Simulation of weak signal detection based on stochastic resonance

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Abstract—The paper introduces the basic theory of stochastic resonance, and through MATLAB simulation study stochastic resonance of nonlinear bi-stable system to detect weak signal with high noise background. Simulation results show that stochastic resonance based on weak signal detection theory is very effective. Most of noise energy transfer to the measured signal after past the nonlinear bi-stable system, so we can detect the weak signal easily. Here we will envisage using this theory to detect signals of gas concentration in mine.

Index Terms—stochastic resonance; weak signal detect; nonlinear bistable system

I. INTRODUCTION

With the development of science and technology, more and more fields have been involving the detection and treatment of weak signal. Such as communication, mechanical fault diagnosis, pathological diagnosis of human organs and so on [3, 8]. Noise is everywhere, however, those traditional methods look so powerless to remove noise form signals. Here we will introduce a more effective method of dealing with weak signals, this method is based on stochastic resonance. The term SR first appeared in1981 [2],when it was proposed as a plausible mechanism for almost periodic occurrences (approximately every 100,000 years) of ice ages on earth during the last 700,000 years. The birth of SR as an experimentally controlled physical phenomenon occurred in 1983[1], after its first Laboratory SR has grown into a rapidly developing, interdisciplinary field of research, with numerous experimental manifestations in biological, laser, electronic, quantum and other systems[5,7]. Many theoretical proposals are still awaiting their experimental verification.

II. PRINCIPLE AND SYSTEM MODEL OF SR

In the bistable nonlinear system, there are three basic conditions of stochastic resonance which are the bistable nonlinear system, weak input signals and noises. In order to realize stochastic resonance which similar to the mechanical resonance conducts, they must achieve the appropriate matching relation on adiabatic conditions[4] (a∈(1, f0), A∈D).Intuitively the idea of SR does not make a great deal of sense, since a signal’s quality would be expected to deteriorate as random noise is added; this is true for linear systems. However, for nonlinear systems

with an input signal which has been cut off in some way the random noise added to the input signal can actually improve the corrupted signal giving a better quality output signal than the input signal. In other words the response of a nonlinear system to a weak input signal is optimized by the presence of a particular non-zero level of noise; or in lay-mans terms, stochastic resonance enables a corrupt or limited signal to become more apparent by adding randomly fluctuating noise to the input signal. The reason why stochastic resonance affects signals in this way can be explained relatively simply if a thresh-old crossing system is used as an example. Assume that a weak signal is not powerful enough to trigger a threshold and noise is then added to it, then the extra little random movements, caused by the noise, can boost the signal enough to exceed the threshold value and hence trigger the threshold. (see Figure 2)This is the basic principle of stochastic resonance and the phenomenon has been widely studied in recent years (see Figure 1).

A. Langevin Equation

Using langevin equation (LE) describes the simplest nonlinear Bi-stable system:

\[ \dot{\chi} = a\chi - b\chi^3 + u(t) + \Gamma(t) \]

In this equation

\[ u(t) = A\cos(2\pi f_0 t) \]

is the input signal, \( A \) is the signal amplitude, \( f_0 \) is the signal frequency. There:

\[ \dot{\Gamma}(t) = 0, \quad \dot{\Gamma}(t-\tau) = 2\delta(t-\tau), \quad \Gamma(t) \]

has nothing to do with random variable force. This kind of noise is called additive noise. The stochastic force is changed with \( \chi \)

\[ \dot{\chi} = a\chi - b\chi^3 + u(t)\Gamma(\chi) \]

This is a multiplicative noise. External noise is usually presented by multiplicative noise, and it has a great influence on system.

Figure 1   System model of Stochastic Resonance
B. Introduction Double-well Potential Equation

Sketch of the double-well potential

\[ v(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 \quad (a > 0, b > 0) \]

The model of the non-linear bistable stochastic resonance system is described in figure 2. In the figure we can see the minima are located at the position \( x = \pm \sqrt{\frac{a}{b}} \). These are separated by a potential barrier with the height given by \( \Delta v = \frac{a^2}{4b} \). This barrier top is located at \( x_0 = 0 \). If we apply a weak periodic forcing to the particle, the double well potential is asymmetrically up and down, periodically raising and lowering the potential barrier. When the signal and noise synergy occurred, the stress of the particles become larger beyond the force potential well, then stochastic resonance occurred.

C. Signal noise ratio (SNR)

Stochastic resonance can be envisioned as a particular problem of signal extraction from background noise. It is quite natural that a number of authors tried to characterize the problem of signal extraction from background noise. It is the performance of the system. Under the adiabatic condition we can get non-linear bistable system output SNR[4, 6].

The signal-to-noise ratio is given by:

\[ \text{SNR} = \frac{2\sigma^2}{4(bD)^2} \left[ 1 - \frac{d^2}{} \frac{(4\pi^2 b D^2)^2 - \frac{a^2}{4b}}{} + \Omega^2 \right]^{-1} \]

Previously, most of the theory and experimental works focused on analyzing and measuring SNR. Many other interesting features arising in the stochastic resonance problem are ignored; among these the behavior of the output noise is the most interesting. In this paper we will experimentally study the characteristic features of the output signal and noise, especially the output noise, and compare the experimental results with the theoretical predictions.

III. BASED ON THE STOCHASTIC RESONANCE DETECTION OF WEAK SIGNALS AND NUMERICAL ANALYSIS

LE is a special kind of differential equation, we use high precision fourth-order Runge-Kutta algorithm to solve the differential equations. In this chapter we use the MATLAB function Runge-Kutta algorithm to do simulation. Here is fourth-order Runge-Kutta algorithm introduction:

\[ \begin{align*}
  x &= ax - bx^3 + u(t) \\
  u(t) &= s(t) + n(t) = A \cos(2\pi f t) + \tau(t)
\end{align*} \]

Then \( x_{n+1} = x_n + \frac{1}{6} \left( k_1 + k_2 + 2k_3 + 2k_4 + k_5 \right) \)

where \( k_1, k_2, k_3, k_4 \) is given by:

\[ \begin{align*}
  k_1 &= h(ax_n - bx_n^3 + u_n) \\
  k_2 &= h[a(x_n + \frac{k_1}{2}) - b(x_n + \frac{k_1}{2})^3 + u_{n+1}] \\
  k_3 &= h[a(x_n + \frac{k_2}{2}) - b(x_n + \frac{k_2}{2})^3 + u_{n+1}] \\
  k_4 &= h[a(x_n + k_3) - b(x_n + k_3)^3 + u_{n+1}]
\end{align*} \]

In those equations \( x_n \) and \( u_n \) is the sampling of \( x(t) \) and \( u(t) \), \( h \) is the iteration step length. Normally, we take \( h \) equal to \( \frac{1}{f_n} \). Experiment shows that change the potential parameter \( a \) (actually reducing barrier height, so that the signal can achieve easier to resonance) and reasonable choice of \( h \) all can optimize the results of simulation.

In this part we make \( a = b = 1 \). In order to make system reach stochastic resonance, we adjust the signal amplitude and noise intensity under the condition of keeping \( a \) and \( b \) unchanged[3]. The weak periodic signal amplitude \( A = 0.4V \), frequency \( f = 0.01Hz \). Sampling frequency \( fs = 5Hz \). The noise intensity is \( D = 0.6V \). There is the simulink result:

The first and most important feature of the amplitude is that it depends on the noise strength \( D \), the periodic response of the system can be manipulated by changing the noise level. The result is changed with the noise strength \( D \). From the MATLAB simulation result we can see that the weak periodic signal amplitude is 0.4V, and its spectrum diagram is shown in Figure 3. And in the figure 4 we added the weak signal with strong noise. Then, in the figure 2, we can see that the weak signal is submerged in noise. On this condition, obviously, it is very difficult for us to separate signal from noise. But we adjusted the system parameters to reach a state of stochastic resonance to detect the signal, after through the stochastic resonance system, output signal power spectrum are shown in Figure 6. Noise energy transferred to the measured signal after past the nonlinear bi-stable system. Signal amplitude have changed from 0.4 to 1.0. So we can easy detect the weak signal.
IV. CONCLUSION

From the simulation results, we can see that adjusting the system parameters to reach a state of stochastic resonance to detect the signal. After through the stochastic resonance system, output signal power spectrum are shown in Figure 6. We can see in this figure that the power spectrum of the signal stand out in 0.01Hz. Most of noise energy transfer to the measured signal after past the nonlinear bi-stable system, signal amplitude have changed from 0.4 to 1.0, enlarged the weak periodic signals and also inhibited the noise.

Recently the frequently happened accidents of gas explosion makes us have to think how to prevent gas explosion. Mine need to monitor the gas concentration, CO concentration, wind speed, etc. These messages all affect the mine's staff safety. A series of mine accidents trembled people's hearts. We can make our efforts to do something let the workers work in a safe and reliable environment. Next I will make further study on theory of stochastic resonance and how to apply it in the gas concentrations detection.

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