Three Dimensional Self Calibration Guidance Law for Guided Munitions

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Abstract—In this paper, one kind of three dimensional guidance law for the air-to–surface attacking is derived based on the variable structure control (VSC) theory, then the relationship between the measurement error and guidance error is investigated and discussed thoroughly, the 3-D guidance law based on VSC is modified in order to be implemented simply, the performance of modified guidance law is validated through the comparison between traditional guidance law and modified guidance law based on the computer simulation.

Index Terms—Measurement error, Guidance error, 3-D guidance law, Variable structure control

I. INTRODUCTION

As to the guidance law of various flying vehicles, many guidance laws have been proposed and applied in the research papers and practical engineering, such as guidance laws based on variable structure control [1-2], optimal control laws [3], differential game laws [4] and known classical guidance laws [6]. All these guidance laws were applied in two dimensional cases and three dimensional scenarios. In order to enhance the efficient of munitions, the development of guidance law research has three typical features: one is more and more constraints were considered in the design of the guidance laws, such as impact angle and multiple target attack [6-7]. The other is modified guidance laws and integrated guidance laws were proposed and simulated, for example, predictive guidance laws based on proportional navigation law [8]. The last one is more and more guidance laws become less depending on the flight time and other uncertainties. Although some uncertainties were taken into consideration during the design period, the disturbances and uncertainties were considered as one undependable term in the formulation, actually these disturbances and uncertainties were related and dependable theoretically and practically, because in the practical application or engineering, various measurement errors are inevitable and these errors have effects on the guidance performance, especially the miss distance. How measurement errors affect the guidance error qualitatively and quantitatively is the main purpose of this paper. It should be noted that in this paper, the measurement errors include the position errors and velocity errors of both munitions and targets. Also in this paper, the guidance errors include the distance error, the light of sight angle errors and rat of light of sight angle errors between the munitions and target.

This paper is organized as follows: the three dimensional guidance law was presented in the section II, error calibration was researched in the section III, modified guidance law was proposed in the section IV, simulation was performed in the section V, and the conclusion was given finally.

II. THREE DIMENSIONAL GUIDANCE LAW

Suppose that \((x_i, y_i, z_i)\) is the nominal position of the munitions in the local level frame, \((x_m, y_m, z_m)\) is the measured position of munitions by the strapdown inertial navigation system boarded on the munitions. \((x_T, y_T, z_T)\) is the target position in the local level frame. From these positions, the nominal distance and related LOS angle can be depicted as

\[
R_i = \sqrt{(x_T - x_i)^2 + (y_T - y_i)^2 + (z_T - z_i)^2}
\]

\[
q_{ei} = \arctan\left(\frac{(y_T - y_i)}{(x_T - x_i)}\right)
\]

\[
q_{pi} = \arctan\left(\frac{z_T - z_i}{x_T - x_i}\right)
\]

From the actual measured data, the measured distance and related LOS angle will be of following forms:

\[
R_m = \sqrt{(x_T - x_m)^2 + (y_T - y_m)^2 + (z_T - z_m)^2}
\]

\[
q_{e\alpha} = \arctan\left(\frac{(y_T - y_m)}{(x_T - x_m)}\right)
\]

\[
q_{p\alpha} = \arctan\left(\frac{z_T - z_m}{x_T - x_m}\right)
\]

In the following discussion, all the variables with subscription i means the nominal values, all the variables with subscription m means the measured values. The nominal value and measured value meet the following constraints:

\[
x_i = x_m - \Delta x \quad y_i = y_m - \Delta y \quad z_i = z_m - \Delta z \quad R_i = R_m - \Delta R
\]
\[ \Delta x, \Delta y, \Delta z, \Delta R \] denote the position errors of munitions and relative distance error between munitions and target.

In the practical scenario, the posot parameters of target have been imbedded in the computer boarded on the munitions, so we can obtain following equations based on equation (1):

\[
\begin{aligned}
\dot{R} &= (x_R - x) \dot{x} + (y_R - y) \dot{y} + (z_R - z) \dot{z} \\
&= -\dot{x} \cos q_x \cos q_p - \dot{y} \sin q_x + \dot{z} \cos q_x \sin q_p \\
q_{\alpha} &= \frac{\dot{x} \sin q_x \cos q_p - \dot{y} \cos q_x - \dot{z} \sin q_x \sin q_p}{R} \\
q_{\beta} &= \frac{\dot{x} \sin q_x + \dot{z} \cos q_p}{R} 
\end{aligned}
\]  (4)

Differentiating equation (4), we can obtain:

\[
\begin{aligned}
\ddot{R} &= R_q^2 \dot{q}_q^2 + R_q^2 \dot{q}_q^2 \cos^2 q_p - \\
&\quad (\ddot{x} \cos q_x \cos q_p + \ddot{y} \sin q_x - \ddot{z} \cos q_x \sin q_p) \\
\ddot{q}_\alpha &= \frac{2 \dot{R} \dot{q}_q}{R} \dot{q}_q \cos q_p \sin q_p + \\
&\quad (\dot{x} \sin q_x \cos q_p - \dot{y} \cos q_x - \dot{z} \sin q_x \sin q_p) \\
\ddot{q}_\beta &= \frac{2 \dot{R} \dot{q}_q}{R} + 2 \dot{q}_q \dot{q}_p \sec q_e + \\
&\quad (\dot{x} \sin q_x \cos q_p + \dot{z} \cos q_p) 
\end{aligned}
\]  (5)

Actually in the racket of equation (5), these terms denote the acceleration projection of munitions in the vision frame:

\[
\begin{aligned}
a_x &= \ddot{x} \cos q_x \cos q_p + \ddot{y} \sin q_x - \ddot{z} \cos q_x \sin q_p \\
a_y &= \ddot{x} \sin q_x \cos q_p - \ddot{y} \cos q_x - \ddot{z} \sin q_x \sin q_p \\
a_z &= \ddot{x} \sin q_x \cos q_p - \ddot{y} \cos q_x + \ddot{z} \cos q_p 
\end{aligned}
\]  (6)

Combining equation (5) and (6), we get:

\[
\begin{aligned}
\ddot{R}_t &= R_c q_e^2 + R_c q_e^2 \cos^2 q_e - a_x \\
\ddot{q}_{\alpha t} &= -\frac{2 \ddot{R} \dot{q}_e}{R_t} - q_e \cos q_e \sin q_e - \frac{a_y}{R_t} \\
\ddot{q}_{\beta t} &= -\frac{2 \ddot{R} \dot{q}_e}{R_t} + 2 \dot{q}_e \dot{q}_p \sec q_e + \frac{a_z}{R_t} \cos q_e 
\end{aligned}
\]  (7)

in the tracking period, munitions must meet \( \ddot{R}_t < 0 \).

Let suppose that

\[
x = [q_e, q_p, \dot{q}_e, \dot{q}_p]^T 
\]

Then the equations discussed above can be depicted as

\[
\dot{x} = f(x) + g(x)u 
\]

As can be seen from the equation (9), this formulation is based on the nominal parameters, actually various measurement errors occurred in the measurement procedure, practical guidance law should be deduced from the actual measurement data. Let suppose

\[
\begin{bmatrix}
x_M \\
\Delta x = x_M - x 
\end{bmatrix} 
\]

Related guidance law equation can be described as

\[
\dot{x}_r = f(x) - \Delta \dot{x} + \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial u} \Delta u 
\]

\[
\Delta f = -\Delta R x / R + 2 x y x t g x + 2 x y z x t g x + 2 x z y x t g x 
\]

\[
\Delta g = \frac{\partial g}{\partial x} \Delta x + \frac{\partial g}{\partial u} \Delta u 
\]

in the equation (12), many high order terms were omitted, the detailed form of \( f(x) \), \( \Delta f \) and \( \Delta g \) will be discussed below.

\[
f(x) = \begin{bmatrix}
x_{r3} \\
x_{r4} \\
-2 \ddot{R} x_{r3} / R - x_{r4}^2 \cos x_{r1} \sin x_{r1} \\
-2 \ddot{R} x_{r4} / R + 2 x_{r3} x_{r4} t g x_{r1} 
\end{bmatrix} 
\]

\[
\Delta f(x, \Delta x) = \frac{\partial f}{\partial x} \Delta x - \Delta \dot{x}_r 
\]

\[
\begin{bmatrix}
0 \\
0 \\
-\Delta q_e - 2 \ddot{R} q_e \Delta q_e \cos q_e / R \\
-\Delta q_p - 2 \ddot{R} q_p / R - 2 \dot{q}_e \dot{q}_p \Delta q_e \\
+2 \dot{q}_p \dot{q}_p \Delta q_p + 2 \dot{q}_e \dot{q}_p \sec^2 q_e 
\end{bmatrix} 
\]

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If we suppose \( \eta = \Delta R / R_m \), then (15) can be rewritten as

\[
\Delta g = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & \frac{\Delta R}{R(1-\eta)} & \frac{\Delta R}{R(1-\eta)\cos^2 q_i} \\
0 & \frac{\Delta R}{R(1-\eta)} & \frac{\Delta R}{R(1-\eta)\cos^2 q_i} \\
\end{bmatrix} \eta 
\tag{16}
\]

Combining the variable structure control theory and equation (9), we get

\[
\begin{align*}
\dot{u}_1 &= (k+2)\left[ R_m \beta_{\dot{q}} - R_m^2 \cos q_i \sin q_i + R_m \epsilon \text{sgn}(\dot{q}_i) \right] \\
\dot{u}_2 &= -(k+2)\left[ R_m \dot{\beta}_{\dot{q}} \cos q_i - 2R_m \beta_{\dot{q}} \dot{q}_i \sin q_i + R_m \epsilon \cos q_i \sin(\dot{q}_i) \right] 
\end{align*}
\tag{17}
\]

As can be seen from the equation (14) and (16), it’s obviously unreasonable to treat the various disturbances and uncertainties as one term in the equation, because the guidance error can be resulted from errors, such as position measurement error, velocity measurement error and acceleration measurement error.

Guidance law deduced from (9) can be seen from many references, in this paper, we propose a guidance law deduced from equation (11).

Suppose the sliding hyper surface is

\[
S_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix} x_M = Cx_M = \begin{bmatrix} \dot{q}_{\text{cm}} \\
\dot{q}_{\beta_{\text{cm}}} \end{bmatrix} 
\tag{18}
\]

According to the VSC theory

\[
\dot{S}_m = -k \left[ R_m \beta \right] S_m - \epsilon \text{sgn}(S_m) 
\tag{19}
\]

Then we get

\[
\begin{align*}
\dot{u}_1 &= (k+2)R_m \left[ \dot{q}_{\text{cm}} - R_m^2 \beta_{\dot{q}} \cos q_i \sin q_i + R_m \epsilon \text{sgn}(\dot{q}_i) \right] \\
\dot{u}_2 &= -(k+2)R_m \left[ \dot{\beta}_{\dot{q}} \cos q_i - 2R_m \beta_{\dot{q}} \dot{q}_i \sin q_i + R_m \epsilon \cos q_i \sin(\dot{q}_i) \right] 
\end{align*}
\tag{20}
\]

From (19), we can get practical guidance law from real measurement data.

**III. ERRORS CALIBRATION**

If we can get some prior knowledge about \( \Delta P, \Delta V \), then we can calibrate guidance error theoretically according to figure 1.

![Figure 1. Guidance error calibration scheme](image-url)
From the equation (22) and (23), we can obviously realize the relationship between the guidance error and measurement errors; furthermore, we can understand the effect of measurement error to the guidance law.

IV. MODIFIED GUIDANCE LAW

The control inputs in the equation (17) and (20) are orthogonal accelerations in the vision frame, these parameters should be related with command angular rate of munitions, whereas the angular rate vector is in the body frame, so transition equations should be given below

\[
\begin{bmatrix}
\sin \beta \cos \gamma - \sin \alpha \cos \beta \\
\cos \alpha \\
\sin \alpha \cos \beta \\
\end{bmatrix}
\]

Equation (25) can be briefly noted as

\[
a_v = R_y^\top R_x^\top a_b
\]  

Equation (25) can be briefly noted as

\[
a_v = R_y^\top \begin{bmatrix} \dot{\theta} & \dot{\psi} & \dot{\gamma} \end{bmatrix}^T
\]  

Where the matrix \( R_{xy} \) is a transition matrix, its elements are functions of attitude angle, sideslip angle, angle of attack and angle of LOS. Its detailed form can be described as

\[
R_{xy} = V \begin{bmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33} \\
\end{bmatrix}
\]  

It should be noted that in the equation (27), each element is very complicated, it’s not easy to handle in the practical engineering, so these formulations should be modified. Traditional proportional guidance law can be noted as

\[
\dot{\theta}_v = k_{\theta_v} \dot{q}_v, \quad \dot{\psi}_v = k_{\psi_v} \dot{q}_v
\]  

Where \( k_{\theta_v} \) and \( k_{\psi_v} \) are proportional coefficients and often these parameters are given certain values. Actually the vector of velocity of munitions and LOS is not in the same plane during the flight of munitions. So we modified equation (24) and present following simplified form

\[
\begin{bmatrix}
a_y \\
a_z
\end{bmatrix} = \begin{bmatrix} (\dot{\theta} - \dot{\alpha})V \cos(q_e - \theta + \alpha) \\
(\dot{\psi} - \dot{\beta})V \cos(q_{\beta} - \psi + \beta)
\end{bmatrix}
\]  

Compared equation (30) with equation (28), we can see several nonlinear modified terms be added to the traditional proportional guidance laws.

V. SIMULATION

It’s well known that the quantitative relationship between measurement error and guidance error depends on the flight parameters of munitions, for the brevity, this paper performed the comparison of the overload in the equation (28) and (30) by the simulation under MATLAB/SIMULINK. In the simulation, the chosen munitions is unpowered flying vehicle, related parameters can be found in the reference [9]. The simulation results can be seen from the figure 2 and figure 3. In the figure 2, the red line denotes overload distribution along Z axis of munitions body frame using the modified proportional guidance law, the black line denotes the overload distribution along Z axis of munitions body frame using the traditional proportional
guidance law, it’s obviously can be seen that modified guidance law is superior to the traditional guidance in the overload distribution along both Z axis and Y axis, in the above simulation, the proportional coefficients are chosen as 3.

From the results of the simulation, we can see that the maximum overload is 10 using modified guidance law, whereas this value becomes 300 when using traditional guidance law, it should be noted that in the above simulation and results discussion, the given overload value is more command overload than actual necessary overload.

VI. CONCLUSIONS

This paper presents three dimensional self calibration guidance law, it has obvious superiority over traditional guidance laws in the overload distribution and effects of measurement errors and coupled relationship between the flight parameters and the measurement errors. It also has some drawbacks such as more information should be supported by SINS or other information center, this inevitable increase the price of the munitions and complexity of flight control system of munitions.

High precision guided munition is one of the key weapons in the future, so much research topics could be deduced from the result of this paper.

REFERENCES