quad (9)$$

$$z_d(n) = \mathbf{b}^T(n) \hat{\mathbf{A}}(n), \quad (10)$$

$$\begin{aligned} z(n) &= (\mathbf{f}^{(1)}(n))^T \mathbf{y}^{(0)}(n) + (\mathbf{f}^{(0)}(n))^T \mathbf{y}^{(1)}(n) \\ &\quad - \mathbf{b}^T(n) \hat{\mathbf{A}}(n), \end{aligned} \quad (11)$$

$$e(n) = z(n) (|z(n)|^2 - R), \quad (12)$$

$$\mathbf{f}^{(m)}(n+1) = \mathbf{f}^{(m)}(n) - \mu_m^{(m)} \mathbf{Q}^{(m)}(n) * (\mathbf{y}^{(m)}(n))^* e(n), \quad (13)$$

$$\begin{aligned} \mathbf{Q}^{(m)}(n+1) &= \frac{1}{1 - \mu_p^{(m)}} \left(\mathbf{Q}^{(m)}(n) - \frac{\mu_p^{(m)} \mathbf{Q}^{(m)}(n) \mathbf{Q}^{(m)}(n)}{1 - \mu_p^{(m)} + \mu_p^{(m)} \mathbf{Q}^{(m)}(n)} \right. \\ &\quad \left. * \frac{(\mathbf{y}^{(m)}(n))^* (\mathbf{y}^{(m)}(n))^T}{(\mathbf{y}^{(m)}(n))^T (\mathbf{y}^{(m)}(n))^*} \right), \end{aligned} \quad (14)$$

$$\mathbf{b}(n+1) = \mathbf{b}(n) - \mu_2 \hat{\mathbf{A}}^*(n) e(n). \quad (15)$$

where $m=0,1$; R is a constant and defined by $R = E[|s(n)|^4] / E[|s(n)|^2]^2$; $\mu_m^{(m)}$ is step-size of subchannel equalizer; $\mu_p^{(m)}$ is step-size of $\mathbf{Q}^{(m)}$ matrix.

From (8)~(15), we can find that WFSD can improve convergence rate, whereas can't correct the channel-introduced carrier phase rotation.

B. T/2 Fractionally Spaced Super Exponential Iteration Blind Equalization Algorithm Based on Orthogonal Wavelet Transform

When the DFE is removed in the WFSD, we can obtain fractionally spaced super exponential iteration blind equalization algorithm based on orthogonal wavelet transform (WFS). Then, the (11) is modified as

$$z(n) = (\mathbf{f}^{(1)}(n))^T \mathbf{y}^{(0)}(n) + (\mathbf{f}^{(0)}(n))^T \mathbf{y}^{(1)}(n). \quad (16)$$

The (8), (9), (12)~(14), and (16) constitute the recursively equations of WFS. Like the WFSD, WFS also can't correct the channel-introduced carrier phase rotation.

C. T/2 Fractionally Spaced Super Exponential Iteration Decision Feedback Equalizer Blind Equalization Algorithm Based on Orthogonal Wavelet Transform and PLL

Owing to the WFSD can't correct the channel-introduced carrier phase rotation, the phase-locked loop (PLL) is introduced to overcome this disadvantage, the principle structure of T/2 fractionally spaced super exponential iteration Decision Feedback Equalizer introduced PLL blind equalization algorithm based on orthogonal wavelet transform (WFSD+PLL) is shown in Fig.2.

Based on the analysis above, WFSD+PLL's equalizer weight vector is updated by

$$\mathbf{f}^{(m)}(n+1) = \mathbf{f}^{(m)}(n) - \mu_m^{(m)} \mathbf{Q}^{(m)}(n) (\mathbf{y}^{(m)}(n))^* e(n) e^{j\theta(n)}. \quad (17)$$

The final output of the equalizer is given by

$$g(n) = z(n) e^{-j\theta(n)}. \quad (18)$$

The (16) is modified as

$$e(n) = g(n) (|g(n)|^2 - R). \quad (19)$$

The (15) is modified as

$$\mathbf{b}(n+1) = \mathbf{b}(n) - \mu_2 \hat{\mathbf{A}}^*(n) e(n) e^{j\theta(n)}. \quad (20)$$

The iteration formula of $\hat{\theta}(n)$ is given by

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \gamma \text{Im}[\hat{s}^*(n) g(n)]. \quad (21)$$

where $\mu_m^{(m)}$ is step-size of feedforward subchannel equalizer, μ_2 is the step-size of feedback filter, γ is iteration step-size, $\text{Im}[\cdot]$ denotes the imaginary projection operator. The (8)~(11), (14), (17)~(21) are the WFSD+PLL's iteration formulas.

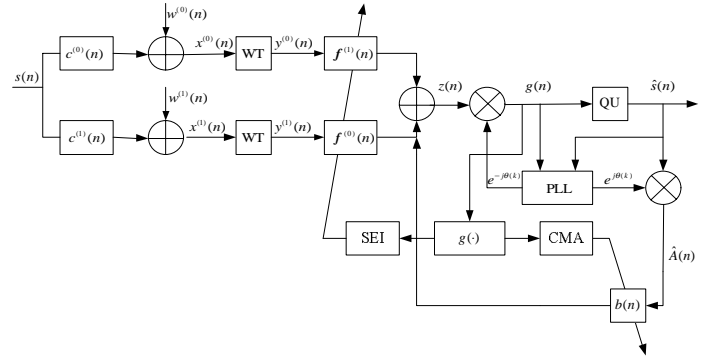


Fig 2. The principle structure of WFSD+PLL

D. T/2 Fractionally Spaced Mixed Blind Equalization Algorithm Based on Orthogonal Wavelet Transform

The WFSD+PLL has high computational complexity because the weight vectors are not rarefied, we can obtain fractionally spaced mixed blind equalization algorithm based on orthogonal wavelet transform (FWSW) through rarefying the WFSD+PLL's weight vectors. First of all, we use the SE (super-exponential) algorithm to estimate the weight vectors, and the SEI algorithm can be quickly initialized by the obtained weight vectors because of SE algorithm only need a few iteration to get convergence, and then the taps are updated as its energy higher than threshold, namely, we compare the energy of every nonzero tap with the threshold, then retain the taps which energy higher than threshold and let rest coefficients to be zero.

Assume that the threshold is E_{th} , the i th tap coefficient is given by

$$f(i) = \begin{cases} f(i), & f^2(i) \geq E_{th} \\ 0, & f^2(i) < E_{th} \end{cases}. \quad (22)$$

III. PERFORMANCE

A. Computation Complexity

We make the outline analysis about computation complexity of FWSW, WFSD and WFS. In the FWSW, the input signals need to be orthogonal wavelet transformed in the weight coefficients iteration, the orthogonal wavelet transform matrix \mathbf{V} is $N \times N$ dimension, so the (8) needs at most N^2 multiplications number. Assume that the nonzero elements number of every row in the matrix \mathbf{V} is L , then calculate (8) needs LN multiplications number. The subchannel taps decrease from L_1 to L_2 and the computation complexity decreases from L_1^2 to L_2^2 . Take the N point data length for example, the three algorithms's computation complexity are $2LN + 2L_2^2(N - L_{Ff}/2)L_{Ff} + 2L_2^2(N - L_{Ff}/2 - L_{Fd})L_{Fd}$, $2LN + 2L_1^2(N - L_{Ff}/2)L_{Ff} + 2L_1^2(N - L_{Ff}/2 - L_{Fd})L_{Fd}$,

$2L_N + 2L_1^2N$ (L_{Ff} is feedforward filter's weight length, L_{Fd} is feedback filter's weight length), respectively. So the computation complexity of FWSW reduces drastically comparison with the WFS's and WFSW's.

B. Simulation Results and Analysis

In this example, 16-QAM data symbols were transmitted through an underwater acoustic channel, which is $c = [1 \ 0 \ 0.3 \cdot \exp(-0.7 \cdot i) \ 0 \ 0 \ 0.2 \cdot \exp(-0.8 \cdot i)]$. The SNR is set to 25dB. In the FWSW, WFSW, and WFS, the equalizers all have 16 taps and each subchannel's fourth tap is initialized to 1; the length of feedback filter is 4 and its taps were initialized to zero; WFS's step-size is 0.00003, the step-size of $Q^{(m)}$ matrix is 0.0002; FWSW's and WFSW's step-size are 0.00005, the step-size of $Q^{(m)}$ matrix are 0.0003, the step-size of feedback filter are 0.00007, FWSW's step-size γ of PLL is 0.00004; the threshold is 0.00003, each subchannel input signal is decomposed by the Db2 wavelet and decomposition level is 3. The results at iteration 1000 are shown in Figs.3.

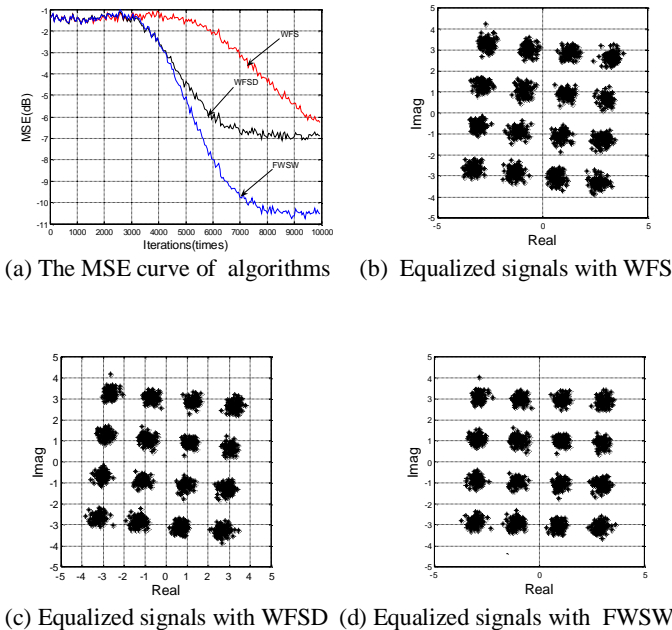


Fig 3. Simulation results

From Figs.3, the MSE of FWSW is decreased by 4dB, 3.5dB comparison with the WFS's, WFSW's, respectively. Further the WFS and WFSW can't correct the phase rotation but the FWSW can completely overcome the phase rotation. From Figs.3(b), (c), (d), the FWSW's constellations are clearer and more focus than WFSW's and WFS's.

IV. CONCLUSIONS

Convergence rate, residual error, ISI are the important indexes which measure good and bad function of equalizer. Aiming to FSE-CMA has slow convergence rate, large MSE and can't correct the channel-introduced carrier phase rotation, fractionally spaced mixed blind equalization algorithm based on orthogonal wavelet transform (FWSW) is proposed, which combines with the orthogonal wavelet transform, FSE, DFE, SEI algorithm and decreases the computation complexity through rarefying the weight vectors, uses the PLL to correct phase rotation. The proposed algorithm not only keeps the well convergence performance of orthogonal wavelet transform, but also absorbs the fast convergence speed of SEI algorithm. Simulation test with underwater acoustic channel shows that the FWSW has characteristic of fast convergence rate, low MSE, which are helpful to the signal real-time recovery and the project implementation.

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