An Approach of time-delay Switch Control for CSC Inventory System

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Abstract— The application of switch control methodology in complex system such as industry of machineries, electronics is widespread. In this paper the robust control system with time-delay based on switch theory (STDRCS) is presented to manage the inventory of cluster supply chains (CSC). The hybrid control approach might be effectively used to overcome model uncertainties and external disturbance. And the robust inventory control strategy is aimed to find the optimal decision variables to weaken the bullwhip effect in cluster supply chains. It is proved that this method is more effective and efficient to tune the controller in face of changing environment.

Index Terms— inventory management; robust inventory; online switched system; cluster supply chains

I. INTRODUCTION

Improved operation of supply chains for manufactured goods is worth billions of dollars to the national economy; effective inventory management plays an important role in this regard. The use of optimization techniques in the management of supply/demand networks began with the development of the classical economic order quantity approach. Later developments include approaches for determining optimal stock levels in “order-up-to” policies. Cluster supply chain (CSC), different from traditional single-chain supply chain, is located in the industrial cluster region, with the relation of “supply-client”, through the link of formal or informal contract of ‘trust and commitment’, formed by organizations containing different firms of the same industry such as research organizations, supplier, manufacturer, wholesaler, retailer, and even end users. Cluster supply chain system is made of a couple of paralleled single supply chains in the agglomeration location, not only do all enterprises in one single supply chain cooperate one another internally, but cooperation and coordination exist across different single supply chains externally as well[1].

Nowadays, numerous literatures have been devoted to study of inventory control models in supply chain, and made fruitful progress. However, few of these involve cross-chain inventory management which objectively exists among cluster supply chain in many industrial cluster locations[2]. In addition, dynamic uncertain environments and integrated control for complex systems require developing a model of combining inventory management and manufacturing process[2][3]. So, the switched robust control under time-delay (STDRCS), one of hybrid controls, is introduced to this paper for establishing the model, therefore, we focus on across-chain inventory in cluster supply chains to analyze its bullwhip effect and approach of implementing it. The following parts are arranged in such way: section II briefly describes switched robust control under time-delay. The section III explore the hybrid optimization and decision approaches. At last, an example is used to show the solving procedure in section IV.

II. SWITCHED ROBUST CONTROL WITH TIME-DELAY

Switch Robust Control with time-delay (STDRCS) is a robust control method controlling system structure uncertainty. The switched controllers considered in this paper are based on the state-space model below:

\[
\dot{x}(k) = A_i \dot{x}(k) + \hat{A}_i \dot{x}(k-h) + B_i \omega(k) + B_0 \theta(k) \quad (1)
\]

Where, \( i \in I, I = \{1,2,\ldots,N\} \) is switched rules, \( A_i \in R^n, \hat{A}_i \in R^n, \) \( h \) is the unknown time-delay integer constant, \( k \geq h \), \( \dot{x}(k-h) \) express unreached goods ordered before the \( k \) period. \( \dot{x}(\cdot) \in R^n \) is state vector, \( \omega(\cdot) \in R^n \) is external disturbance.

Given the general definition of quadratic cost function

\[
J = \sum_{k=0}^{\infty} \left[ \dot{x}^T(k)Q \dot{x}(k) + \dot{u}^T(k)R \dot{u}(k) \right] \quad (2)
\]

Where, \( 0 < Q = Q^T \in R^{n \times n} \) and \( 0 \preceq R = R^T \in R^{n \times n} \) are respectively state and control weight matrix.

Define the subsidiary output sign as the following

\[
z(k) = C \hat{x}(k) + D \hat{u}(k) \quad (3)
\]

Where, \( C = [Q^{1/2} \ 0]^T, \ D = [0 \ R^{1/2}]^T \), then the quadratic index (2) can be represented as

\[
J = \sum_{k=0}^{\infty} z^T(k)z(k) = \|z\|^2 \quad (4)
\]

For the uncertain discrete system (1) and (3), introducing the state feedback control as

\[
\hat{u}(k) = K_{ij} \hat{x}(k) + K_{ij} \hat{x}(k-h) \quad (5)
\]

Accordingly, the closed-loop system may be represented:

\[
\left\{ \begin{array}{l}
\dot{x}(k+1) = \sum_{i=1}^{N} \theta_i(A_i \dot{x}(k) + (\hat{A}_i + B_i \omega(k)) \dot{x}(k-h) + B_0 \theta(k) \\
\hat{x}(k) = (C + DK_{ij}) \hat{x}(k) + DK_{ij} \hat{x}(k-h) 
\end{array} \right. \quad (6)
\]

In above, the SDRC controller selects the input \( u(k) \) by solving the following optimization problem.

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\[ J = \min \left( \sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 \|w\|_2^2 \right) \]  

(7)

Definition: for time-delay switched system (6), if the selected Lyapunov function \( V(\hat{x}(k)) \) along the difference of system (6) satisfy \( \Delta V(\hat{x}(k)) < 0 \), then the system is robust quadratically stable

**Theorem 1** for time-delay switched system (5), and given constant \( \gamma \), If there existing positive definite matrix \( W_i \) and \( V \), existing matrix \( G_{i1} \) and \( G_{i2} \), thus \( \forall (i, j) \in I = \{1, 2, \ldots, N\} \)

\[
\begin{bmatrix}
-A & W_i & 0 & 0 & 0 & 0 \\
W_i^T & -W_i & 0 & 0 & (AW_i + B_G)^T & (CW_i + DG_i)^T \\
0 & 0 & -2W_i & 0 & (\hat{A}W_i + B_G)^T & (\hat{D}W_i + DG_i)^T \\
0 & 0 & 0 & -2I & 0 & 0 \\
0 & (AW_i + B_G) & (\hat{A}W_i + B_G) & B_i & -W_i & 0 \\
0 & (CW_i + DG_i) & DG_i & 0 & 0 & -I
\end{bmatrix} < 0
\]

(8)

then the existing system state-setback control law with memory (formula (5)), \( K_i = G_iW_i^{-1}, K_{i2} = G_iW_i^{-1} \) and for arbitrary switched signal, the closed-loop system has the H-performance \( \gamma \).

**Proof:** for subsystem \( i \), defy Lyapunov function as

\[ V(k) = \hat{x}^T(k)P_i\hat{x}(k) + \sum_{r=1}^{k} \hat{x}^T(r - \tau)\Gamma\hat{x}(r - \tau) \]

where, \( P_i, \Gamma \) is positive definite matrix, then \( V(k) \) is positive definite function, By definition and formula (5), the sufficient condition that the closed-loop system (6) is robust quadratically stable and having the H-performance \( \gamma \). Thus

\[
\dot{V}(k+1) - V(k) + 2z^T(k)\dot{z}(k) - \gamma^2 \alpha^T(k)\alpha(k)
\]

\[ = \dot{z}(k)\dot{z}(k) - z(k)^T\Gamma z(k) - \gamma^2 \alpha^T(k)\alpha(k) \]

\[ = \begin{bmatrix} \dot{\alpha}(k) \\ \dot{\alpha}(k) \\ \dot{\alpha}(k) \end{bmatrix} - \begin{bmatrix} -P + I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix} \begin{bmatrix} C + DK_i \\ DG^T + D_K \\ D_K \end{bmatrix} \begin{bmatrix} \dot{\alpha}(k) \\ \dot{\alpha}(k) \end{bmatrix} < 0 \]

By the Shur performance of matrix, existing

\[
\begin{bmatrix}
-P + I & 0 & 0 & (A + B_K)^T & (C + DK_i) \\
0 & -I & 0 & (A + B_K)^T & (D_K)^T \\
0 & 0 & -I & 0 & 0 \\
(A + B_K) & (\hat{A} + B_K) & B_i & -W_i & 0 \\
C + DK_i & D_K & 0 & 0 & -I
\end{bmatrix} < 0
\]

(9)

multiply the above formula from left and right separately with matrix \( \text{diag}(\{P_i^{-1}, P_i^{-1}, I, I, I\}) \), and let \( W_i = P_i^{-1}, W_f = P_f^{-1}, V = I^{-1} \), then existing

\[
\begin{bmatrix}
-W_fW_i & 0 & 0 & W_i(A + B_K)^T & W_i(C + DK_i) \\
0 & -W_f & 0 & W_i(A + B_K)^T & W_i(D_K)^T \\
0 & 0 & -2I & 0 & 0 \\
W_i(A + B_K) & W_i(A + B_K) & B_i & -W_f & 0 \\
W_i(D_K)W_i & D_K & 0 & 0 & -I
\end{bmatrix} < 0
\]

(10)

For \( V > 0 \), there is \((V - W_f)^T(V - W_f) \geq 0\), therefore there is \( +W_f^{-1}W_f \geq 2W_f - W_f \). So there exists

\[
\begin{bmatrix}
-W_fW_i & 0 & 0 & W_i(A + B_K)^T & W_i(C + DK_i) \\
0 & -W_f & 0 & W_i(A + B_K)^T & W_i(D_K)^T \\
0 & 0 & -2I & 0 & 0 \\
W_i(A + B_K) & W_i(A + B_K) & B_i & -W_f & 0 \\
W_i(D_K)W_i & D_K & 0 & 0 & -I
\end{bmatrix} \leq 0
\]

In the above formulation, if the right matrix is positive definite, then the left one must be passive definite. So for arbitrary switched signal, the right matrix is a sufficient condition of robust quadratic stability and meeting H-performance \( \gamma \) for system (6). By Schur performance of matrix, the following formula can be made.

Thus

Let \( G_i = K_iW_i, G_{i2} = K_{i2}W_i \), then formula (8) can be obtained. Then the theorem is proved.

**III. INVENTORY CONTROL IN CLUSTER SUPPLY CHAINS**

The inventory system of cluster supply chain in this section is composed of two single-chain supply chains which encompass one manufacturer and one retailer (showed in fig.1) and manufacture character-equal substitutable product.

Suppose \( x_1, x_2 \) represent inventory level of retailer and manufacturer in SC1 respectively, \( x_1, x_2 \) represent inventory level of retailer and manufacturer in SC2 respectively. Suppose \( a_1, a_2 \) represent order of retailer and manufacturer in SC1 respectively, whereas, \( a_1, a_2 \) represent order in SC2 respectively. \( \xi, \xi \) represent the market demand of SC1 and SC2.
always maintains small level in the system; when the uncertain demand from customer of retailer 2 increases sharply and suddenly, then the retailer 2 may transship inventory to retailer 1 for its emergent need, the supply quantity as \(\hat{a}_x(0 < a \leq 1)\); when the uncertain demand from retailer to customer 2 increases sharply and suddenly, vice versa, the supply quantity as \(\hat{b}_x(0 < b \leq 1)\).

Thus, regarding the inventory state as the state variable, and order (control variable) are parts of being certain and uncertain ones:

\[
\hat{\xi}(k) = d(k) + \omega(k)
\]

Thus, in seek of demand disturbance, the inventory model may be defined as

\[
x(k + 1) = A_x x(k) + \hat{B}_x \hat{\xi}(k) + B_x u(k)
\]

or

\[
x(k + 1) = A_x x(k) + \hat{B}_x A_x x(k) + B_x u(k)
\]

The demand from the customer is divided into two parts of being certain and uncertain ones:

\[
\hat{\xi}(k) = d(k) + \omega(k)
\]

The switched rules are set as

\[
\begin{cases}
a = 0, 0 < b \leq 1 \quad \text{when } \omega_{2x} > 2S_{2i} & \text{and } \omega_{3x} < 2S_{3i} \\
\hat{x}_{1x} > S_{1i} & \text{and } \hat{x}_{3x} > S_{3i} \\
S = \begin{cases}
0 = 0, 0 < a \leq 1 \quad \text{when } \omega_{2x} > 2S_{2i} & \text{and } \omega_{3x} < 2S_{3i} \\
\hat{x}_{1x} > S_{1i} & \text{and } \hat{x}_{3x} > S_{3i}
\end{cases}
\end{cases}
\]

Where, \(S_{1i}, S_{2i}, S_{3i}\) are respectively secure inventory level of retailers in SC1 and SC2. The system satisfies \(a \neq 0\) at any time, that is to say, the transshipment between the retailers may not happen, but they cannot mutually replenish at the same time.

The bullwhip effect is described as the proportion of the sum of inventory and order fluctuation to the terminal demand fluctuation, the definition may be showed in [4] namely

\[
r_i = (\hat{x}_1(k)^T Q \hat{x}_1 + (\hat{u}_1(k)^T R \hat{u}_1) / (\omega_1(k)^T S \omega_1)
\]

Where \(Q, R, S\) are set symmetrical positively definite weighted matrix. The parameter \(r_i\) describes bullwhip effect in cluster supply chains. The bullwhip effect becomes stronger with increase in \(r_i\) while weaker with decrease in \(r_i\).

For some external disturbance \(\omega(k)\), if the controlled output \(z(k)\) always maintains small level in the system, then the system with such index present “better” performance. In this case, the controlled output is less influenced by both external disturbance, and the capacity of restraining disturbance in the system appears stronger.

Thereby, the solver satisfying performance index (EQ. 8) surely guarantees minimizing bullwhip effect. The solving process of robust controller may be obtained with given model in section 2. However, how to switch among multiple modes must depend on one online decision-making system which will be introduced in section 4.

IV. SIMULATION TEST CASE

Two-echelon cluster supply chains are taken as instance in fig.1 and the switched robust inventory model (EQ. 1 and EQ.3) is given in the former section.

Suppose the standard system of cluster supply chains are

\[
x_* = [1.2, 1.3, 1.5, 1.55]'kton,
\]

\[
u_* = [1.2, 1.35, 1.6, 1.7]'kton
\]

And suppose the initial values of order error are zero, but the initial values of stock error are

\[
\hat{x}_* = [-0.1, 0.1, 0.35, 0.1]'kton
\]

The demand disturbances in the two supply chains are random. Suppose the system face demand disturbance shown in Fig.3. The retailer in SC1 has larger need at period of \(k = 0\) and \(k = 15\) (namely, \(a = 0, \omega_3 > 2S_{3i}\)). The other supply chain will provide emergent inventory supply for the retailer in SC1 by contract when meeting the condition of inventory transshipment. Suppose the supply ratio is \(a = 0.8\). The supply chain 2 faces larger demand fluctuation at \(k = 14\) in Fig. 3, but the across-chain transshipment condition is not met (namely, \(\omega_3 < 2S_{3i}\)), so supply chain 2 replenishes inventory only by single channel.

The subsystem 1 is set as system without across-chain inventory cooperation, the according parameters are

\[
A_1 = \begin{bmatrix}
0.8 & 0 & 0 & 0 \\
0 & 0.8 & 0 & 0 \\
0 & 0 & 0.8 & 0 \\
0 & 0 & 0 & 0.8 \\
\end{bmatrix}, \quad \hat{A}_1 = \begin{bmatrix}
0.2 & 0 & 0 & 0 \\
0 & 0.2 & 0 & 0 \\
0 & 0 & 0.2 & 0 \\
0 & 0 & 0 & 0.2 \\
\end{bmatrix}
\]

The subsystem 2 is set as system with across-chain inventory cooperation (the retailer of SC2 supplies inventory for retailer of SC1), the according parameters are
\[
A_2 = \begin{bmatrix}
0.8 & 0 & 0.8 & 0 \\
0 & 0.8 & 0 & 0 \\
0 & 0 & 0 & 0.8 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 1
\end{bmatrix},
\]

\[\tilde{A}_1 = \tilde{A}_2, \tilde{A}_3 = A_3.\]

\[B_1 = \text{diag}([-1.0, -1.0]), \text{suppose } Q = R = S = I_4.\]

The system is switched to mode 2 (subsystem 2 is determined) in periods of \(k = 1, k = 16\), while mode 1 in other periods by results of online decision-making module. Set \(\gamma = 3\), according to the condition that the system has certain performance of restraining disturbance and must satisfy robust stability (EQ. 8, 9).

In supply chain, the inventory fluctuation and order fluctuation at every tier can only maintain stable within a small scope in seek of demand disturbance. The simulation and computing are carried out by emulating online decision in this section, given the real demand disturbance shown in Fig. 3. In order to analyze the different varying trend of related parameters between considering time-delay and without considering time-delay, the system made simulation and computing separately in the two situations (Fig.4).

![Fig. 3 Demand disturbance in CSC](image)

![Fig. 4 Variation tendency of bullwhip effect](image)

It is shown from Fig.4 that the system may tend to be stable within a small scope by exerting \(H_\infty\) control with time-delay considered, while the effect is better when existing across-chain inventory cooperation. One evident reason for this result is that the order fluctuation is largely dwindled by across-chain inventory cooperation, thus the bullwhip effect can be better weakened.

Additionally, it is proved by simulation in this section that the online decision-making system of cluster supply chain with time-delay consideration can optimize the whole system in multiple periods, and then take on quicker response to complex market.

It is also shown from the figures that the varying trend of some variables in the system with time-delay consideration is inferior slightly to that of without time-delay because the system decision was conservative for time-delay. On the other hand, the transship quantity across chains is less than that of none time-delay, because the decision is based on the current inventory fluctuation and without stock in transit considered. However, the phenomenon of time-delay is objectively existing and the system with time-delay consideration conforms to reality. Thus it is necessary that the across-chain transshipment problem of stock with time-delay consideration is discussed in multiple views.

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