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Mathematical model analysis and improvement of Lightning simulation

Liu Xiaojing[1], Deng Chang[1], and Liu Caihong[2]
[1] Jiaxing University, Zhejiang Jiaxing, P.R.China
Email: liuxiaojing99999@163.ocm
[2] Jiaxing University, Zhejiang Jiaxing, P.R.China
Email: liuch0001@163.com

***Abstract*—Due to the complexity of the physical mechanism of lightning and mathematical formula, Parameter range and large amount of calculation, at present the mathematical model of lightning simulation is always empirical or inconsistent with the physical properties of lightning. In order to bring down the complexity, and to advance physical properties, of the traditional lightning model the paper carries on the study of improving DBM model of lightning simulation. Firstly, the improvement of DBM model is researched. Secondly, the improved DBM model is applied to simulate a variety of effects images of lightning by computer. The research shows that the improved DBM model can cut time and space complexity, and save the workload of calculation.**

***Index Terms*—Lightning, Lightning process, Simulation, DBM model .**

I. INTRODUCTION

Lightning is a complex and a large-scale electrostatic natural phenomenon. As lightning (arc discharge) can bring so strong visual impacts that films and television industry have been studied it for long time. Lightning simulation has been the important part of the visual simulation[1]. Although the effect of lightning has been well known that it did not conduct an in-depth study earlier, especially from the physical characteristics. Most of them generated randomly lightning looks like a tree-shaped (linear or branched lightning) using the experience method[2].

Nowadays the lightning model generates firstly the main lightning in computer graphics, and then a lightning branch iteratively. Lightning simulation methods are usually based on the particle system which simulates lightning by the means of controlling every movement and changing of a particle. So the track of Particle movement constitutes Lightning. Currently in the field, most research has focused on the rendering algorithm; less research-generated model curve Lightning[3-4].

Although there are many different controversies and ambiguities for the physical mechanism in the formation of lightning, One thing is certain, like other nature, such as crystals, moss and other material varies with the whole phenomenon sharing the same fractal dimension, lightning (arc discharge) the fractal dimension is about 1.7D. These phenomena have a similar fractal nature known as the Laplace growth of the phenomenon. The following three models can achieve the growth of the Laplace phenomenon with different effect for different targets and focusing[5-6].

1) DLA (Diffusion Limited Aggregation)
2) HLCM (Hastings-Levite conformal mapping)
3) DBM (Dielectric Breakdown Model)

Although DLA model has certain simple physical meaning, it only can generate smaller-scale fractal structure than lightning (lightning has some lager-scale fractal structures); HLCM model, from the algorithm vision, is the most common, and has the closest relation to mathematics, but it is very difficult to elaborate its physical meaning; comparatively, the DBM model has both physical meaning and enough growth structure (lightning is actually a conductor transformed from insulator).

In addition, methods such as fractal geometry(lightning has the dimension of fractal geometry) 、L－Systems、Cellular of self-organizing、Poisson Growth Model 、filtering theory and so on are also simulation methods. According to research content, the paper focuses on the analysis and improves of DBM model.

II. DBM SIMULATION MODEL AND ITS INSUFFICIENCY

A. DBM simulation model

At present, although the Poisson growth algorithm and Laplacian growth algorithm are the most commonly modeling of lightning curve, for the physical meaning and Mathematics significance, DBM model is used to simulate the arc discharge in the form of the bifurcation appropriately with the following three key steps[7]:

1） According to boundary conditions and Laplace equation: $\nabla^2\phi = 0$ *, the signal of " ϕ " marking potential in Grid will be calculated.

And the equation- "$\nabla^2\phi = 0$" results from the differential form of gauss' theorem of electrostatic field: $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ ("ρ" signs the charge density;" ε_0 " presents the free-space permittivity) and $E = -\nabla\phi$ in an ideal medium hypothesis, namely ρ =0(no free charges).this equation is taken aim at calculate the potential of every point in space.

2） In accordance with the corresponding potential" ϕ ", "the growth spots (puncture spots)"of grid are selected.

3） The selected "growth spots (puncture spots)" are added to the boundary conditions.

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Through the three steps iteration operation above-mentioned, the track of lightning (the shape of arc discharge) is formed as follows:

As the Figure 1 shows: the initial boundary conditions in 2D plane are assumed: the red area presents $\phi = 0$ (corresponding to negative charge); the blue area marks $\phi = 1$ (equivalent to positive charge); the middle white box On behalf of potential is obtained by $\nabla^2 \phi = 0$ (according to the above boundary conditions given: $\phi = 0$ and $\phi = 1$).

$$\phi = \begin{cases} \phi = 0 & \text{negative charge in red region} \\ \phi = 1 & \text{positive charge in blue area} \end{cases}$$

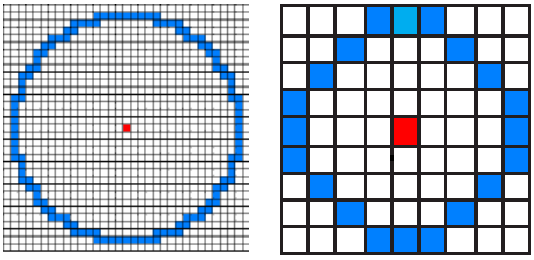

Figure 1

After potential "ϕ" is obtained, the next step is to choose "the growth spots (puncture spots)". Spots around negative charge are seen as the waiting puncture spots being given different weights by formula: $p(i) = \frac{(\phi_i)^\eta}{\sum_{j \in N} (\phi_j)^\eta}$ (in which, "N" being half of the set of all of growth spots; "η" is employed to control the fractal dimension of lightning shape). Then, one is selected from these waiting breakdown spots, and its potential will be amended as $\phi = 0$ (namely, the initial boundary conditions is modified), at the same time, $\phi = 0$ will be taken as the boundary conditions choosing breakdown spots for the next iteration.

B. The insufficiency of DBM model[7-8]

The deficiencies of DBM model in lightning simulation are as follows:

1） Potential as a scalar has not the actual physical meaning until zero potential surfaces in the reference model is considered. But the matter has not been considered in DBM model. Because the electric field must be strong enough to breakdown in physical, so a high Potential of a spot does not mean the large probability of being broken-down for it, in other words, the probability of a spot being punctured have no direct relationship to Potential.

2） In the above-mentioned model, large amount of computing time (at the same time a lot of memory space occupied) was spent in the first step. Moreover when a new waiting-puncture spot is joined into puncture spots set, it must be done that all the potential of spots in set is to be calculated by Laplace equation- $\nabla^2 \phi = 0$ under the new boundary conditions of new spot, and the process is complexity and spend much time.

3） The model is built on a wholly different (completely symmetric) plane of 2D. But the actual environment of lightning existing is impossible symmetrical completely, that is to say that some other obstacles are existing in a circular area with radius of 2 to 3 km around lightning, and the air density is uneven.

III. IMPROVING AND OPTIMIZING TO DBM MODEL

The steps to improve and optimize the above-mentioned insufficiency of DBM model are taken as follows.

1) firstly ,for the initial position of lightning occurring is usually at the bottom, with negative charge, of clouds, so a initial puncture spot with negative charge is selected from grid of the 2D plane.

Because of 95 percent of lightning statistically is occurred between clouds and ground accompanying charge moving from the cloud to earth, so the ground with positive charge and the bottom of clouds With negative charge are assumed as the actual situation for simulation. As shown in Figure 2, the gray region shows positive charge, and potential is zero ($\phi = 0$); the blue region indicates negative charge, and potential is $\phi = \phi_0 < 0$.

$$\phi = \begin{cases} \phi_0 & \text{in the blue region}(\ \phi_0 < 0) \\ 0 & \text{in the gray region} \end{cases}$$

2) As is shown in Figure 3 that spots around this spot discussed above are made as the candidate spots (waiting- puncture points).

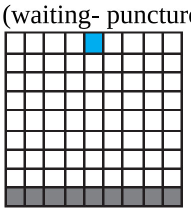

Figure 2

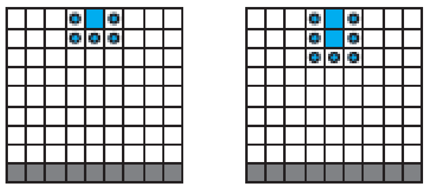

Figure 3

3) According the formula (1), the electric potential and electric field strength of each grid point is calculated.

$$\phi_i = \sum_{j \in N\ j \neq i} \frac{k}{r_{ij}} (k = 9 \times 10^9 m/f, k = \frac{1}{4\pi \varepsilon_0})$$

$$\underset{i \in N}{E} = \frac{\Delta \phi_i}{\Delta x_i} \qquad (1)$$

The main Parameters and applying of the formula are set forth as follows:

a） The signal of "N" means negative charge set (all puncture spots). "i" represents the set of all the candidate's spots in the surrounding of spots with negative charge. The potential of candidates spots will be calculated, then the field strength (field strength is the opposite of the number of potential gradient) of each of candidate's spots is known as $\underset{i\in N}{E} = \frac{\Delta \phi_i}{\Delta x_i}$ by equation of $E = -\nabla \phi$.

b) The formula (2) from grid will be used to calculate potential.

$$\phi_i = \sum_{j\in N\ j\neq i} \frac{k}{r_{ij}} (k = 9\times10^9 m/f, k = \frac{1}{4\pi \varepsilon_0}) \qquad (2)$$

To assume the space exists n-charges that are made up of q_1, $q_2 \cdots q_n$. and the force of any charge-q_j is forcing by all other charges, **on the basis of** Coulomb's theorem, obtaining from $\vec{F}_j = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_j q_i}{r_{ji}^3}\vec{r}_{ji}$ is named as a principle of superposition. For $\vec{F} = q_j \vec{E}(\vec{x})$ ($\vec{x}$ is the location vector of q_j; $\vec{F}(\vec{x})$ is some vector function for $\vec{x}$), so, according to Coulomb's theorem $\vec{E}(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i(\vec{x}-\vec{x}_i')}{|\vec{x}-\vec{x}_i'|^3}$ can be reduced to $\vec{E}(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{ji}^3}\vec{r}_{ji}$, then $\phi_i = \sum_{j\in N\ j\neq i}\frac{k}{r_{ij}}\times q_i$ by $\vec{E}(\vec{x}) = -\nabla\phi$. Here we use the unit charge, thus the Simplified formula of $\phi_i = \sum_{j\in N\ j\neq i} \frac{k}{r_{ij}} (k = 9\times10^9 m/f, k = \frac{1}{4\pi \varepsilon_0})$ is formed to calculate potential of each spots conveniently and dependably.

c) As mentioned above, a lot of time and space will cost for calculating electricity potential by Laplace equation.

Since the ground (conductor) has been supposed as zero-potential surface, here we can compute the electric potential-ϕ of each grid spot by the method of the plane mirror in electrostatic field.

The method of the plane mirror is based on the Gelin'equivalent layer theorem which guides that charges having potential on surface of conductor can be substituted by potential equivalent charges (image of charge) in conductor. That is to say that when the charge of a spot is regarded as the object and the earth (conductor) is seen as the mirror, then the sensor charge of conductor or the polarization charge of media will be taken as image. And the potential of object and image in field can be superposed. So, the calculation of electric potential can be transformed into the calculation of electric field superposition.

Figure 4 is formed by formula (3) under the method of the plane mirror. As shown in Figure.2-4 that each blue spot charge corresponds to a isometric heterogeneous charge (red spots in Figure.2-4). As a result, the potential of the field is getting from superposition of all these spot charges including positive and negative in the assuming of cloud with negative charges and earth being as the symmetric plane.

$$\phi_i = \sum_{j\in N\ j\neq i} k\frac{1}{r_{ij}} (k = 9\times10^9 m/f, k = \frac{1}{4\pi \varepsilon_0}) \qquad (3)$$

(In which, symbols of "N" and "i" are same to the above.)

The simulation of the charge distribution in the entire field is shown in Figure 4.

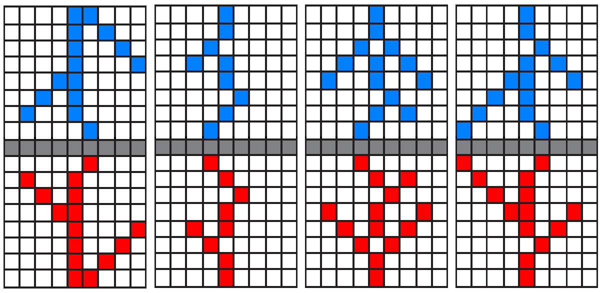

Figure 4

The Iterative algorithm for this simulation example is as follows:

1) According to the formula: $p(i) = \frac{(E_i)^\eta}{\sum_{j\in N}(E_j)^\eta}$, a "growth spot (puncture spot)" is selected. The probability method of selecting puncture spot is improved to make lightning curve vividly.

2) A new spot charge is added to the set of "growing point (puncture spot)".

3) As a result of a new puncture spot generating in each iteration operation that the potential and field strength of each candidate spot must to be recalculated by the formula(4).

$$\phi_i = \phi_i + \frac{k}{r_{i,j}} (k = 9\times10^9 m/f, k = \frac{1}{4\pi \varepsilon_0})$$

$$\underset{i\in N}{E} = \frac{\Delta\phi_i}{\Delta x_i} \qquad (4)$$

According to formula(4) and formula of Plane mirror method of Green's, potential of all candidate spots will be renown accompanying with each puncture spot added into.

4) At the same time the "new growth spots(puncture spots)" of choice, the spots around the spot charge, will be added to the candidate spots(waiting puncture spots).

5) Because new waiting puncture spots should to be added to the puncture spots, so these new spots and old puncture spots will be put together to carry on the operation of plane mirror, in light of superposition principle of electric potential and the formula(5).

$$\phi_i = \sum_{j\in N\ j\neq i} \frac{k}{r_{ij}} (k = 9\times10^9 m/f, k = \frac{1}{4\pi \varepsilon_0}) \qquad (5)$$

(in which, “j” represents the set of puncture spots; ”i” symbols all the new waiting puncture spots.)

6） the above iteration operations are repeating till the "growing spots (puncture spots)" reach the earth (namely, zero potential surfaces).

7） A set of "growing spots (puncture spots)" will be generating.

8） The spots set of lightning discharge trunk(key recursive of lighting) should to be find by backtracking in puncture spots, and the branch spots will be selected from the key recursive of lighting.

9） According to the found puncture spots, the lightning trunk and the branch curve are to be produced.

The above steps can be used to generate the backbone curve，at the same time, lightning branch curve can be obtaining from the way of modifying the above model. The process of simulating lightning is based on Microsoft Visual C + + 6.0 development environment by mean of OpenGL 、Object-oriented programming and visualization technology.

After the trunk and branch curve of lightning is drawn, and then plays up the curves of lightning which includes the color, glow, halo and dye effects. The effect of map is as shown in Figure 5 finally.

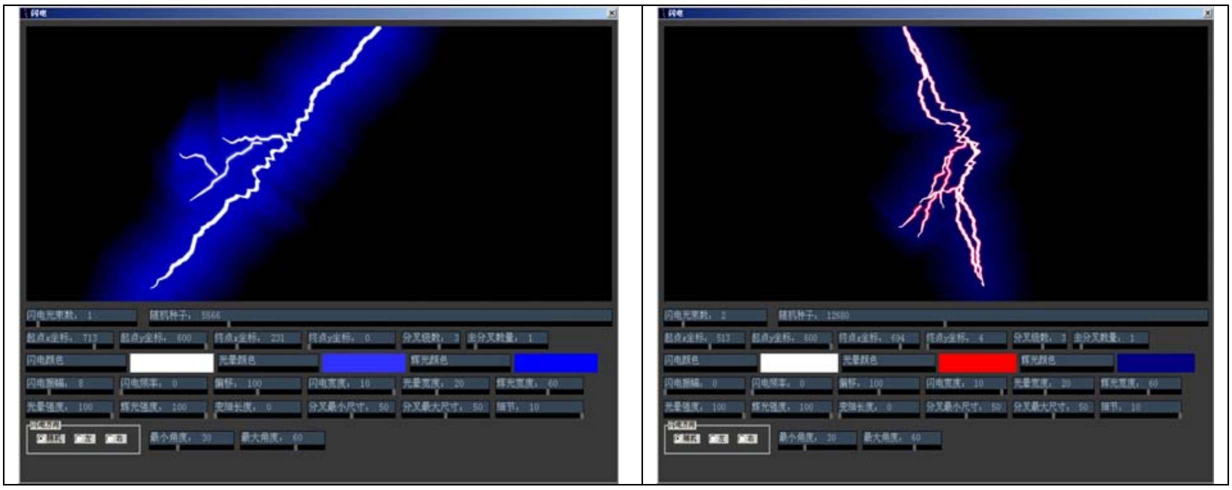

Figure 5

Ⅳ. Conclusion

From the view of meteorology or the perspective of computer graphics, the research on lighting computer simulation not only contributes to the realism of virtual world, but also to games, images, graphics and video work. A reasonable, visual, images realistic image of lightning can be f simulated **clearly** in the improved methods of DBM model. The results also show that the improved methods of DBM model have smaller spatial and temporal complexity, less calculation. But it did not consider that lightning hits the shape of objects such as factors. But the improved DBM model also runs short of only assumption the ground as being unity zero, in order to create a more realistic atmosphere the lightning hits the shape of objects, and other constitutes material factors in the model simulation should be considered, and the further study should be taken to make up for.

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