Design of the Evolutional Group Buying Auction in Business to Business Electronic Commerce

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Abstract—Group buying auction (GBA) is seen as an effective form of B2C electronic commerce. GBA could not be directly applied to B2B transactions. This paper establishes the evolutional group buying auction (EGBA) for B2B commodity trading based on GBA. First, the EGBA model is presented, and the key algorithms are introduced. Then, the Complicity Problem in EGBA is discussed. Finally, we validate the EGBA model through some examples.

Index Terms—electronic commerce; business to business; online auction; evolutional group buying

I. INTRODUCTION

The Internet’s computational power and flexibility have made auctions a widespread and integral part of both consumer and business markets [1]. In recent years, online auctions are popular both in the business-to-business (B2B) and the consumer markets. Auction played an important role in commerce as an effective price discovering mechanism. Johnson et al. [2] point out that the online consumer auction sales in the US will reach $65 billion by 2010, accounting for nearly one-fifth of all online retail sales.

The traditional group buying auction (GBA) mainly aimed at B2C transactions [3, 4, 5]. Buyers can only bid for one commodity, and as the network characteristics, it may appear that many bidders successfully bid the same commodity at the last-minute, but GBA did not have a good solution to this shortcoming. So, due to model deficiencies, GBA could not be directly applied to B2B transactions. Here, the evolutional group buying auction (EGBA) mainly aims at B2B commodity trading, which is the improvement based on group buying auction (GBA).

The rest of the paper is organized as follows. In Section 2, the EGBA model is presented, and the key algorithms are introduced. Complicity Problem in EGBA is discussed in Section 3. We validate the EGBA model and give some examples in Section 4. Finally, Section 6 draws some conclusions and future work.

II. EGBA MECHANISM DESIGN

A. Model description

EGBA is an extension of the traditional discount sales method, which was based on homogeneous multi-items auction. The popularity of the traditional quantity discount has been studied thoroughly [6, 7, 8]. All bidders were a group. The more bidders, the more numbers of goods were sold at the lower the price, so EGBA is suitable for large B2B commodity trading. The basic EGBA model is established as the following:

There are one seller and n-buyers. Sellers set the price ladder in accordance with the quantity commodity quantity, and each buyer is an independent bid. The seller is auctioning a total of $L$ items. The auction start time is recorded as 0, and set the auction end time which is recorded as $T$. The auction is ended according to Rule 2 and Rule 3 in the following parts.

Set on the amount purchased by the seller in accordance with the price ladder, ladder price vector which the seller sets is denoted by

$$S = \begin{pmatrix} s_1 & s_2 & \ldots & s_i & \ldots & s_m \end{pmatrix}^T$$

Where $i = 1, 2, \ldots, m$. The price vector satisfied $s_1 > s_2 > \ldots > s_m$ and $0 < l_1 < l_2 < \ldots < l_m \leq L$. Row vector $(s_i, l_i)$ indicated that the total purchase amount $Y$ to satisfy $l_i \leq Y < l_{i+1}$ at the price of $s_i$.

Each buyer at moment $t_j$ bid $b_j$ and decided to purchase the number of $x_j$. All buyer's bid vector is denoted by

$$B = \begin{pmatrix} t_1 & t_2 & \ldots & t_j & \ldots & t_n \end{pmatrix}^T$$

Where $j = 1, 2, \ldots, n$, that a total of n of buyers bid, and $t_j \in (0, T)$. The total amount of goods are recorded as $X = \sum_{j=1}^n x_j$ (Not necessarily equal to the last traded quantity).

The EGBA sets the relevant rules as follows:

**Rule 1** Priority principle. In the following three cases, the buyer $i$ gives priority to the seller $j$ access to goods: (1) $t_i < t_j$ ; (2) $t_i = t_j$, $x_i > x_j$ ; (3) $t_i = t_j$, $x_i = x_j$. 

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\(b_j > b_j \). That is in accordance with the time priority, the number of priority, and the price priority.

**Rule 2** The first condition to end the auction. Under the conditions of the rule 1, the auction time is taken to reach \(T\), then the auction is ended.

**Rule 3** The second condition to end the auction. In time \(T\), under the conditions of the rule 1, if the \(X \geq L\) then the auction is also ended at this time.

### B. Algorithm Design

In the EGBA auction mechanism, in the number of bidders to determine the circumstances, the need to determine the final transaction price and transaction volume, as well as the winning buyer. Here the algorithm which we design is to determine the final transaction price, the successful bidders set and the winning number of commodities.

1) **Determine the transaction price algorithm**

Denote the transaction price based on \(B\) for \(P(B)\). Then, \(P(B) \in \{s_1, s_2, \ldots, s_r, \ldots, s_m\}\). In order to determine the final transaction price, first of all define functions \(\Phi(\cdot)\):

\[
\Phi(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0 
\end{cases}
\]

Consider

\[
i^* = \text{arg}(\max_{1 \leq j \leq n} \sum_{j=1}^{n} x_j \cdot \Phi(b_j - s_j))
\]

\(s.t. \sum_{j=1}^{n} x_j \cdot \Phi(b_j - s_j) \geq l_i, \ \forall i \in [1, m]\)

Denote the target subscript for \(i^*\), then the final transaction price \(P(B) = s_{\mu}\), corresponding to the price ladder \((s_{\mu}, l_{\mu})\).

Specially, in accordance with Rule 2 of the end of the auction, the transaction volume is \(L\). It is satisfied \(L \geq l_{\mu}\), so that \(P(B) = s_{\mu}\). By the above algorithm (1) we can also get the same results.

2) **The algorithm to calculate the successful bidder set and the corresponding winning number of commodities.**

In the circumstances of getting the transaction price \(P(B)\), the next step is to determine the successful bidder set and the corresponding winning number of commodities. In order to facilitate the expression, we denote the corresponding successful bidder’s subscript set for \(J\).

According to the different auction ending conditions, it can be divided into two kinds of calculation:

a) **In accordance with Rule 2 to end the auction**

In this case, the auction time reached \(T\). It is clearly \(X < L\) Therefore the tender price which was greater than the transaction price for each buyer can receive the desired number of commodities. The tenderer’s set \(J\) could be obtained using the following formula:

\[
J = \{j \mid b_j - P(B) \geq 0, j \in [1, n]\}
\]

The final successful bidder \(j\) got \(x_j\) units commodities, Where \(j \in J\).

b) **In accordance with Rule 3 to end the auction**

In this case, the tender commodities was greater than or equal to the number of the total number of auction items, namely \(X \geq L\). Here comes down to how to allocate the commodities. Then we must follow the rules 1 to re-row the row vector of the bid vector \(B\), and one by one according to the new order to allocate goods. That is, at first in accordance with the tender \(t_{j}\) re-arranged from small to large order, then if the times was the same, we re-arranged in accordance with \(x_j\) descending order.

If both the time and the purchase of the same amount were the same, we re-arranged by \(b_j\) with descending order. Finally, we get the re-arranged bid vector

\[
B' = \left( \begin{array}{c} 
t_{k_1} \\
t_{k_2} \\
\vdots \\
t_{k_n} \\
x_{k_1} \\
x_{k_2} \\
\vdots \\
x_{k_n} 
\end{array} \right)^T
\]

According to the row sequence of vector \(B'\), the successful bidder from the first line start in this order, for example, \(k_1\) is the first successful, and the second one is the \(k_2\), and so on. Denote the last row of the successful bidder by \(r^*\), then last one successful bidder is \(k_{r^*}\). It can be obtained using the following formula:

\[
r^* = \text{arg}(\min_{1 \leq j \leq n} \sum_{j=1}^{n} x_{k_j})
\]

\(s.t. \sum_{j=1}^{n} x_{k_j} \geq L, \ \forall r \in [1, n]\)

A collection of the successful bidder for the \(J = \{k_j \mid j \in [1, r^*]\}\)

In this case, not all of the successful tenderer can obtain the desired number of goods, specifically, the last one successful bidder \(k_{r^*}\) may not get the amount of the bid amount of goods. Denote the last one successful bidder quantity of goods the last one successful bidder got by \(x_{r^*}\), then \(x_{r^*} \leq x_{r^*}\). We can get

\[
x_{r^*} = L - \sum_{j=1}^{r^*-1} x_{k_j}
\]

Aside from the successful bidder \(k_{r^*}\), The remaining quantity of goods was distributed to the other successful bidders for the \(x_{k_j}\), where
\[ k_j \in J' = \{ k_j \mid j \in [1, r^* - 1] \} . \]

c) Algorithmic process

According to auction rules and the above algorithm we can get the specific algorithm processes in the following:

Step 1 To determine whether the auction time to reach \( T \), if the time taken to reach \( T \), the auction ended, and if not, skip step 2 to step 3;

Step 2 Calculate \( X = \sum_{j=1}^{N} x_j \). To determine whether \( X \geq L ? \) If met, then the auctions have ended, go to step 5. If the auction does not end, then jump to Step 1;

Step 3 Calculate \( P(B) \) according to the formula (1);

Step 4 Calculate \( J \) according to the formula (2) if the auction was ended at Rule 2. All the buyers in set \( J \) are successful bidders, the winning number of commodities for their own bid amount \( x_j \ ( j \in J ) \). Go to step 7;

Step 5 If the auction was ended at Rule 3 , then \( P(B) = s_m \). According to the formula (3) get the re-arranged bid vector \( B' \).

Step 6 After getting \( B' \), according to the formula (4) to calculate the last one successful bidder in the line \( r^\ast \), then the collection of the successful bidder \( J = \{ k_j \mid j \in [1, r^\ast] \} \). According to the formula calculation (6) to obtain \( x_{r^\ast} \), the remaining amount of the winning buyers got bids when the number of commodities were \( x_j \), where

\[ k_j \in J' = \{ k_j \mid j \in [1, r^* - 1] \} ; \]

Step 7 end.

III. COMPLICITY PROBLEM

Comspiracy is a common phenomenon in the auction, and conspiracy, some or all of the co-ordination among the bidders bid their own actions in order to get in the auction, when acting alone higher than the respective gains. Conspiracy may appear to be a clear agreement: Which field in which bidders win the auction, it may appear to be consistent bidders were secretly down their bids. In order to facilitate the description of the bidders valuation of goods, as well as the relationship with bid price and the price ladder, we define the function \( \theta(\cdot) \):

\[
\theta(v) = \begin{cases} 
  s_1 & v \geq s_1 \\
  s_i & s_i \leq v < s_i-1 \\
  s_m & v = s_m \\
  0 & v < s_m 
\end{cases}
\]

To prevent complicity in the bidding vector \( B \) Where in the line \( k_j \). The Group's bid collusion with the total number of goods expressed as \( X_{\Omega} \), \( X_{\Omega} = \sum_{j=\Omega} x_j \), The seller set the corresponding price vector for the \((s, l)\). That satisfy \( l_i \leq X_{\Omega} < l_{i+1} \). And seek a final transaction price under

\[ P(B) \leq s_j \]. Because they do not complicity of the tender price \( b_j = \theta(v_j) \geq s_i \) (Because if \( \theta(v_j) < s_i \), The number of other bidders in uncertain circumstances, the conspirators of the final winning bid gains may be negative) otherwise they would not form a conspiracy group, according to auction rules in the price of a principle of giving priority conspirator in determining the \( P(B) \leq s_j \). Will not reduce the tender price, otherwise you will reduce the possibility of winning. Q.E.D.

Theorem 1 In EGBA mechanism, the conspiracy within the group of bidders will not reduce the tender price.

Proof: \( q \) bidders form a conspiracy group \((1 < q \leq n , n \) is the total number of bidders), where the conspirators set denoted \( \Omega_q = \{ k_j \mid j \in [1, q] \} \). Member of the complicity in the bidding vector \( B \) Where in the line \( k_j \).

The Group's bid collusion with the total number of goods expressed as \( X_{\Omega} \), \( X_{\Omega} = \sum_{j=\Omega} x_j \), The seller set the corresponding price vector for the \((s, l)\). That satisfy \( l_i \leq X_{\Omega} < l_{i+1} \). And seek a final transaction price under

\[ P(B) \leq s_j \]. Because they do not complicity of the tender price \( b_j = \theta(v_j) \geq s_i \) (Because if \( \theta(v_j) < s_i \), The number of other bidders in uncertain circumstances, the conspirators of the final winning bid gains may be negative) otherwise they would not form a conspiracy group, according to auction rules in the price of a principle of giving priority conspirator in determining the \( P(B) \leq s_j \). Will not reduce the tender price, otherwise you will reduce the possibility of winning. Q.E.D.

Theorem 2 In EGBA mechanism, the buyer’s complicity will not damage the interests of the seller.

Proof: According to Theorem 1, the conspirators will not reduce the tender. Therefore, according to formula (1) the final transaction price will not be reduced. And then according to formula (2) \(- (6) \) we can see the number of goods in a conspiracy case the ultimate buyer will not reduce the total number of successful products (or even likely to increase). So the buyer’s complicity will not damage the interests of the seller. Q.E.D.

IV. VALIDATION OF THE MODEL

Suppose that one seller has a total of \( L = 300 \) tons commodities for sale, and the auction time is set to \( T = 5 \). Ladder price vector is

\[
S = \begin{pmatrix} 
1000 & 900 & 800 & 700 \\
1 & 51 & 101 & 201 
\end{pmatrix}^T
\]

It can be described as Figure 1.

- \[ 1 \sim 500 \text{ton} \]
- \[ 51 \sim 1000 \text{ton} \]
- \[ 101 \sim 2000 \text{ton} \]
- \[ 201 \sim 3000 \text{ton} \]

Figure 1. Examples
A. The example to end the auction in accordance with Rule 2

At the end of time corresponding to all buyers bidding vector is as follows:

\[
B = \begin{pmatrix}
    t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\
    b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\
    x_1 & x_2 & x_3 & x_4 & x_5 & x_6 
\end{pmatrix}^T
\]

\[
= \begin{pmatrix}
    1 & 1.5 & 2 & 2.5 & 3 & 4 \\
    1000 & 900 & 900 & 800 & 700 & 700 \\
    40 & 60 & 10 & 20 & 20 & 10 
\end{pmatrix}^T
\]

Since the end of the auction in accordance with Rule 2, directly in accordance with the algorithmic process steps 3 and 4 can be calculated,

Step 3: Calculate the transaction price, according to equation (1):

\[
i^* = \arg\max \sum_{j=1}^{n} x_j \cdot \Phi(b_j - s_j)
\]

\[
s.t. \sum_{j=1}^{n} x_j \cdot \Phi(b_j - s_j) \geq l_i, \quad \forall i \in [1, m]
\]

We can get \(i^* = 3\), and \(P(B) = s_3 = 800\).

Step 4: Calculate the successful bidder collection,

\(J = \{j \mid b_j - 80 \geq 0, j \in [1, 6]\} = \{1, 2, 3, 4\}\),

That is, 1 to 4 winning bidder to obtain the number of goods followed by 40, 60, 10, 20.

B. The example to end the auction in accordance with Rule 3

Reach the time \(t_7 = 3.5\). Vector corresponding to the tender is as follows:

\[
B' = \begin{pmatrix}
    t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\
    b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\
    x_1 & x_2 & x_3 & x_4 & x_5 & x_6 
\end{pmatrix}^T
\]

\[
= \begin{pmatrix}
    1 & 1.5 & 2.5 & 2.5 & 3.5 & 3.5 & 3.5 \\
    700 & 900 & 700 & 800 & 800 & 700 & 800 \\
    40 & 80 & 50 & 40 & 40 & 20 & 20 
\end{pmatrix}^T
\]

According to algorithmic process in step 2,

\[
X = \sum_{j=1}^{6} x_j = 320 > L,
\]

The auction ended, and then follow steps 5 and 6:

Step 5: We got \(P(B) = s_m = 700\), and the Re-arranged bid vector was

Step 6: According to the formula (4), we got \(r^* = 6\) and \(J = \{k_j \mid j \in [1, r^*]\} = \{1, 2, 4, 3, 6\}\). So successful bidder 1, 2, 4, 3, 6 to obtain the number of goods 40, 80, 50, 80, 40, and the last one successful bidder \(k_r = 5\) got the number

\[x_{r'} = L - \sum_{j=1}^{r-1} x_{k_j} = 300 - 290 = 10\]

V. CONCLUSIONS

In this paper, we design the evolutional group buying auction for business to business electronic commerce, which is the improvement of the traditional evolutional group buying auction. The basic model is firstly established. Then we design several algorithms to determine the final transaction price, the successful bidders set and the winning number of commodities. At last, some examples are given to validate the above model.

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