Optimization of Grey Derivative in GM (1, 1) Based on the Discrete Exponential Sequence

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Abstract—One improved approach of grey derivative in the direct GM (1, 1) is presented in this paper, which raises the modeling precision once again. The new model has been proven strictly to have the property of exponent, coefficient and translation constants superposition. The results of data simulation and model comparison show that the improved model in this paper raises the accuracy of background value, the fitting precision and forecasting precision. Moreover, it is not only suitable for the low growth sequence, but also suitable for the high growth sequence. What’s more, it is suitable for the nonhomogeneous exponential sequence.

Index Terms—GM (1, 1), grey derivative, optimization, exponential model

I. INTRODUCTION

GM (1, 1) is the foundation and core of grey system prediction theory [1~2]. And it has widely applied in numerous fields, such as agriculture, industry, meteorology, electric power, economy, society and so on. It regards a system as the exponential function which changes with the time variation, and does not need the massive time series data to establish the forecast model. The calculating simpleness for GM (1, 1) has been accepted by people. However, there are still some deficiencies in grey system theory, the accuracy of model need to be further improved. Many scholars have done a lot of research in improving the model accuracy [3~7]. References [3], [4] and [7] do some improvement from the angle of optimizing background value, making the white differential equation and grey differential equation even more matched, so the modeling precision are greatly raised. Reference [5] did not use the accumulation, but directly set up the GM (1, 1), and presents the direct method that using the weighted average between the forward difference quotient and the backward difference quotient to replace the grey derivative. According to the principle of GM (1, 1), which regards a system as the exponential function which changes with the time variation, this paper presents a new method to establish the direct model through optimizing the grey derivative. The results of data simulation and model comparison show that the improved model in this paper improves the accuracy of background value, the fitting precision and forecasting precision in comparison with the original GM (1, 1) and reference [7]. So it has a high theoretical worth and the value of application.

II. OPTIMIZATION OF GREY DERIVATIVE IN GM (1, 1) BASED ON THE DISCRETE EXponential SEQUENCE

A. Optimization of Grey Derivative

Theorem 1 Let \( x^{(0)}(0) = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \) be the original data sequence, \( \alpha^{(1)} x^{(0)}(0) \) be the 1-IAGO series of \( x^{(0)} \). We suppose that \( x^{(0)} \) satisfies the exponential form \( y = x^{(0)}(t) = Be^{\alpha(k-1)} + C \), then \( x^{(0)} \) and \( \alpha^{(1)} x^{(0)}(0) \) have the same exponent.

Proof:
If the sequence \( x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \) satisfies the exponential form \( y = x^{(0)}(t) = Be^{\alpha(k-1)} + C \), that is \( x^{(0)}(k) = Be^{\alpha(k-1)} + C \), then
\[
\alpha^{(1)}(x^{(0)}(k)) = x^{(0)}(k) - x^{(0)}(k-1) = Be^{\alpha(k-1)} + C - (Be^{\alpha(k-2)} + C) = B(1 - e^{-\alpha})e^{\alpha(k-1)}.
\]
Let \( D = B(1 - e^{-\alpha}) \), we have \( \alpha^{(1)}(x^{(0)}(k)) = De^{\alpha(k-1)} \).

If \( \alpha^{(1)}(x^{(0)}(k)) = De^{\alpha(k-1)} \), then
\[
x^{(0)}(k) = \sum_{i=2}^{k} \alpha^{(1)}(x^{(0)}(i)) + x^{(0)}(1) = B(1 - e^{-\alpha})(e^{\alpha} + e^{2\alpha} + \cdots + e^{(k-1)\alpha}) + x^{(0)}(1) = Be^{\alpha(k-1)} + C.
\]
That is to say, \( x^{(0)} \) and \( \alpha^{(1)} x^{(0)}(0) \) have the same exponent.

The above theorem shows that the discrete exponential function and its 1-IAGO sequence have the same exponent.

Theorem 2 Let \( x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\} \) be the original data sequence,

i) If \( x^{(0)} \) satisfies the nonhomogenous exponential form \( x^{(0)}(k) = Be^{\alpha(k-1)} + C \), then \( \alpha^{(1)} x^{(0)}(k) = Be^{\alpha(k-1)} + C \), then \( A = \ln \left( \frac{\alpha^{(1)}(x^{(0)}(k))}{\alpha^{(1)} x^{(0)}(k-1)} \right) \).

\[ k \geq 3. \]
B. Modeling with the Optimized Grey Derivative

As is well known, the grey differential equation is
\[ z(k) = A(x^{(0)}(k) - C) \]
Put \( z(k) = A(x^{(0)}(k) - C) \) into (1), we have
\[ A(x^{(0)}(k) - C) + ax^{(0)}(k) = b, \]
and
\[ Ax^{(0)}(k) + ax^{(0)}(k) = b + AC. \]
Let \( b + AC = b' \) and \( Ax^{(0)}(k) = z(k) \), so we know
\[ z'(k) + ax^{(0)}(k) = b'. \]

Example 1: With regard to the standard exponential series \( x^{(0)}(k + 1) = e^{-ak} \), let \(-a = 0.1, 0.2, 0.4, 0.6, 0.8, 1, 0, 1.5, 2, 0.2, 2.5, 3, 0\), then we can get the original series \( x_i^{(0)} = \{x_i^{(0)}(1), \ldots, x_i^{(0)}(6)\}, k = 0, \ldots, 5 \). See table 1.

We use the model of paper [4] (M1), the model of paper [7] (M2) and this paper model (M3) to simulate and predict, and compare with [7]. See table 2.

From the table 2, it is obvious that according to the average relative error and the average absolute error of the model, the new GM (1, 1) in this paper is superior to other models. Actually, it does not have the model error, but the approximate calculation brings counting error.

Example 2: original series \( x^{(0)}(k) = [2.7180, 7.3883, 20.0835, 54.5925, 148.3978, 403.3870, 1096.5] \), this is a high growth series. We establish the model on the base of the former five data and leave the other two data to forecast, and then we obtain: \( a = -1.0000 \), \( b = 5.8141 \), and \( z(k) = [2.7180, 7.3883, 20.0835, 54.5926, 148.3981, 403.3878, 1096.5] \). The relative error and average relative error of the model refer to table 3.

From the table 3, we can find that the new model in this paper keeps high precision. Comparatively, the simulation and forecasting precision of the new model is higher than the model in paper [7] all the time, the simulation and forecasting precision of the new model is nearly reach 100%.

IV. Conclusions

One improved approach of grey derivative in GM (1, 1) is presented in this paper, and then we get a new GM (1,
1). The new model has been proven strictly to have the property of exponent, coefficient and translation constants superposition. The simulation results for both the standard exponential sequence and non-normal exponential sequence show that the new model has very high simulation and forecasting precision and the accuracy of background value, therefore the applicable scope has been expanded compared with the original model. It not only is suitable for the low growth sequence, but also is suitable for the high growth. What's more, it is suitable for nonhomogeneous exponential sequence. So it has a high theoretical worth and the value of application.

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REFERENCES


TABLE I  THE ORIGINAL SERIES

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1.5 The new model has been proven strictly to have the property of exponent, coefficient and translation constants superposition. The simulation results for both the standard exponential sequence and non-normal exponential sequence show that the new model has very high simulation and forecasting precision and the accuracy of background value, therefore the applicable scope has been expanded compared with the original model. It not only is suitable for the low growth sequence, but also is suitable for the high growth. What’s more, it is suitable for nonhomogeneous exponential sequence. So it has a high theoretical worth and the value of application.

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