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A Novel Multi-Passive-Sensor Target Tracking Algorithm Based On Gaussian Filter

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***Abstract*—This paper presents a new multi-passive-sensor target tracking algorithm which yields a nonlinear state estimator called Gaussian filter based on deterministic sampling. Firstly, this state estimator employs a deterministic sample selection scheme, where a parametric density function representation of the sample points is employed to approximate the cumulative distribution function of the prior Gaussian density. The performance of the filter is more accurate than the extended Kalman Filter (EKF) and the unscented Kalman Filter (UKF) in nonlinear dynamic system. Secondly, in order to avoid the unobservability problem of passive target tracking, a nonlinear measurement model of multiple passive sensors is founded. Finally, the algorithm performance has been verified by illustrating some simulation results.**

***Index Terms*—Gaussian Filter, Target Tracking, Multi-Passive-Sensor**

I. Introduction

Accurate estimation of state variables of systems is important in many fields of science and engineering. The Bayesian framework is the most commonly used method for the study of these dynamic systems [1]. The best known algorithm to solve the problem of nonlinear filtering is the extended Kalman filter [2] which is based upon the principle of linearizing the measurements and evolution models using the first order term of the Taylor series expansions. Linearization in EKF introduces errors in the corresponding state estimations. Unlike the EKF, the Unscented Kalman Filter [3] uses the intuition that it is easier to approximate a probability distribution function than to approximate an arbitrary nonlinear function or transformation. It does not approximate the nonlinear process and observation models; it is specified using a minimal set of carefully chosen sample points to capture the true mean and covariance of the Gauss random variable. When propagated through the nonlinear system, it captures the posterior mean and covariance of the states accurately to the 2nd order, with errors only introduces in the 3rd and higher orders. However, adapting parameters in the UKF gets more involved or scaling is not guaranteed.

Recently, a new approach GF (Gaussian Filer) for nonlinear Bayesian estimation is proposed that is based on a deterministic sample selection scheme [4], where the sample points are represented by a Dirac mixture density, whose deviation to the true density is minimized. The estimator represents the state estimate by means of a multivariate Gaussian density. Compared to the popular UKF, the sample points of the GF are placed more systematically. We demonstrate the performance benefits in passive tracking, and we argue that the ease of the implementation and more accurate estimation features of the new filter recommend its use over the EKF and the UKF.

The structure of this paper is as follows. Section II describes briefly the sample selection scheme. Section III discusses the GF algorithm in more detail. In Section IV we introduce the observation model of the multi-passive sensors. And the simulation is performed in Section V. Finally we conclude our results in Section VI.

II. Deterministic Sampling

The one-dimensional random variable x with mean $\hat{x}=0$ and covariance $\sigma_x^2=1$ can be approximated by L weighted samples. We consider its true density function $\tilde{f}(x)$ .To approximate the true density function in both density and moments, consider an analytic and parametric form by means of a Dirac mixture given by

$$f_X(x,\underline{\mathbf{\eta}})=\sum_{i=1}^{L}\omega_i\cdot\delta(x-\mu_i) \tag{1}$$

where the weighting factors ω_i are assumed to be equal and given by $\omega_i=1/L$. The parameter vector $\underline{\mathbf{\eta}}$ contains the positions of the individual Dirac functions according to $\underline{\mathbf{\eta}}=[\mu_1,\mu_2,\cdots,\mu_L]^T$. Altogether, it is assumed that the positions of the samples are ordered according to $\mu_1<\mu_2<\cdots<\mu_{L-1}<\mu_L$. Our goal is to minimize a certain distance measure between the true density $\tilde{f}(x)$ and its approximation $f_X(x,\underline{\mathbf{\eta}})$. According to Cramér-von Mises distance [5], [6], quantifying the divergence between the distribution functions, the sample points can be concluded in the Lagrange multiplier approach. The sample points are symmetrically distributed and chosen with equal weights about $\hat{x}$, the sample mean is obviously guaranteed and the variance is exactly captured. In doing so, the deterministic sample points can list as follows in Table I.

To approximate any arbitrary Gaussian function $\tilde{f}_x(\mathbf{x})=\mathbf{N}(\mathbf{x}-\hat{\mathbf{x}},\mathbf{P}_x)$, the resulting sample points are scaled and rotated by employing the eigenvalue decomposition

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$$\mathbf{P}_x = \mathbf{V}\mathbf{D}\mathbf{V}^T \quad (2)$$

where $\mathbf{V}$ is the orthogonal matrix of eigenvectors and $\mathbf{D}$ is a diagonal matrix of eigenvalues of $\mathbf{P}_x$.

TABLE I.
SAMPLE POSITIONS FOR SEVERAL NUMBERS OF SAMPLES

L	3	5	7
μ_1	-1.2247	-1.4795	-1.6346
μ_2	0	-0.5578	-0.8275
μ_3	1.2247	0	-0.3788
μ_4	–	0.5578	0
μ_5	–	1.4795	0.3788
μ_6	–	–	0.8275
μ_7	–	–	1.6346

The n-dimensional random variable $\mathbf{x}$ with mean $\hat{\mathbf{x}}$ and covariance $\mathbf{P}_x$ can be approximated by the following $n \cdot L$ weighted sample points, $i = 1, 2, ..., L$, $j = 1, 2, ..., n$.

$$\underline{\boldsymbol{\mu}}_{i,j} = \hat{\mathbf{x}} + \mu_i \cdot (\mathbf{V}\sqrt{\mathbf{D}})_j \quad (3)$$

$$\omega_{i,j} = \frac{1}{1 + n \cdot (L-1)} \cdot \begin{cases} \frac{1}{n}, & i = \left\lceil \frac{L-1}{2} \right\rceil + 1 \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

where $(\mathbf{V}\sqrt{\mathbf{D}})_j$ is the jth column of the matrix and μ_i is the deterministic sample points in the above mentioned method.

III. GAUSSIAN FILTER BASED ON DETERMINISTIC SAMPLING

This note investigates the problem of the state estimation for nonlinear discrete-time stochastic system of the type:

$$\mathbf{x}(k+1) = \mathbf{f}(k, \mathbf{x}(k)) + \mathbf{v}(k) \quad (5)$$

$$\mathbf{y}(k) = \mathbf{h}(k, \mathbf{x}(k)) + \mathbf{w}(k), k \geq 0 \quad \mathbf{x}(0) = \mathbf{x}_0 \quad (6)$$

where $\mathbf{x}(k) \in \mathbf{R}^n$ is the dynamic system state, $\mathbf{y}(k) \in \mathbf{R}^q$ is the measured output, $\mathbf{f}: \mathbf{Z}^+ \times \mathbf{R}^n \to \mathbf{R}^n$, $\mathbf{h}: \mathbf{Z}^+ \times \mathbf{R}^n \to \mathbf{R}^q$ are time-varying smooth nonlinear maps, denoted state-transition map and state-output map, respectively. The initial state $\mathbf{x}_0$ is a random vector independent of both the noise sequences. It is assumed that the noise vectors are uncorrelated white Gaussian processed with expected means and covariance

$$E[\mathbf{w}(k)\mathbf{w}^T(j)] = \mathbf{Q}(k)\delta_{k,j}, E[\mathbf{v}(k)] = 0 \quad (7)$$

$$E[\mathbf{v}(k)\mathbf{v}^T(j)] = \mathbf{R}(k)\delta_{k,j}, E[\mathbf{v}(k)\mathbf{w}^T(j)] = 0 \quad (8)$$

Since the GF shares a number of similarities such as point-wise evaluation of nonlinearities and weighted sum statistics, it can be considered as a new member of the family of sigma point filters. The GF designs an optimization problem, where the sample points are interpreted as analytic density function and are represented as the distribution function of the prior Gaussian density. This is different from the unscented Kalman filter or particle filters. Thus, the Gaussian filter can approximate true posterior density function more accurately than UKF.

The GF, according to [4], can be summarized as follows.

(1) Initialize with:

$$\hat{\mathbf{x}}_0 = E[\mathbf{x}_0]$$

$$\mathbf{P}_0 = E[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T]$$

$$\hat{\mathbf{x}}_0^a = E[\mathbf{x}^a] = [\hat{\mathbf{x}}_0^T \ \ \mathbf{0} \ \ \mathbf{0}]^T$$

$$\mathbf{P}_0^a = E[(\mathbf{x}_0^a - \hat{\mathbf{x}}_0^a)(\mathbf{x}_0^a - \hat{\mathbf{x}}_0^a)^T] = \begin{bmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}$$

(2) For the time step $k = 1, 2, ...$ do:

a. According to $\hat{\mathbf{x}}_k^a$ and $\mathbf{P}_{k-1}^a$, determine sample points with weights: $\left\{\underline{\boldsymbol{\mu}}_{k-1,l}^a, \omega_l, l = 1, 2, ..., n \cdot L\right\}$

b. Time update step:

$$\underline{\boldsymbol{\mu}}_{k|k-1,l}^x = \mathbf{f}(\underline{\boldsymbol{\mu}}_{k-1,l}^x, \underline{\boldsymbol{\mu}}_{k-1,l}^v)$$

$$\hat{\mathbf{x}}_{k|k-1} = \sum_l \omega_l \underline{\boldsymbol{\mu}}_{k|k-1,l}^x$$

$$\underline{\boldsymbol{\mu}}_{k|k-1,l}^y = \mathbf{h}(\underline{\boldsymbol{\mu}}_{k|k-1,l}^x, \underline{\boldsymbol{\mu}}_{k-1,l}^w)$$

$$\hat{\mathbf{y}}_{k|k-1} = \sum_l \omega_l \underline{\boldsymbol{\mu}}_{k|k-1,l}^y$$

$$\mathbf{P}_{k|k-1} = \sum_l \omega_l [\underline{\boldsymbol{\mu}}_{k|k-1,l}^x - \hat{\mathbf{x}}_{k|k-1}][\underline{\boldsymbol{\mu}}_{k|k-1,l}^x - \hat{\mathbf{x}}_{k|k-1}]^T$$

c. Measurement update step:

$$\mathbf{P}_k^y = \sum_l \omega_l [\underline{\boldsymbol{\mu}}_{k|k-1,l}^y - \hat{\mathbf{y}}_{k|k-1}][\underline{\boldsymbol{\mu}}_{k|k-1,l}^y - \hat{\mathbf{y}}_{k|k-1}]^T$$

$$\mathbf{P}_k^{xy} = \sum_l \omega_l [\underline{\boldsymbol{\mu}}_{k|k-1,l}^x - \hat{\mathbf{x}}_{k|k-1}][\underline{\boldsymbol{\mu}}_{k|k-1,l}^y - \hat{\mathbf{y}}_{k|k-1}]^T$$

$$\mathbf{K}_k = \mathbf{P}_k^{xy} \mathbf{P}_k^{y-1}$$

$$\hat{\mathbf{x}}_k = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1})$$

$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{P}_k^y \mathbf{K}_k^T$$

where the augmented state is $\hat{\mathbf{x}}_k^a = [\hat{\mathbf{x}}_k^T, \mathbf{v}_{k-1}^T, \mathbf{w}_k^T]^T$, its corresponding augmented error covariance $\mathbf{P}_k^a = \begin{bmatrix} \mathbf{P}_k & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}$, and the sample points of the augmented state are $\underline{\boldsymbol{\mu}}_{k,l}^a = [(\underline{\boldsymbol{\mu}}_{k,l}^x)^T, (\underline{\boldsymbol{\mu}}_{k,l}^v)^T, (\underline{\boldsymbol{\mu}}_{k,l}^w)^T]^T$

IV. MULTI-PASSIVE-SENSOR OBSERVATION MODLE

The emergence of passive sensors has a significant impact on the design of weapons system that depended on radars. When the passive sensor platform is allowed to move freely, it is possible to recover range observability by selecting an appropriate path for the platform [7]. For a stationary passive sensor system, to avoid observability problems, the range must be acquired through multiple passive sensors. This paper considers multiple passive sensors system; the observation vector can be defined as

$$\mathbf{z}(k) = [\alpha_1, \alpha_2, \cdots, \alpha_N, \beta_1, \beta_2, \cdots, \beta_N]^T \tag{9}$$

$$\alpha_i = \arctan \frac{z_T - z_i}{\sqrt{(x_T - x_i)^2 + (y_T - y_i)^2}} \tag{10}$$

$$\beta_i = \arctan \frac{x_T - x_i}{y_T - y_i} \tag{11}$$

where N is the number of the passive sensors, (x_T, y_T, z_T) is the position of the target, and (x_i, y_i, z_i) is the position of the ith passive sensor.

V. EXPERRIMENTS

In this section, simulation results are reported which compare the performance of the GF algorithm with the EKF algorithm and the UKF algorithm. Consider the following simulation trajectory scenario as an illustrative example

$$\mathbf{x}(k) = \mathbf{f}(\mathbf{x}(k-1)) + \mathbf{v}(k-1) \tag{12}$$

where $\mathbf{x}(k) = [x_k \; y_k \; z_k \; \dot{x}_k \; \dot{y}_k \; \dot{z}_k]^T$ with x, y and z denotes the Cartesian coordinates of the target with velocities $\dot{x}$, $\dot{y}$ and $\dot{z}$ in x, y and z directions, and the sampling period is $1.0s$, $\mathbf{v}(k)$ is zero-mean Gaussian white noise with its covariance $\mathbf{Q}(k)$. $\mathbf{w}(k)$ is zero-mean Gaussian white noise with its covariance $\mathbf{R}(k) = diag([0.1mrad^2 \; 0.1mrad^2 \; 0.1mrad^2 \; 0.1mrad^2 \; 0.1mrad^2 \; 0.1mrad^2])$. The positions of three passive sensors are assumed to be $S_1(-20km, 0km, 0km)$, $S_2(0km, -20km, 0km)$, $S_3(-20km, -20km, 0km)$. The initial state is a Gaussian random vector with mean $\mathbf{x}_0$ and variance $\mathbf{Q}_0$.

$$\mathbf{x}_0 = [50km \; 60km \; 40km \; -300m \; -400m \; 0m]^T$$

$$\mathbf{Q}_0 = diag(10^{-6} km^2 s^{-4} \times [100 \; 100 \; 100 \;\; 64 \; 64 \; 64])$$

The performance is analyzed with 100 MC runs. The simulated target is tracked over 30 steps. Its motion model is assumed as follows:

$$\mathbf{f}(\mathbf{x}_{k-1}) = \begin{bmatrix} x(k-1) + \dot{x}(k-1) \\ y(k-1) + \dot{y}(k-1) \\ z(k-1) + \dot{z}(k-1) \\ \dfrac{14\dot{x}(k-1)}{15} + \dfrac{0.1\dot{x}(k-1)}{\sqrt{\dot{x}(k-1)^2 + \dot{y}(k-1)^2}} \\ \dfrac{14\dot{y}(k-1)}{15} + \dfrac{0.1\dot{y}(k-1)}{\sqrt{\dot{x}(k-1)^2 + \dot{y}(k-1)^2}} \\ \dot{z}(k-1) \end{bmatrix} \tag{13}$$

The real target motion scenario is given by Fig. 1. Fig. 2 and Fig. 3 compares the position and the velocity root mean square error (RMSE) of three filtering methods ($L=3$) when the process noise covariance $\mathbf{Q}_1(k) = diag(10^{-6} km^2 s^{-4} \times [400 \; 400 \; 400 \;\; 400 \; 400 \; 400])$ and $\mathbf{Q}_2(k) = diag(10^{-4} km^2 s^{-4} \times [64 \; 64 \; 64 \; 64 \; 64 \; 64])$ respectively. We can see that GF outperforms EKF and UKF in position and velocity RMSE in two situations, while the computation time is almost identical.

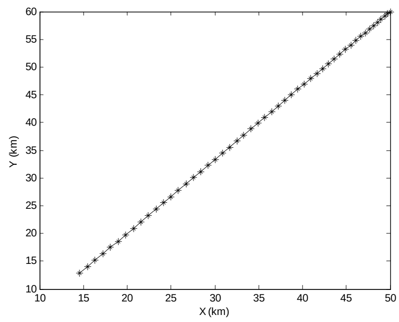
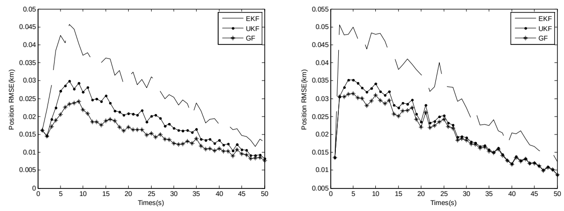
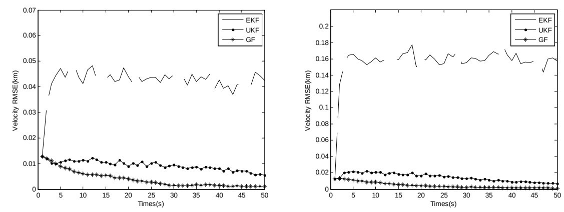


Figure 1. Target motion scenario



(a) $\mathbf{Q}_1(k)$ (b) $\mathbf{Q}_2(k)$

Figure 2. Comparison of position RMSE among EKF, UKF, and GF filter (L=3)



(a) $\mathbf{Q}_1(k)$ (b) $\mathbf{Q}_2(k)$

Figure 3. Comparison of velocity RMSE among EKF, UKF, and GF filter (L=3)

VI. CONCLUSIONS

For tracking a target in the multi-passive-sensor system, we propose a new filtering algorithm, GF, and compare it with the EKF and UKF algorithm. All these filtering methods share the same manner to update state but differ in the way to obtain the prediction terms. The EKF may introduce large estimation errors due to filter divergence when the filtering problem is highly nonlinear and the local linearization assumption breaks down. The UKF adopts some parameters which get more involved or scaling is not guaranteed. In this paper, the idea of the unscented transform is extended by interpreting the sample points as analytic density function, namely a Dirac mixture. Altogether, higher-order information of the Gaussian density is implicitly incorporated and the nonlinearities of the state transformation are captured more accurate. In contrast the UKF, the GF does not require any further tuning parameters, which eases its use. In summary, the structure and the computational complexity of the GF is comparable to the UKF, whereas its superior estimation accuracy has been demonstrated by means of simulations. Considering the theory analysis and simulation study together, we conclude that the GF is a good alternative for the multi-passive-sensor target tracking problem being studied.

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