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Correlation Function based the Blind Equalization Algorithm

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***Abstract*—Based on analyzing blind equalization algorithm, Correlation Function based the Blind Equalization Algorithm (CFBEA) is proposed. The proposed algorithm is based on the processing of correlation function of the input signal, instead of the processing of input signal itself, a new error function is defined, The proposed algorithm is derived according to correlation function mean square error criteria defined by Asharif and Newton's instantaneous gradient descent algorithm, correlation ratio coefficient is introduced to improve convergence performance of the algorithm. Simulation tests with underwater acoustic channel indicate that the proposed algorithm has better convergence performance.**

***Index Terms*—blind equalization, correlation function, correlation ratio coefficient, underwater acoustic channel**

I. Introduction

Blind equalization techniques do not need to launch periodic training sequences, automatically track changes in channel and save the bandwidth, so it is an effective method to overcome intersymbol interference. In the blind equalization algorithm, constant modulus algorithm(CMA) is widely used due to its simple structure, stable performance, a small amount of computation. However, its convergence rate is slow and mean square error is big after convergence [1-3]. In references [4-8] correlation function is introduced to adaptive filter algorithm and applied to adaptive echo cancellation system, effectively improve convergence performance of the algorithm and overcome the shortcomings of echo cancellation system in conventional algorithm.

Correlation function is introduced to blind equalization algorithm in this paper , directly processing correlation function of the input signal , a new error function is defined, correlation function based the blind equalization algorithm is proposed. The algorithm is derived according to correlation function mean squarecorrelation ratio coefficient is introduced and used to adjust the equalizer weight vector.

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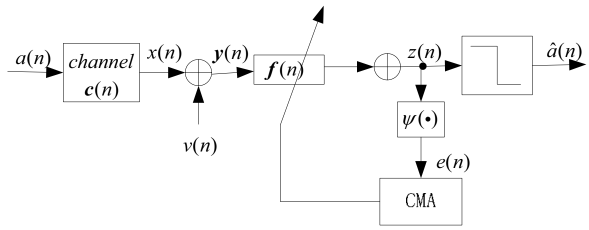


Figure 1. CMA structure model

II. CMA Algorithm

CMA structure is shown in Figure 1.

Where, $\boldsymbol{a}(n)$ is the i i d. launching signal sequence; $\boldsymbol{c}(n)$ is the impulse response of the channel; $v(n)$ is an equalizer input sequence; $\boldsymbol{f}(n)=[f(n),f(n),\cdots,f_{L-1}(n)]^{T}$ is the equalizer weight vector with the length of L (L is positive integer); $\psi(\cdot)$ is memoryless nonlinear function and used to generate error function; $z(n)$ is the equalizer output sequence; $\hat{a}(n)$ is estimate of $z(n)$.

CMA is the most commonly used blind equalization algorithm. Its error function is

$$e(n)=R_2-|z(n)|^2 \tag{1}$$

Where, $R_2=E\{|a(n)|^4\}/E\{|a(n)|^2\}$ is real constant which dependents on higher-order statistics of source sequence.

The equalizer weight vector updating equation

$$\boldsymbol{f}(n+1)=\boldsymbol{f}(n)+\mu z(n)\boldsymbol{y}^{*}(n)e(n) \tag{2}$$

Where, μ_i is iterative step. (1-2) constitute CMA algorithm, its structure is simple, performance is stable , a amount of computation is small, but its convergence rate is slow and mean square error is big after convergence.

III. Correlation Function based the Blind Equalization Algorithm

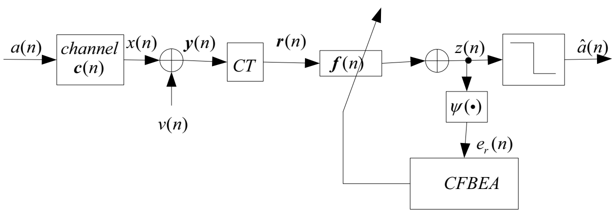


Figure 2. Correlation function based the blind Equalization algorithm structure

Correlation function based the blind Equalization algorithm is correlation transformation to input signal at first, correlation sequence of input signal as blind equalizer input and the part of equalizer weight vector update equation, and correlation ratio coefficient is used to adjust the equalizer weight vector. Its structure is shown in Figure 2.

Where, $\boldsymbol{a}(n)$ is the i i d. launching signal sequence; $\boldsymbol{c}(n)$ is the impulse response of the channel; $v(n)$ is an additive white Gaussian noise; $\boldsymbol{y}(n)$=[$y(n)$,$y(n-1)$,⋯, $y(n-L_f+1)]^T$ ($(\cdot)^T$ denotes transpose of a vector) is the equalizer input sequence; $\boldsymbol{f}(n)$=[$f(n)$,$f(n)$,⋯, $f_{L-1}(n)]^T$ is the equalizer weight vector with the length of L (L is positive integer); $\boldsymbol{r}(n)$ is correlation sequence constituted $\boldsymbol{y}(n)$ via correlation transformation; $\psi(\cdot)$ is memory less nonlinear function and used to generate error function; $z(n)$ is the equalizer output sequence; $\hat{a}(n)$ is estimate of $z(n)$; CT is the abbreviation of correlation transform , CFBEA is the abbreviation of correlation function blind equalization algorithm.

The expression of $\boldsymbol{r}(n)$

$$\boldsymbol{r}(n) = [r_{yy}(n,0), r_{yy}(n,1), \cdots, r_{yy}(n,L-1)]^T \quad (3)$$

Where, $r_{yy}(n,k) = E[y(n)y(n-k)]$ ($k = 0,\cdots,L-1$) is correlation function of the input signal.

The assumption that correlation function of the input signal and output signal is $r_{zy}(n)$, Then

$$r_{zy}(n) = E[z(n)x(n)] = E[\sum_{i=0}^{L-1} f_i(n)x(n-i)x(n)]$$
$$= \sum_{i=0}^{L-1} f_i(n)\cdot E[x(n)x(n-i)] = \sum_{i=0}^{L-1} f_i(n)\cdot r_{yy}(n,i)$$
$$= \boldsymbol{f}^T(n)\cdot r_{yy}(n,i) = \boldsymbol{f}^T(n)\cdot \boldsymbol{r}(n) \quad (4)$$

Error function of the algorithm is defined as $e_r(n)$

$$e_r(n) = R_2 - r_{zy}(n) = R_2 - \boldsymbol{f}^T\boldsymbol{r}(n) \quad (5)$$

Correlation function mean square error criteria defined by Asharif

$$\boldsymbol{J}_r(n) = E[\{e_r(n)\}^2] \quad (6)$$

In practical applications, common using the instantaneous gradient $\hat{\nabla}_f \boldsymbol{J}_r(n)$ of (6) to update equalizer weight vector, equalizer weight vector updating equation

$$\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \mu\hat{\nabla}_f \boldsymbol{J}_r(n) \quad (7)$$

Where, μ_i is iterative step.

$$\hat{\nabla}_f \boldsymbol{J}_r(n) = \frac{\partial}{\partial \boldsymbol{f}(n)}\{e_r(n)\} = 2e_r(n)\frac{\partial e_r(n)}{\partial \boldsymbol{f}(n)} \quad (8)$$

Order $\Phi = E[\boldsymbol{r}(n)\boldsymbol{r}^T(n)]$, then equalizer weight vector updating equation

$$\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \mu\Phi^{-1}e_r(n+1)\boldsymbol{r}(n+1) \quad (9)$$

$e_r(n) = R_2 - \boldsymbol{f}^T\boldsymbol{r}(n)$ is substituted into the upper equation and collated, then

$$\left[\boldsymbol{I} + \mu\Phi^{-1}\boldsymbol{r}(n+1)\boldsymbol{r}^T(n+1)\right]\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \mu\Phi^{-1}R_2\boldsymbol{r}(n+1) \quad (10)$$

Matrix Φ is used to multiply on both sides, then

$$\left[\Phi + \mu\boldsymbol{r}(n+1)\boldsymbol{r}^T(n+1)\right]\boldsymbol{f}(n+1) = \Phi\boldsymbol{f}(n) - \mu R_2\boldsymbol{r}(n+1) \quad (11)$$

That is,

$$\boldsymbol{f}(n+1) = \left[\Phi + \mu\boldsymbol{r}(n+1)\boldsymbol{r}^T(n+1)\right]^{-1} \bullet [\Phi\boldsymbol{f}(n) - \mu R_2\boldsymbol{r}(n+1)] \quad (12)$$

According to matrix inversion theorem: $(\boldsymbol{A} + \boldsymbol{DCD}^T) = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{D}\cdot(\boldsymbol{C}^{-1} + \boldsymbol{D}^T\boldsymbol{A}^{-1}D)^{-1}\boldsymbol{D}^T\boldsymbol{A}^{-1}$, obtained

$$\left[\Phi + \mu\boldsymbol{r}(n+1)\boldsymbol{r}^T(n+1)\right]^{-1} = \Phi^{-1} - \Phi^{-1}\boldsymbol{r}(n+1)$$
$$[\boldsymbol{r}^T(n+1)\Phi^{-1}\boldsymbol{r}(n+1) + \mu^{-1}]^{-1}\boldsymbol{r}^T(n+1)\Phi^{-1} \quad (13)$$

(13) is substituted into the (12) and collate, then

$$\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \frac{\mu\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n+1)e_r(n+1)}{1 + \mu\boldsymbol{r}^T(n+1)\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n+1)} \quad (14)$$

The correlation ratio coefficient $\rho_r = \boldsymbol{r}(n)/\boldsymbol{r}(n-1)$ is introduced and used to adjust the equalizer weight vector，then (14) becomes

$$\boldsymbol{f}(n+1) = \boldsymbol{f}(n) - \frac{\rho_r\mu\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n+1)e_r(n+1)}{1 + \mu\boldsymbol{r}^T(n+1)\boldsymbol{\Phi}^{-1}\boldsymbol{r}(n+1)} \quad (15)$$

The correlation ratio coefficient expresses the correlation between input sequence, and is variable, equivalent to a varying step, via change of the correlation ratio coefficient to update adaptively weight vector. (3-15) is called Correlation Function based the Blind Equalization Algorithm (CFBEA).

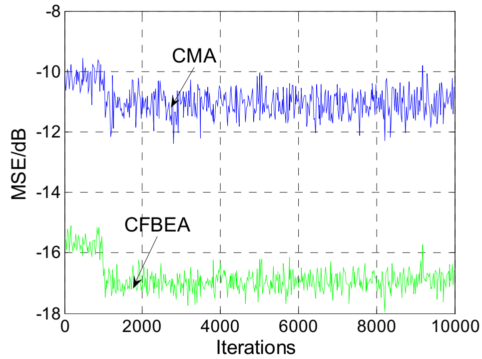
IV. SIMULATION TEST

In order to verify performance of CFBEA, it is simulated and researched with deep-sea sound channel axis underwater acoustic, and compare with the traditional CMA algorithm. Channel is $C(z) = 0.076 + 0.122z^{-1} + z^{-1000}$ [9]; The length of equalizer weight is 16 ; SNR is 20dB.

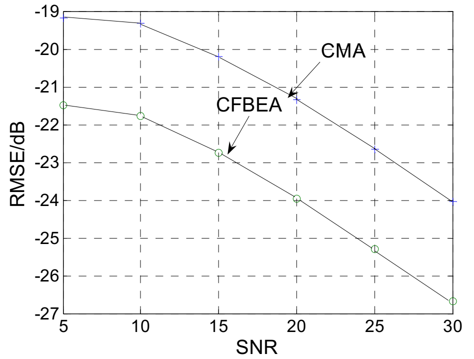
Simulation test 1: Emission signal is 4QAM, other parameters such as Table 1, 200 times' Monte Carlo simulation results as shown in Figure 3.

TABLE I.
SIMULATION PARAMETER VALUES

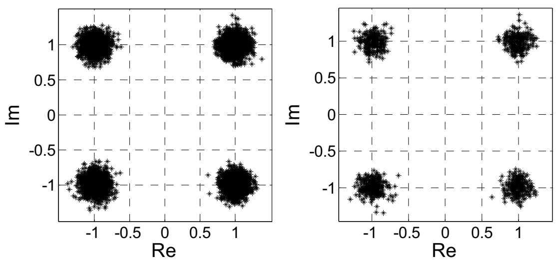
algorithm	simulation step size	correlation ratio coefficient
CMA	$\mu = 0.000015$	
CFBEA	$\mu = 0.000001$	$\rho_r = 0.001$



(a) mean square error curve



(b) root mean square error



(c) CMA output (d) CFBEA output

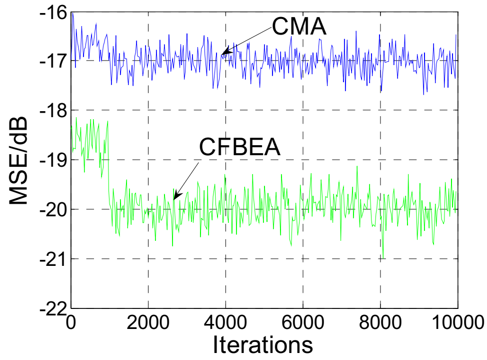
Figure 3. simulation test 1 results

Figure 3 (a) shows that the mean square errors of CFBEA is smaller than CMA about 6 dB; convergence rate of CFBEA is faster than CMA about 200 steps; Fig 3 (b) is relationship curve between the root-mean-square and signal to noise ratio (SNR), and shows that when SNR is the same root mean square errors of CFBEA is smaller than CMA, and at any SNR, root mean square errors of CFBEA is smaller than CMA; Fig 3 (c-d) show that constellation of CFBEA is clearer than CMA.

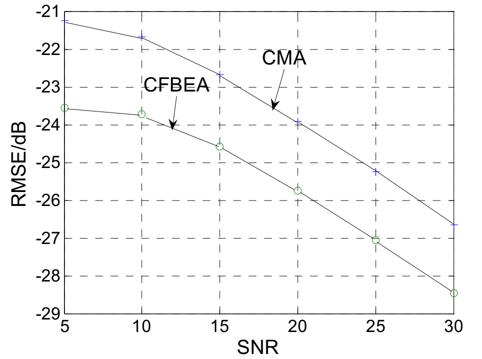
Simulation test 2: Emission signal is 8PSK, other parameters such as Table 1, 200 times' Monte Carlo simulation results as shown in Figure 4.

TABLE II.
SIMULATION PARAMETER VALUES

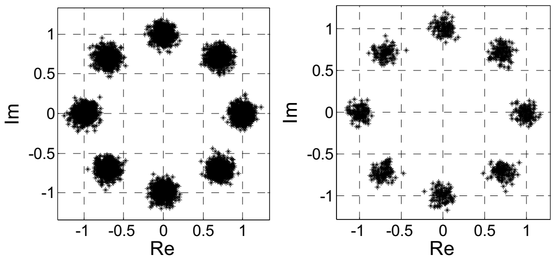
algorithm	simulation step size	correlation ratio coefficient
CMA	$\mu = 0.000008$	
CFBEA	$\mu = 0.000001$	$\rho_r = 0.001$



(a) mean square error curve



(b) root mean square error



(c) CMA output (d) CFBEA output

Figure 4. simulation test 2 results

Figure 4 (a) shows that the mean square errors of CFBEA is smaller than CMA about 3 dB; convergence rate of CFBEA is faster than CMA about 300 steps; Fig 4 (b) is relationship curve between the root-mean-square and signal to noise ratio (SNR), and shows that when SNR is the same root mean square errors of CFBEA is smaller than CMA, and at any SNR, root mean square errors of CFBEA is smaller than CMA; Fig 4(c-d) show that constellation of CFBEA is clearer than CMA.

V. CONCLUSIONS

Correlation function is introduced to blind equalization algorithm in this paper , directly processing correlation function of the input signal , correlation function based the blind equalization algorithm is proposed and the algorithm is derived according to correlation function mean square error criteria defined by Asharif and Newton's instantaneous gradient descent algorithm，At the same time correlation ratio coefficient is used to adjust the equalizer weight vector. Simulation tests with underwater acoustic channel indicate that convergence performance and mean square error CFBEA has mean square error and faster convergence performance compared with CMA . Thus, CFBEA is more conducive to resumption of real-time signal, and it provides a way to improve the performance of blind equalizer.

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