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Skeleton Extraction of Cerebrovascular Image Based on Topological Nodes

Jian Wu[1,2], Guangming Zhang[2], Jie Xia[2] and Zhiming Cui[1,2]
[1] JiangSu Province Support Software Engineering R&D Center for Modern Information Technology Application in Enterprise, Suzhou 215104, China;
[2] The Institute of Intelligent Information Processing and Application, Soochow University, Suzhou 215006, China
Email: szjianwu@163.com, szzmcui@suda.edu.cn

***Abstract*—Skeleton extraction is a very challenging subject, and has an important application value. Because of the ambiguity and complexity of cerebrovascular image, the skeleton gained by conventional skeleton algorithms is discontinuous. This paper proposes a cerebrovascular image skeleton extraction algorithm based on topological nodes. This algorithm first determines the important topological nodes from the starting point of the skeleton, then calls the extraction algorithm for a single skeleton from every topological node until the arrival of the source point. This algorithm avoids the locating and classifying of the skeleton connection points which guide the skeleton extraction. Because all its parameters are gotten by the analysis and reasoning, no artificial interference is needed.**

***Index Terms*—skeleton extraction, cerebrovascular image, topological nodes, Level Set model**

I. Introduction

Medial axial and skeleton are considered the presentation of compressed shape in the case of maintaining the topology information, and the goal of skeletonization is to reduce the dimension of shape. Zhou and Toga [1] proposed a pixel decoding technology, discrete wavefront spread over the entire object beginning with a hand-selected control point. Bitter [2] proposed a punishment distance algorithm to extract the skeleton line. Extraction of the medial axis uses the Dijkstra shortest path algorithm [3]. Bouix [4] extract medial axis using the Euclidean distance gradient vector field of the average outward flux across the Jordan curve to the border, and use calculation of the gradient of the distance map and a threshold to calculate the skeleton. Deschamps and Cohen [5] associate skeleton extraction with finding the shortest path. First slove short-term equation with fast marching method, and then follow the sudden drawdown between the two user-selected points, and finally find the shortest path.

The existing methods of skeleton extraction more or less have the following deficiencies: (1) the usage of different pruning techniques to extract skeleton from the mesial surface; (2) high computational complexity; (3) the need of manually select starting point of each skeleton; (4) lack of robustness; (5) noise-sensitive of the edges. This paper proposes a skeleton extraction algorithm using Level Set model, which can avoid the above shortcomings. The main idea is to capture the topological information of objects by spreading a wavefront with a medium velocity to with skeleton point (source point), and second spread high-velocity wavefront to start with these topology nodes, in which the skeleton point is the points with greatest curvature value on spreading peak surface. Through the solution of ordinary differential equations, the skeleton points can be identified.

II. Formula Expressions of Level Set

Fast Marching Method (FMM) is a technology using fully binary tree method to find the point of the least arrival time by sorting all the arrival time in narrow-band region, assuming that the pixel number of narrow band is N . Every step must be to do this, so the time complexity is $O(N\log_2 N)$.

In order to reduce the computational cost, Kim proposed an effective $O(N)$ Group Marching Method (GMM) [6]. Eikonal type equations are:

$$\left|\nabla T_\tau(x_\zeta, x)\right|^2 = 1 \big/ v^2(x) \tag{1}$$

Where $T_\tau(x_\zeta, x)$ denotes the time from x_ζ to x on curve, $v^2(x)$ denotes the velocity at the peak x.

GMM is to fine a group of points to move forward at the same time, rather than finding the point of least arrival time by sorting points of all solutions.

III. GMM High Speed Model

This paper uses a new cost function, that is, the lowest cost path between two central points is a skeleton line, which is defined as follows.

$$W(x) = e^{-\lambda\mu_n(x)} \qquad \lambda > 0 \tag{2}$$

Parameter λ is the control coefficient of peak surface crown on skeleton points, $\mu_n(x)$ is a function proportional to the normalized minimum distance domain to the edge of the target. In order to make algorithm realize automation without artificial interference, the following the analysis of λ and selection of intermediate function $\mu(x)$ is shown as follows.

A. Value Selection of λ

Consider the following figure.

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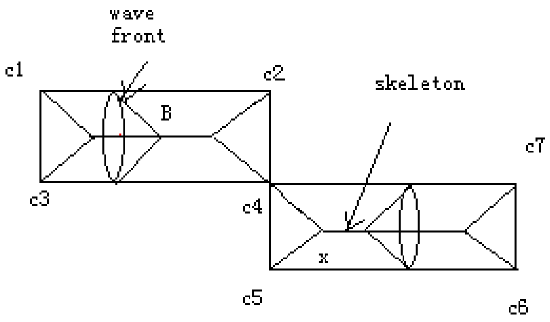


Figure 1. Skeleton and evolutive peak surface

Suppose the skeleton in Figure 1 crosses the center of B and x. Suppose G_i is non-central point. Supposing the source point P_s, from which a monotone-forward wavefront will spread along the normal direction. If the peak surface encounter skeleton points before encountering non-skeleton points (that is, peak spreads fastest at skeleton points); skeleton can be interpreted as the point with the largest curvature.

Since the energy function $w(x)$ is interactive with speed function, the skeleton points assign the lowest cost path, so the skeleton line between B and x is the lowest cost path [7]. This argument must be satisfied the following inequalities.

$$t_x = \frac{\nu(O,x)}{F(x)} < t_{G_i} = \frac{\nu(O,G_i)}{F(G_i)} \tag{3}$$

t_x and t_{G_i} are the arrival time when the wave arrive x and G_i, then,

$$\frac{\nu(O,x)}{\nu(O,G_i)} < \frac{F(x)}{F(G_i)} \tag{4}$$

ν is the euclidean distance, supposing function $F(x):\Omega \to R_+$

$$F(x) = \eta(\mu(x)) \tag{5}$$

$\mu(x):\Omega \to R_+$ is the intermediate function with heavier weight assigned to skeleton points, $\mu(x) > \mu(G_i)$。 Suppose

$$\mu(x) = \varpi\ , \quad \mu(G_i) = \varpi - \theta_i \tag{6}$$

h and θ_i are positive real number, θ_i donotes the absolute difference between two nearhood points. When

$$\nu(0,x) = \sqrt{\Delta^2 a + \Delta^2 b} \tag{7}$$

$$\nu(0,G_i) = \min(\Delta a, \Delta b) \tag{8}$$

Proportion $\nu(0,x)/\nu(0,G_i)$ is largest, function F must satisfy:

$$\frac{F(x)}{F(G_i)} > \frac{\sqrt{\Delta^2 a + \Delta^2 b}}{\min(\Delta a, \Delta b)} \tag{9}$$

Therefore,

$$\lambda > \frac{1}{\theta}\ln(d) = \frac{1}{\theta}\ln\left(\frac{\sqrt{\Delta^2 a + \Delta^2 b}}{\min(\Delta a, \Delta b)}\right) \tag{10}$$

is the necessary condition.

B. Selection of Intermediate Function $\mu(x)$

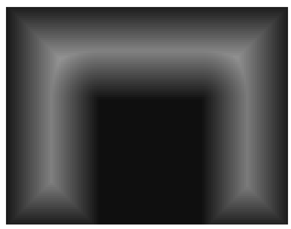
Minimize the following function:

$$E(Z) = \iiint \upsilon |\nabla Z|^2 + |\nabla f|^2 |Z - \nabla f|^2 dx \tag{11}$$

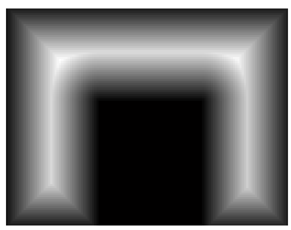
Obtain gradient vector flow (GVF) of the vector domain $Z(x)$. Where, $x = (a,b)$, υ is a regularization parameter. $g(x)$ is from the edge image of $I(x)$. Because boundary movement of GVF is very slow, $|Z|$ points to the center of objects, value of $|Z|$ is very small, therefore, the impact of $|Z|$ to distinguish between focal point and non-focal point is insufficient, as shown in Figure 2(b). Therefore, the intensity of the following middle function is controlled by the domain intensity r.

$$\mu(x) = 1 - \left(\frac{|Z(x)| - \min|Z|}{\max|Z| - \min|Z|}\right)^r \quad 0 < r < 1 \tag{12}$$

(a) a connected n-shaped graph



(b) function $Z(x)$



(c) intermediate function $\mu(x)$

Figure 2. Intermediate function of n-shaped graph

Figure 2(c) is the new intermediate function. Aiming at our problem, this paper does some improvements to GVF: $g(x) = I(x)$ .Vector domain point points to the center of object, the calculation of GVF was limited to the internal objects, so that the calculation is more effective than the original GVF, GVF domain does not maintain the properties of the middle function as the original GVF.

IV. Skeleton Extraction Using Level Set Model

In order to get the complete skeleton of object, we must first determine the important topological nodes from the starting point of the skeleton, call the extraction algorithm for a single skeleton at the beginning from every topological node until the arrival of the source points P_S or the extracted skeleton line, and prevent the appearing of overlapping paths.

A. Extraction of Topological Nodes

If an object can be denoted with a graph, its important topological nodes can be easily identified. Through the way of transforming the object into a graph, distinguish the topological nodes of the salient part. The generation of graph is controlled by the parameter χ .

We propose the following approach to extract the topological nodes automatically. Firstly, we compute the shortest distance domain $D(x)$ using the evolutionary from the boundary to the center with a medium-speed wave. Then we automatically select a skeleton point as the source point P_S , through which spreading an evolutive peak surface χ -surface with middle speed. The movement of the surface peak is controlled by the short-term distance equation, and its solution is a new distance domain $D_1(x)$. The velocity of the peak surface is given by the following formula.

$$F(x) = e^{\chi D(x)} \qquad \chi > 0 \tag{13}$$

B. Extraction of Single Skeleton Line

In order to extract skeleton line between the two skeleton points A and B , initialize the spreading time of A to zero, and then select the high-speed evolutive function of GMM to reach the B point. Finally, we backtrack from B to A along the ∇T . The extraction process is the solving the following ordinary differential equation:

$$\frac{dL}{dt} = -\frac{\nabla T}{|\nabla T|}, L(0) = B \tag{14}$$

Solving the equation (36) can depict the skeleton line with $L(t)$, the error is $O(h^2)$, h is the integration step. $G_i = [a_i, b_i]^T$, and

$$f(G_i) = -\frac{\nabla T(G_i)}{\|\nabla T(G_i)\|}, k_1 = hf(G_i) \tag{15}$$

$$G_{i+1} = G_i + hf(G_i + \frac{k_1}{2})) \tag{16}$$

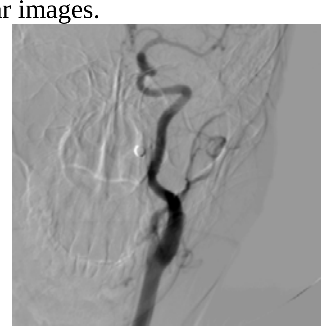
In order to ensure the connectivity of skeleton points, we choose $h = 0.1$.

V. Experimental Result and Analysis

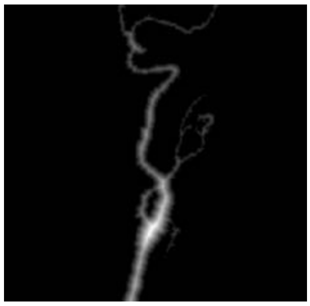
A. Experimental Effect Figure

Use the algorithm proposed in this paper to extract skeleton from a number of cerebrovascular images, and the extraction effect is shown as follows.

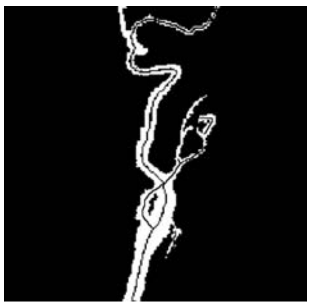
Figure 3 is the skeleton extraction of two-dimensional cerebrovascular images.



(a) the original two-dimensional cerebrovascular image



(b) intermediate function $\mu(x)$



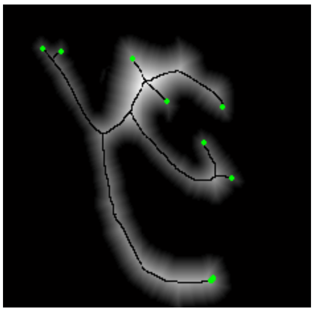
(c) the skeleton of two-dimensional cerebrovascular image

Figure 3. The skeleton extraction example of two-dimensional cerebrovascular image

Figure 4 is the skeleton extraction of three-dimensional cerebrovascular images.

(a) the original three-dimensional cerebrovascular image

(b) intermediate function $\mu(x)$



(c) the skeleton of three-dimensional cerebrovascular image

Figure 4. The skeleton extraction example of three-dimensional cerebrovascular image

B. Time Complexity Analysis

The core of algorithm is GMM, and this model can be used to calculate all the distance domains. The time complexity of calculating n points is $O(N \log_2 N)$, so the algorithm is efficient. If there is no ring-structure in the object, its time complexity is $O(3n \log n)$ in the worst case, otherwise it is $O((3+k)n \log n)$, and k is the number of rings.

VI. CONCLUSION

This paper proposes a skeleton extraction algorithm using Level Set model. The skeleton line points are automatically selected by the global maximum of Euclidean distance to the border, as the source point P_s. Firstly, the source point P_s spreads a middle-speed wave to scan the individual domain, and extract the topological information of the target. Afterwards, spread new peak surface from the topological nodes, and the spreading velocity of peak surface at the skeleton points is faster than the non-skeleton points. At this time, the skeleton points intersect with the evolutive peak surface at the point of maximum positive curvature. The skeleton of the target can be obtained by tracking from each topological node to the arrival of source point P_s, and use the efficient numerical solution to solve ordinary differential equation. In this paper, the time complexity of the algorithm is small, which is suitable for dealing with the target object with complex topological structure, and satisfies the characteristics of skeleton, that is, in the middle of the target, continuous, single-pixel width, and not sensitive to boundary noise. In addition, LSG consisting of extracted paths doesn't increase extra overhead.

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