ISBN 978-952-5726-02-2 (Print), 978-952-5726-03-9 (CD-ROM)
Proceedings of the 2009 International Symposium on Information Processing (ISIP'09)
Huangshan, P. R. China, August 21-23, 2009, pp. 033-036

Possibility Degree of Interval-valued Intuitionistic Fuzzy Numbers and its Application

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***Abstract*—Interval-valued intuitionistic fuzzy number uses membership degree and non-member ship degree to express a decision-maker's hesitation, which make decision-maker express his/her opinion in uncertainty decision-making environment easily. The comparison between some interval-valued intuitionistic fuzzy numbers can not be undertaken because of its limitation. In application such as weak consistency judgment of an interval-valued intuitionistic fuzzy judgment and multi-attribute decision-making, the comparison operator is needed. To expand its application, new possibility degree functions of interval-valued intuitionistic fuzzy numbers were proposed. The possibility degree of interval-valued intuitionistic fuzzy numbers was applied to multi-attribute decision-making and an example was used to illustrate it.**

***Index Terms*—interval-valued intuitionisitc fuzzy number, possibility degree, multi-attribute decision making, comparison, possibility degree**

I. INTRODUCTION

In Ref[1], Antanassov (1986) generalized the notion of Zaheh's fuzzy set to the concept of the intuitionistic fuzzy set(IFS), which was composed of the membership degree, non-membership degree, hesitation degree of a element x in a set A. It was more flexible in manipulating fuzziness and uncertainty, and was applied in many fields such as medical diagnosis, logical planning [2,3]. However, sometimes the membership degree and non-membership degree is difficult to give using exact numbers. Antanassov and Gargov (1989) extended the intuistionistic fuzzy set and put forward the concept of interval-valued intuitionistic fuzzy set (IVIFS) [4]. Bustince and Burillo (1995) introduced the concepts of correlation and correlation coefficient of interval-valued intuitionistic fuzzy sets and their properties [5]. Hong (1998) generalized the concepts of correlation and correlation coefficient of interval-valued intuitionistic fuzzy sets in a general probability space and introduced three more decomposition theorems of interval-valued intuitionistic fuzzy sets in terms of the correlation of interval-valued fuzzy sets [6]. Mondal and Samanta (2001) defined topology of interval-valued intuitionistic fuzzy sets and studied some properties of it [7]. Xu (2007) defined the concept of the degree of similarity between interval-valued intuitionistic fuzzy sets and defines some distances measures between two IVIFSs and proposed an approach for decision making with interval-valued intuitionistic fuzzy information [8,9]. Xu and Chen (2007) developed the ordered weighted aggregation operator and the hybrid aggregation operator for aggregating interval-valued intuitionistic fuzzy information, and put forward a new group decision-making method based on interval-valued intuitionistic judgement matrix [10].

Antassov had defined the comparison of two interval-valued intuitionistic fuzzy numbers. Some interval-valued intuitionistic fuzzy numbers can't be compared used his definition for its limitation, do not meet the comparing condition.

Possibility degree reflects the probability of one fuzzy number is larger than another fuzzy number and can be used to compare two fuzzy numbers. Zhang and Le(1982) introduced the possibility degree to describe the fuzzy true-value logic with some operators, and studied some properties of possibility degree[11]. Zhang, Fan and Pan proposed the rational formula to compute the possibility degree of interval-valued fuzzy numbers and put forward a new multi-attribute decision making sorting approach based on the possibility degree[12]. Xu and Da (2003) had developed a possibility degree formula for the comparison between two interval fuzzy numbers and proved the three formulas' equivalence [13]. Extended the concept of possibility degree of Interval numbers, we defined the possibility degree formula for the comparison of two intuitionistic fuzzy numbers was proposed, and was used to solve ranking alternatives in multi-attribute decision making

II. SOME PRELIMINARIES

Definition 1 Let X be a universe of discourse. An IVIFS A over X is an object having the form [4]:

$$A = \{< x, \tilde{\mu}_A(x), \tilde{v}_A(x) > | \, x \in X\} \qquad (1)$$

where $\tilde{\mu}_A(x) \subset [0,1]$ and $\tilde{v}_A(x) \subset [0,1]$ are intervals, and for every $x \in X$:

$$\sup \tilde{\mu}_A(x) + \sup \tilde{v}_A(x) \le 1 \qquad (2)$$

Especially, if $\mu_A(x) = \inf \tilde{\mu}_A(x) = \sup \tilde{\mu}_A(x)$ and $v_A(x) = \inf \tilde{v}_A(x) = \sup \tilde{v}_A(x)$, then, the given IVIFS A is reduced to an ordinary intuitionistic fuzzy set.

The ordered $(\tilde{\mu}_A(x), \tilde{v}_A(x))$ is composed by the membership degree $\tilde{\mu}_A(x)$ of element x belonging to X and the non-membership degree $\tilde{v}_A(x)$ of element x belonging to X. And $(\tilde{\mu}_A(x), \tilde{v}_A(x))$ was called an interval-valued intuitionistic fuzzy number[8]. To make it more clearly, we denote an interval-valued intuitionistic fuzzy number as $([a,b],[c,d])$. And we denote Ω as the collection set of all interval-valued intuitionistic fuzzy numbers.

Let $\tilde{\alpha}_1 = ([a_1, b_1],[c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2],[c_2, d_2])$ are two interval-valued intuitionisitc fuzzy numbers in Ω. The following expressions are defined [4,8].

$$\tilde{\alpha}_1 \le \tilde{\alpha}_2 \Leftrightarrow a_1 \le a_2, b_1 \le b_2, c_2 \le c_1, d_2 \le d_1 \quad (3)$$

$$\tilde{\alpha}_1 = \tilde{\alpha}_2 \Leftrightarrow a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2 \quad (4)$$

$\tilde{\alpha}_1^c$ is the complement of $\tilde{\alpha}_1$,

$$\tilde{\alpha}_1^c = ([c_1, d_1],[a_1, b_1]) \quad (5)$$

$$\tilde{\alpha}_1 + \tilde{\alpha}_2 = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2],[c_1c_2, d_1d_2]) \quad (6)$$

$$\tilde{\alpha}_1 \cdot \tilde{\alpha}_2 = ([a_1a_2, b_1b_2],[c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2]) \quad (7)$$

$$\lambda\tilde{\alpha}_1 = ([1-(1-a_1)^\lambda, [1-(1-b_1)^\lambda],[c_1^\lambda, d_1^\lambda]) \quad (8)$$

$$\tilde{\alpha}_1^\lambda = ([a_1^\lambda, b_1^\lambda],[1-(1-c_1)^\lambda, [1-(1-d_1)^\lambda]) \quad (9)$$

Definition 2[9] Suppose $\tilde{\alpha}_j\,(j = 1,2,\cdots,n)$ are interval-valued intuitionistic fuzzy numbers in Ω and $f : \Omega^n \to \Omega$, if

$$f_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = \sum_{j=1}^n \omega_j \tilde{\alpha}_j = ([1 - \prod_{j=1}^n (1-a_1)^{\omega_j}, 1 - \prod_{j=1}^n (1-b_1)^{\omega_j}], [\prod_{j=1}^n c_1^{\omega_j}, \prod_{j=1}^n d_1^{\omega_j}]) \quad (10)$$

and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{\alpha}_j\,(j = 1,2,\cdots,n)$, $\omega_j \in [0,1]$ and $\sum_{j=1}^n \omega_j = 1$, then f is the weighted arithmetic average operator.

Definition 3 [9] Suppose $\tilde{\alpha}_j\,(j = 1,2,\cdots,n)$ are interval-valued intuitionistic fuzzy numbers in Ω and $f : \Omega^n \to \Omega$, if

$$f_\omega(\tilde{\alpha}_1, \tilde{\alpha}_2, \cdots, \tilde{\alpha}_n) = ([\prod_{j=1}^n a_1^{\omega_j}, \prod_{j=1}^n b_1^{\omega_j}], [1 - \prod_{j=1}^n (1-c_1)^{\omega_j}, 1 - \prod_{j=1}^n (1-d_1)^{\omega_j}]) \quad (11)$$

and $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of $\tilde{\alpha}_j\,(j = 1,2,\cdots,n)$, $\omega_j \in [0,1]$ and $\sum_{j=1}^n \omega_j = 1$, then f is the geometric arithmetic average operator.

III. POSSIBILITY DEGREE OF INTERVAL-VALUED INTUIONISITC FUZZY NUMBERS

Let $\tilde{\alpha}_1 = ([a_1, b_1],[c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2],[c_2, d_2])$ are two interval-valued intuitionisitc fuzzy numbers in Ω. From formula (3), we can compare two interval-valued intuitionistic fuzzy numbers if $a_1 \le a_2, b_1 \le b_2, c_2 \le c_1, d_2 \le d_1$. Not every two interval-valued intuitionistic fuzzy numbers however, meet the condition. For example, $\tilde{\alpha}_1 = ([0.4, 0.5],[0.3, 0.4])$ and $\tilde{\alpha}_2 = ([0.4, 0.6],[0.2, 0.4])$ do not meet the comparing condition.

Xu and Chen defined the score function and accurate function of interval-valued intuitionistic fuzzy numbers to sort the interval-valued intuitionistic fuzzy numbers.

Let $\tilde{\alpha}_1 = ([a_1, b_1],[c_1, d_1])$ be an interval-valued intuitionistic fuzzy number, then $S(\tilde{a})$ is called the score function of it, and $H(\tilde{a})$ is called the accurate function of it.

$$S(\tilde{a}) = \frac{a - c + b - d}{2} \quad (12)$$

$$H(\tilde{a}) = \frac{a + b + c + d}{2} \quad (13)$$

Let $\tilde{\alpha}_1 = ([a_1, b_1],[c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2],[c_2, d_2])$ be two interval-valued intuitionistic fuzzy numbers, the sorting method of any interval-valued intuitionistic fuzzy numbers is as follows:

$$\tilde{\alpha}_1 \le \tilde{\alpha}_2 \Leftrightarrow S(\tilde{a}_1) \le S(\tilde{a}_2) or S(\tilde{a}_1) = S(\tilde{a}_2), H(\tilde{a}_1) \le H(\tilde{a}_2) \quad (14)$$

Obviously, $S(\tilde{a})$ represented the deviation the membership degree and non-membership degree of an interval-valued intuitionistic fuzzy number, $H(\tilde{a})$ represented the mean values of membership degree and non-membership degree of an interval-valued intuitionistic fuzzy number. Consider the formula (12) and (13), we can get the following formula:

$$H(\tilde{a}) = a + b - S(\tilde{a}) \quad (15)$$

If the score functions of two interval-valued intuitionistic fuzzy numbers are equal, then the comparison between the two interval-valued intuitionistic fuzzy numbers depends on the mean values of its membership degree. We thought the accuracy function of an interval-valued intuitionistic fuzzy number should be related to the mean value of its membership degree and the deviation value between its membership degree and non-membership degree. Thus, a new accuracy function of an interval-valued intuitionistic fuzzy number was proposed as follows.

$$A(\tilde{\alpha}_1) = \lambda \cdot \frac{a + b}{2} + (1 - \lambda)\frac{a - c + b - d}{2} \quad (16)$$

where $\lambda \in [0,1]$ and it represents the preference on the mean valued of its membership degree.

Let us consider the following example: if interval-valued intuitionistic fuzzy values for two alternatives are $\tilde{\alpha}_1 = ([0.4, 0.5],[0.3, 0.4])$ and $\tilde{\alpha}_2 = ([0.4, 0.6],[0.2, 0.4])$, $S(\tilde{a}_1) = 0.2$ and $S(\tilde{a}_2) = 0.2$, $H(\tilde{a}_1) = 0.8$ and $H(\tilde{a}_2) = 0.85$, so we can say $\tilde{a}_1 \le \tilde{a}_2$. Using formula (16) and let =0.5, then we got $A(\tilde{\alpha}_1) = 0.35$ and $A(\tilde{\alpha}_2) = 0.375$, and got the same result $\tilde{a}_1 \le \tilde{a}_2$. So we can use the new accuracy function to compare two interval-valued intuitionistic fuzzy numbers and the comparison operator of two interval-valued intuitionistic fuzzy numbers can be expressed as follows.

$$\tilde{\alpha}_1 \le \tilde{\alpha}_2 \Leftrightarrow A(\tilde{\alpha}_1) \le A(\tilde{\alpha}_2) \quad (17)$$

Possibility degree reflects the probability of one fuzzy number is larger than another fuzzy number. Possibility degree can be used to compare two fuzzy numbers[12,13]. Possibility degree of two fuzzy numbers not only reflects the comparison of two fuzzy numbers, but also reflects the degree of one fuzzy number is bigger than another fuzzy number.

The accuracy function reflects the true-value of an interval-valued intuitionistic fuzzy number, so it can be used to describe the probability degree of one interval-valued intuitionistic fuzzy number is larger than another interval-valued intuitionistic fuzzy number. Thus, we got the following definition.

Definition 4 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ are two interval-valued intuitionisitc fuzzy numbers in Ω. The possibility degree of two interval-valued intuitionisitc fuzzy numbers can be denoted as follows.

$$p_1(\tilde{\alpha}_1, \tilde{\alpha}_2) = \max(\min(A(\tilde{\alpha}_1) - A(\tilde{\alpha}_2) + 0.5, 0), 1) \quad (18)$$

Obviously, the above formula (18) has the following properties:

(1) $0 \le p_1(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) \le 1$;

(2) $p_1(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) = 1 \Leftrightarrow A(\tilde{\alpha}_1) - A(\tilde{\alpha}_2) \geqslant 0.5$;

(3) $p_1(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) = 0 \Leftrightarrow A(\tilde{\alpha}_1) - A(\tilde{\alpha}_2) \leqslant -0.5$;

(4) $p(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) + p(\tilde{\alpha}_2 \ge \tilde{\alpha}_1) = 1$.

The membership degree and non-membership degree of an interval-valued intuitionistic fuzzy number are expressed by interval-value fuzzy numbers, and there were research on possibility degree of interval-valued fuzzy numbers[11-13]. We thought the possibility degree formula proposed by Xu and Da appropriate because it can deal with any two interval-valued fuzzy numbers, and it was equivalent with other formulas proposed by other scholars.

Based on the possibility degree of two interval numbers, we extended it and got the concept of possibility degree of two interval-valued intuitionistic fuzzy number.

Definition 5 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ are two interval-valued intuitionisitc fuzzy numbers in Ω . $p(\tilde{\alpha}_1 \ge \tilde{\alpha}_2)$ is called as the possibility degree of two interval-valued intuitionistic fuzzy numbers.

$$\begin{aligned} p_2(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) &= \gamma \min(\max(\frac{b_1 - a_2}{b_1 - a_2 + b_2 - a_2}, 0), 1) \\ &+ (1-\gamma)\min(\max(\frac{d_2 - c_1}{d_1 - c_1 + d_2 - c_2}, 0), 1), \gamma \in [0,1] \end{aligned} \quad (19)$$

where $\gamma \in [0,1]$ and reflects the decision maker's preference on membership degree or non-membership degree. When The decision maker is optimal, $\gamma > 0.5$. When the decision maker is pessimistic, then $\gamma < 0.5$.

Theorem 1 Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$, $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$, $\tilde{\alpha}_3 = ([a_3, b_3], [c_3, d_3])$ are three interval-valued intuitionisitc fuzzy numbers in Ω . Then the following holds:

(1) $0 \le p_2(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) \le 1$.

(2) $p_2(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) = 1 \Leftrightarrow b_2 \le a_1$ and $d_1 \le c_2$.

(3) $p_2(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) = 0 \Leftrightarrow b_1 \le a_2$ and $d_2 \le c_1$.

(4) $p_2(\tilde{\alpha}_1 \ge \tilde{\alpha}_2) + p_2(\tilde{\alpha}_2 \ge \tilde{\alpha}_1) = 1$.

Ⅳ. MADM BASED ON POSSIBILITY DEGREE OF INTERVAL-VALUED INTUITIONISTIC FUZZY NUMBERS

For a multi-attribute decision making problem, let A={A_1, A_2, … , A_n} is the set of alternatives and X={X_1,X_2, …, X_m} is the set of attributes. $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ is the weight vector of attributes X_1,X_2, …, X_m, $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Suppose the characteristic information of alternative A_i over attribute X_j is represented by interval-valued intuitionistic fuzzy number $\tilde{\alpha} = ([a,b],[c,d])$, and [a,b] represents the fuzzy membership degree of the alternative A_i over attribute X_j, [c,d] represents the fuzzy nonmembership degree of the alternative A_i over attribute X_j. Then the judgment matrix $\tilde{M}$ is obtained.

$$\tilde{M} = \begin{bmatrix} ([a_{11}, b_{11}], [c_{11}, d_{11}]) & \cdots & ([a_{11}, b_{11}], [c_{11}, d_{11}]) \\ \vdots & \vdots & \vdots \\ ([a_{n1}, b_{n1}], [c_{n1}, d_{n1}]) & \cdots & ([a_{nm}, b_{nm}], [c_{nm}, d_{nm}]) \end{bmatrix}$$

Ranks of the alternatives in the multi-attribute decision making case can be solved by using possibility degree of interval-valued intuitionistic fuzzy numbers. The decision making process can be illustrated as follows:

Step 1 Use weighted arithmetic average operator (10) or geometric average operator (11) to aggregate the evaluation of each alternative, IC(A_1), IC(A_2), … , IC(A_n) .

Step 2 Apply formula (13) to compute the possibility degree of interval-valued intuitionistic fuzzy numbers IC(A_1), IC(A_2), … , IC(A_n), and get the possibility degree matrix P.

Setp 3 We use simple formula of RMM(row mean method) to calculate the comprehensive evaluation of each alternative.

$$w = (w_1, w_2 \cdots, w_n), \; w_i = \frac{1}{n-1} \sum_{j=1, j \ne i}^{n} a_{ij} \quad (14)$$

Step 4 Rank $w_1, w_2 \cdots, w_n$ in order $w'_1, w'_2 \cdots, w'_n$. Then we can rank the alternative as $A'_1 \underset{p(A'_1, A'_2)}{\ge} A'_2 \underset{p(A'_2, A'_3)}{\ge} \cdots \underset{p(A'_{n-1}, A'_n)}{\ge} A'_n$.

Ⅴ. ILLUSTRATED EXAMPLE

In supply chain management, the core enterprise will focus on many factors when it comes to select partners. The key factors that affect the choice of partners are response time and supply capability (y_1), quality and technique(y_2), price and cost(y_3), and services(y_4) [12]. In Ref [10], three experts were asked to give their preferences over the above four factors, using the possibility degree we got the weight of each factor is $\omega = (\omega_1, \omega_2, \omega_3, \omega_4)^T = (0.34, 0.33, 0.17, 0.16)^T$. There are three selected company denoted as A_1, A_2, A_3 . The matrix of the alternatives over the above four factors M is:

$$M^T = \begin{bmatrix} ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.7],[0.1,0.3]) \\ ([0.4,0.6],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) \\ ([0.4,0.5],[0.2,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.3,0.4],[0.4,0.5]) \\ ([0.8,0.9],[0,0.1]) & ([0.7,0.8],[0.1,0.2]) & ([0.6,0.8],[0,0.2]) \end{bmatrix}$$

Firstly, compute the comprehensive evaluation of each company and got: $IC(A_1)=([0.6024,0.7370],[0,0.2192])$, $IC(A_2)=([0.6362,0.7375],[0.1257,0.2286])$, $IC(A_3)=([0.5254,0.6837],[0,0.3067])$.

Secondly, compute the possibility degree of each company's interval-valued intuitionistic fuzzy number evaluation and the possibility degree matrix is:

$$P = \begin{bmatrix} 0.5 & 0.5685 & 0.6528 \\ 0.4315 & 0.5 & 0.6295 \\ 0.3472 & 0.3705 & 0.5 \end{bmatrix}$$

Thirdly, we got the comprehensive evaluation of each company is $HC(A_1)=0.6106$, $HC(A_2)=0.5805$, $HC(A_3)=0.3509$.

Thus, the select order of partner is A1, A2,A3.

VI. CONCLUSION

Xu and Chen defined the score function and accuracy function to compare two interval-valued intuitionistic fuzzy numbers. We analyzed the score function and accuracy function, and find that there was some relation between them. So we defined a new accuracy function of an interval-valued intuitionistic fuzzy number, which can be used to compare two interval-valued intuitionistic fuzzy numbers and simplify the comparison operator. We also extended the concept of possibility degree of interval fuzzy numbers to possibility degree of interval-valued intuitionistic fuzzy number and proposed serveral formulas to computer the possibility degree of two interval-valued intuitionistic fuzzy numbers. Based on the possibility degree of two interval-valued intuitioistic fuzzy numbers, it put forward a multi-attribute decision making approach for ranking alternatives in interval-valued intuitionistic fuzzy environment.

ACKNOWLEDGMENT

This work is supported by Zhejiang Philosopher and Social Foundation under Grant by 08CGYD030YBQ and is also supported by Jiaxing Technology Division of China under Grant 2008AY2015.

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