Design and Optimization of Reentry Trajectory of Maneuverable Warhead

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Abstract—A strike scheme is presented by restricting the entry velocity, flight path angle and introducing the conception of the best attack line. Based on this scheme, a design method of reentry trajectory is proposed when the earth is assumed to be non-rotating. The method is based on the reentry with maximum left-to-drag, but a small attack angle flight phase and a terminal adjust flight phase are included in. A feasible reentry trajectory with two or three skips can be get by adjust the small attack angle flight time. Based on this trajectory, a standard trajectory considering earth rotation and practical parameters on entry point.

Index Terms—Maneuverable Warhead, strike scheme, reentry trajectory

I. INTRODUCTION

With the development of the anti-missile system, the existing probability of the ballistic missile is gradually decreased[1] as for its long duration of the booster phase and inertia flight phase. Comparatively speaking, the maneuvering reentry vehicle(MaRV) has its brilliant combat capability. MaRV could be launched as space operation vehicle(SOV) from the near surface trajectory or launched as a small sized carrier missile from ground and guide it to a sub-orbital trajectory. MARV flies into the dense atmosphere by inert, afterwards depending on its own high lift and resistance ratio gliding maneuvering to the designated position and release its military carriers. The anti-surface flight process of MaRV launched from SOV could be seen from Fig. 1.

II. REENTRY ATTACKING PROGRAM

From the MaRV operation process, one can figure out that the reentry point is the connecting point between the free flight phase and reentry phase, at the departure point applied different departure speed gain Δν in order to obtain different transfer trajectory, corresponding to the reentry point parameter, and affecting the gliding maneuverability at the layers of atmosphere. This article mainly discusses the simplification of the reentry strike scheme by way of limiting the parameter of reentry point.

A. parameter of reentry point on the supposition that the earth is static

The magnitude of the reentry point speed νₑ and the track bearing γₑ has great affect to the reentry flight of the MaRV, similarly, the max available departure speed gain Δν_max of MaRV is fix, the value of this two parameter has certain limitations. Reference[4]take the reentry angle within the range of [2° ~ 6°] while doing the attacking simulation. While reference[5] pointed out that via simulation the best reentry angle should limit to the range of [5.15° ~ 6°]. This article will tries to determine the reentry point parameter by means of the following simulation methods. Considering the reentry attacking process regardless of the change of the trajectory plane. As per one time impulse departure, applied different braking direction and breaking speed gain at the departure point, different reentry point parameter could be obtained, regarding the specific mathematical expectations of the MaRV, calculate the max lateral maneuvering flight performance corresponding to the inner layer of the atmosphere. Since the max lateral maneuvering range is more at the inner layer of atmosphere, the flight time at the outer layer of the atmosphere is short, the required speed gain of departure is less, a better group of reentry point parameter could be selected from the multi simulation results.

Regarding the different original trajectory of the close ground MaRV, the different reentry point parameter could be determined via simulation methods. This article is on the supposition that the MaRV is moving along the round trajectory at the height of 3000 km, the max seed gain of MaRV is Δν_max = 1000 m/s, by doing a great amount of departure simulations and experiments of the reentry flying vehicles the selected reentry parameter are νₑ = 6464000 m, γₑ = 7550 m/s, γₑ = -4.8°.

B. The reentry attacking planning process based on the optimal attacking line
On the supposition that the earth is static, once the \( r_e, v_e, \) and \( \gamma_e \) are determined, the approximately lateral max maneuvering range \( \phi_{\text{max}} \) of the MaRV at the inner layer of the atmosphere and the corresponding longitudinal flight distance \( \theta_{\text{max}} \) can be obtained by using the max lift and resistance ratio, further more, the flight distance \( \theta_{\text{ex}} \) from the departure point and to the reentry point (expressed by the geocentric angle), flight time \( t_{\text{ex}} \) and the max variation angle \( \beta_{\text{max}} \) of the trajectory plane at the outer layer of the atmosphere are becoming the rated values. The overall lateral max maneuvering range \( \varphi_{\text{max}} \) of the warhead section and its corresponding longitudinal range \( \theta_{\text{max}} \) from the departure point can be known by taking the use of the numerical value considering the maneuverability at both inner and outer layer of the atmosphere.

Taking the advantage of the lateral and longitudinal maneuvering performance of MaRV after entering into the atmosphere from a certain point, the targets within a large range could be attacked. As the Fig 2 shows: taken the lateral maneuvering range at the inner layer of atmosphere as the radius to make a circle, then there will be great maneuverability for MaRV to attack the targets within the circle at a longer distance, so usually this circle is named as the optimal attacking circle, the center point of this circle is the optimal attacking point. While carrying out the ground target attacking, the targets should be closer to the center area of this circle. This is the starting point of the reentry attacking process. Generally, \( \beta_{\text{max}} \) is less, optimal attacking point of same departure point and different reentry point is approximately at one straight line, it is named as the optimal attacking line.

After guided to the concept of “optimal attacking line” taken the time while target is passing through the optimal line as the departure attacking time of MaRV, the intercept range \( d \) of the target at the optimal attacking line is determined, (shown in Fig.2) supposed that the intercept range \( d_{\text{max}} \) at the attacking line of the trajectory under the dotted the lines after the trajectory plane of MaRV change \( \beta_{\text{max}} \) angle, if the \( \beta_{\text{max}} \) is less, the variation angle of the required trajectory plane at the departure point of MaRV is:

\[
\beta = \begin{cases} \\
\frac{d}{d_{\text{max}}} \beta_{\text{max}} & d < d_{\text{max}} \\
\beta_{\text{max}} & d \geq d_{\text{max}}
\end{cases}
\]

The conclusion of to-the-ground attacking planned process on the supposition that the earth is static is as follows:

- Calculate \( d_{\text{max}} \), determine the intercept range \( d \) and time \( \tau \) when the target point passing through the optimal attacking line.
- Determine position \( P_e \) at the departure point according to \( \tau \), according to (1) formula determine \( \beta \) so as to obtain the reentry point position \( P_e \).
- Select the proper guidance method at the departure section, guide the MaRV entering into the free flight trajectory.

Based on the reentry point parameter and the terminal guidance initial parameter design the trajectory of the reentry section.

If the earth is rotating, the optimal attacking line is perpendicular to the initial trajectory of the dotted line trace of MaRV, neglect the deviations value \( d_{\text{max}} \), \( \varphi_{\text{max}} \) result from the earth rotation, and convert the reentry point parameter into the earth coordinate, then the above mentioned planned process can still be in use.

III. THE DESIGN OF THE STANDARD TRAJECTORY AT THE REENTRY SECTION

The main objective of the standard trajectory design at the reentry section is to get the trajectory of the reentry section which meets the requirement of the reentry terminal restriction, under restriction of the aerodynamic heating, pressure, overload. By means of simulation, the MaRV at the inner layer of the atmosphere, according to the max lift and resistance ratio flight obtains the max lateral maneuvering trajectory which meets the requirement of the reentry flight restrictions.

A. terminal parameter of the reentry section

The terminal parameter of the MaRV reentry section relates with the releasing conditions, mainly includes height, speed, track bearing, course angle and missile-target range. This article directly selects the following terminal status parameter: height \( h_f = 25000 m \), track bearing \( \gamma_f = 0^\circ \), speed \( v_f = 1500 m/s \), course angle pointed at the target point, missile and target range is \( L_f = 100 km \).

B. the design of the reentry trajectory on the supposition that the earth is static

Reentry speed of MaRV is close to the approaching trajectory speed, and the speed to release the carrier is comparatively slow, so it is required to consume lots of...
initial energy of MaRV to the atmosphere at the reentry section. In a respect of energy, the essence of the reentry trajectory design is the control energy consumption under the condition of a certain restrictions.

In order to have a visual understanding about the max lift and resistance ratio, Fig.3 shows the comparison between the MaRV plane reentry and the max lateral maneuvering reentry. The Fig. illustrates: the first wave trough of the plane reentry should be higher than the max lateral maneuvering reentry. the later has the rolling angle which result in the decrease of the lift component on the vertical plane.

Based on the above mentioned simulating result, this article tries to explain by reducing the flight attacking angle when MaRV first reaches the wave trough at a certain time to prolong the flight time of MaRV at the dense atmosphere so as to consume the mechanical energy effectively. To be conclude, to control the consumption of the energy of MaRV by adjusting the small attacking angle flight time. Suppose that the small attacking angle initial fight time $t_{af} = 75\, s$, the small attacking angle is $\tilde{\alpha} = 6^\circ$, Fig. 4 is the reentry comparison of about the sustaining time of the small attacking angle $t_{af} = 80\, s$, $t_{af} = 100\, s$ and $t_{af} = 120\, s$.

The multi simulation shows it is effective to consume the mechanical energy by reduce the flight time of the attacking angle while reaching the wave trough. The thermal power, pressure, overload peak and overall thermal energy of the reentry process are close to the max lateral maneuvering reentry value, furthermore, $t_{as}, \tilde{\alpha}, t_{af}$ these three parameter could be used to adjust the energy consumption, among of which $t_{as}$ has the most affect on the energy consumption., $t_{af}$ is second. If $t_{as}$ is not properly selected, there is no way to adjust the energy consumption effectively to another to parameter. So, usually, $\tilde{\alpha}$ is a constant, and while MaRV is making lateral maneuvering reentry adjust $t_{as}$:

$$t_{as} = t_{asp} + d'(t_{10} - t_{asp}) / \varphi_{max}$$

In this formular $t_{asp}$ is the initial moment of the small attacking angle when making the plane reentry, $t_{10}$ is the max lateral maneuvering reentry flight time.

$$d' = \begin{cases} 0 & d \leq d_{max} \\ d - d_{max} & d > d_{max} \end{cases}$$

The terminal speed of the reentry section should be in align with the target. That is the terminal rolling angle is $\sigma = 0^\circ$. Based on this restrictions, to carry out the corrections of max lateral maneuvering rolling angle, is to make MaRV as zero when it is at the $L_f$ the optimal attacking line. MaRV 's longitudinal flight distance are all $\theta_{max}$ within the atmosphere, so the variation law of the rolling angle could be expressed by the longitudinal flight range at the inner layers of atmosphere. when MaRV is the max reentry lift and resistance ratio, the max lateral maneuvering rolling angle could be expressed in this way.

$$\sigma_{c_{max}}(\theta) = \begin{cases} \sigma_{c_{max}}(0) & \theta < \theta_z \\ 0 & \theta \geq \theta_z \end{cases}$$

Among of which $\theta_z = \theta_{1max} - 57.3L_f / R_E$

Rolling angel is the one which corresponding to the max lateral maneuvering in a fix proportion, that is :

$$\sigma(\theta) = p\sigma_{c_{max}}(\theta)$$

In this formula, $p$ is the proportional coefficient, $p \in [0,1]$. Once the variation law of the attacking angle is determined, regarding the target at the intercept from the
optimal attacking line, by using the mathematical calculation the corresponding \( p \) value could be obtained in order to guide the MaRV dotted trajectory passing through the target point. So, once the target at the intercept from the optimal attacking line, only by adjusting the attacking flight time \( t_{af} \), the energy consumption could be controlled. The flight trajectory of MaRV which was passed by the dotted trajectory after adjusting the small attacking angle is named as the small attacking angle flight trajectory.

Considering the reentry restriction of the terminal height, speed, course, and on the basis of the small attacking angle flight trajectory, the terminal adjusting flight section is discussed here. The ground projection of the terminal adjusting flight section is the dotted trace of the small attacking angle flight trajectory. To be sure that the flight status parameter has the secondary continuity, the projection curve on the longitudinal flight plane of this trajectory is the third curve, at the deduction section of the third and fourth flight trajectory finds out the starting point of terminal adjusting phase. Take the geocentric range as the cubic polynomial of the longitudinal flight distance:

\[
  r(\theta) = a_0 \theta^3 + a_1 \theta^2 + a_2 \theta + a_3 \quad (6)
\]

The equation coefficient can be determined by the following conditions:

\[
  \begin{align*}
    3\theta_0^2 a_3 + 2\theta_0 a_2 + a_1 &= r_0' \\
    6\theta_0 a_3 + 2a_2 &= r_0'' \\
    \theta_0^3 a_3 + \theta_0^2 a_2 + \theta_0 a_1 + a_0 &= r_0 \\
    3\theta_0^2 a_3 + 2\theta_0 a_2 + a_1 &= r_1'
  \end{align*}
\]  

In this equation \( \theta_0, \theta_1 \) is the starting point and terminal point of the terminal adjusting phase (starting point positions at the deduction section of the third and forth trajectory, terminal point is the starting point of the terminal guidance) \( r_0', r_0'' \) is the first and second order derivative of the longitudinal range corresponding to the geocentric range at the starting point, by MaRV movement parameter calculation could be obtains, \( r_1' \) is the first order derivative of the longitudinal range corresponding to the geocentric range at the terminal point under the terminal restrictions.

So far the non linear relations between the small attacking angle flight time and terminal speed have established. According to the several times adjustment of the small attacking angle flight time while the target at the optimal attacking line, could determine, the starting point at the terminal adjusting phase is at the third deduction section or the forth, similarly the small attacking angle flight time interval which includes the terminal restricted speed is determined. What is more, as this design phase regardless the rotation of the earth, so the various flight time interval could be tested at the different intercept. Since this is known, by using the non linear equation the small attacking angle flight time which terminal restricted speed is 1500m/s could be searched, the reentry trajectory with terminal restrictions can be obtained.

C. the standard trajectory design on the supposition that the earth is rotating

Supposed that the reentry trajectory is \( \text{trj}^0 \) under the condition of the earth is static, the control quantity of trajectory \( \text{trj}^0 \) (angle of attack, roll angle) within time area parameterized. And convert the optimal control into the parameter optimization, the time range is more than the actual control time, the small angle flight section is the important one, the parameter optimization after conversion could be expressed in the following way:

\[
  \min f(x) \quad x \in [a,b]
\]

s.t. \( g(x) - g_{\max} \leq 0, \ h(x) - h_j = 0 \)  

Among of which \( x = [a_1, \ldots, a_N, \sigma_1, \ldots, \sigma_N]^T \) as the parameter variations, \( a_{\min} \leq a_i \leq a_{\max}, N \cdot a_{\min} \cdot a_{\max} \cdot \sigma_{\min} \cdot \sigma_{\max} \) the dispersion point within a certain time range and the limit of attacking angle and roll angle respectively, is \( g_{\max} \cdot g_j(x) \) the \( J \)TH restricted value and the actual value of \( X \) in the course of the reentry. \( h_j, h_j(x) \) is the \( K \)TH terminal restricted value and actual value \( x \) at the reentry process if the optimal parameter is \( x \).

Based on the strict restrictions of the optimization, the initial parameter which meet the restriction requirement is difficult to determine, only take the non-feasible region as the optimal initial value, the feasible region which meet the restriction requirement is very limited, and within this limit feasible region it is difficult to increase the performance index. So the main objective of this article is: starting from some non-feasible initial value, tries to find out the feasible solution which meet the restriction requirement, while taking the optimal index as the min reentry flight time.

Take the above parameter as the initial value, considering the earth is rotating, reentry restrictions and terminal restrictions, adopting the parameter optimization solution (8) to get the reentry standard trajectory \( \text{trj}^0 \). But the traditional optimal method is sensitive to the initial value. After the control quantity of trajectory \( \text{trj}^0 \) is changed into parameters, optimization trajectory \( \text{trj}^1 \) is obtained on the supposition that the earth is static, then take this value as the initial value, then convert this solution into the rotating conditions obtain \( \text{trj}^0 \).

IV. CALCULATION OF THE REENTRY ATTACK PLAN

Supposed that the MaRV is moving on the round trajectory with the altitude of 300km, trajectory deflection angle is 5\( ^\circ \), right ascension of the ascending node(RAAN) 0\( ^\circ \) is. the argument of perigee is 0\( ^\circ \). the true anomaly at the initial moment

\[
  \left[ \begin{array}{c}
    3232 \\
    3222 \\
    3212 \\
    3202 \\
  \end{array} \right]
\]

\[
  \left[ \begin{array}{c}
    1312 \\
    1321 \\
    1331 \\
    1341 \\
  \end{array} \right]
\]

\[
  \left[ \begin{array}{c}
    03020100 \\
  \end{array} \right]
\]

\[
  \left[ \begin{array}{c}
    13121111 \\
    13211111 \\
    13311111 \\
    13411111 \\
  \end{array} \right]
\]

\[
  \left[ \begin{array}{c}
    03020100 \\
    13121111 \\
    13211111 \\
    13311111 \\
    13411111 \\
  \end{array} \right]
\]
is $0^\circ$ . $\phi_{max} = 14^\circ, \theta_{max} = 35^\circ$ the longitudinal range between the optimal attacking line and MaRV is $\theta' = 70.17^\circ$ . The ground target longitude is $\theta = 165^\circ$ , latitude is $\phi = 35^\circ$ .

<table>
<thead>
<tr>
<th>TABLE I. PARAMETERS ON THE PROCESS OF REENTRY ATTACKING PLAN</th>
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<tbody>
<tr>
<td>$\beta_{max}$</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>6.7793$^\circ$</td>
</tr>
<tr>
<td>$\gamma_\gamma$ ($\text{t/s}$)</td>
</tr>
<tr>
<td>7294.8</td>
</tr>
</tbody>
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Based on above mentioned plan, part of the parameter will be listed in the table 1, here $f_c$ is the true anomaly of MaRV departure point at the original trajectory, $T_{orb}$ is the moving time of the MaRV at the original trajectory from the initial time till the attacking time $\left[\gamma_\gamma, r_\gamma, \psi_\gamma, r_\gamma, \theta_\gamma, \phi_\gamma\right]$ are the reentry point parameter in the earth coordinate. Reentry trajectory design takes $t_{asp} = 75s$ , $t_{t_0} = 90s$ , $\tilde{a} = 6^\circ$ while standard trajectory design adopts the secondary solution. Fig 5a and Fig 5b is the comparison of the parameter variations on the supposition that the earth is static, Fig 6 is final result of the reentry standard trajectory.

**V. CONCLUSION**

This article mainly discusses about the operational plan of the MaRV, the specific attacking plan and scheme on the ground target has been addressed here, based on the attacking scheme, the author proposed a certain design method regarding the reentry standard trajectory. By using the specific calculations the accuracy at the reentry terminal section increases.

**REFERENCES**


