Property Preservation and Application of a kind of Petri net Synthesis

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Abstract—A Petri net synthesis method is proposed, which is the key method to ensure the synthesis net preserving well behaved properties. Conditions of structural liveness preservation of ordinary Petri synthesis net are proposed.

Index Terms—Petri nets; property preservation; synthesis; structural liveness; system modeling

I. INTRODUCTION

Petri nets are well known for their graphical and analytical capabilities in specification and verification, especially for concurrent systems. Many properties can be analytically defined and many techniques are available for development and verification. In particular, the approach based on property-preserving transformations will be described in more detail in this paper.

Usually, a design may be subject to many transformations, such as synthesis, refinement, reduction, etc. A transformation may be used for system generation or system verification. It is important that a transformation should not destroy or create those properties under investigation.

Petri net synthesis is an important transformation. In the literature, there exist many methods to solve net synthesis problem. For example, Agerwala et al. [1] presented a synthesis rule for concurrent systems and proved the preservation of P-invariants under the 1-way merge. An effective solution to the net synthesis problem for path-automatic specifications is presented in [2]. A set of module nets was presented in [3]. The module net composed of some kinds of small nets which got synchrony by sharing transitions. A kind of ST-net was defined in [4], under some conditions, these ST-nets can be used to model some systems. Franceschinis [5] presented the application of a compositional modeling methodology to the re-engineering of stochastic Well Formed Net (SWN) models of a contact center, the advantages are that this approach, based on the definition of classes and instances of submodels, can provide to the application of SWN to complex case studies. A Petri net based approach was presented in [6] to solve the digital system’s high-level synthesis problem. The refinement and abstract representation method of Petri net is proposed [7], which is the key method to ensure the synthesis net preserving the well-behaved properties. Xia [8] investigated property preservations of a kind of synthesis Petri net. With some additional constrains, liveness and boundedness are preserved after merging some sets of subnets.

This paper investigates one type of transformations and its property-preserving approach for verification. A kind of sharing Single-Link subnet synthesis method is proposed. Conditions of structural liveness preservation of ordinary Petri synthesis net are proposed.

The remainder of this paper is organized as below. Section II presents basic definitions. Section III investigates structural liveness preservation of the ordinary synthesis net. Sufficient conditions of synthesis FC (AC) net is FC (AC) net are presented in Section IV. Section V studies property preservation of the synthesis FC (AC) net. Section VI gives an application example. Section VII concludes this paper.

II. BASIC DEFINITIONS

In this section we will quickly review key definitions.

Definition 2.1 Let \( N = (P, T; F, W) \) be a net, \( \Sigma = (N, M_0) \) be a net system,

(1) Transition \( t \in T \) is said to be enabled under \( M \), if \( \forall p \in T, M(p) \geq W(p, t) \), namely \( M[t] \geq . \)

(2) If \( t \) is enabled under \( M \), then \( t \) can be fired. \( M \) can be transformed to \( M' \).

Definition 2.2 Let \( \Sigma = (N, M_0) \) be a net system.

(1) Transition \( t \in T \) is live, if \( \forall M \in R(M_0) \), \( \exists M' \in R(M) \), \( M'[t] > . \)

(2) \( \Sigma \) is live, if \( \forall t \in T \), \( t \) is live.

(3) \( \Sigma \) is structural live, if there exists \( M_0 \), such that \( (N, M_0) \) is live.

Definition 2.3 Let \( N \) be a net,

(1) \( N \) is said to be a free choice (FC) net, if \( \forall p \in P, |p*| \leq 1 \) or \( *(p*) = \{p\} \).

(2) \( N \) is an asymmetric choice (AC) net, if \( \forall p_1, p_2 \in P, p_1 \wedge p_2 \neq \phi \Rightarrow p_1 \subseteq p_2 \) or \( p_2 \subseteq p_1 \).

Definition 2.4 Let \( N = (P, T; F, W) \) and \( N_0 = (P_0, T_0; F_0, W_0) \) be two nets. If

(1) \( P_0 \subseteq P, T_0 \subseteq T \) and \( P_0 \neq \phi, T_0 \neq \phi \),

(2) \( F_0 = F \cap ((P_0 \times T_0) \cup (T_0 \times P_0)) \),

then \( N_0 \) is said to be a subnet of \( N \).
Here, we present a kind of T-type subnet (Fig.1).

![Fig.1 An example of T-type subnet]

Note that, except $t_x, t_y$, every node of the T-type subnet does not connect nodes outside the subnet.

**Definition 2.5** Let $N = (P, T; F, W)$ be a net, $N_0 = (P_0, T_0; F_0, W_0)$ be a subnet of $N$. If

1. $P_0 \cup P \subseteq T_0$,
2. $T_0 = T_0 - \{t_x, t_y\}, T_0 \cup T \subseteq P_0$ and $t_x \in P - P_0$,
3. $t_y \in P - P_0, t_x \neq \phi, t_y \neq \phi$,

then $N_0$ is said to be a T-type subnet of $N$.

**Definition 2.6** Let $N = (P, T; F, W)$ be a net, if

$$\exists x, y \in P \cup T, v = 1, 2, ..., n$$

such that

$$x_i = \{x_1, ..., x_i, ..., x_n\}, y_i = \{x_1, ..., x_{i-1}, ..., x_n\},$$

$$x_i = \{x_2, ..., x_{i-1}, y_i\}, y_i = \{x_2, ..., x_{i-1}\},$$

then $x_i \rightarrow y_i \rightarrow x_{i+1} \rightarrow ... \rightarrow y_j (1 \leq i \leq n)$ is said to be a Single-Link road.

There are 4 kinds of Single-Link roads:

- **Single-Link road I**: $t_1 \rightarrow p_1 \rightarrow t_2 \rightarrow p_2 \rightarrow ... \rightarrow p_{i-1} \rightarrow t_i$,

  \[ (p_j \in P, j = 1, 2, ..., i - 1; t_k \in T, k = 1, 2, ..., i) \]

- **Single-Link road II**: $p_1 \rightarrow t_1 \rightarrow p_2 \rightarrow t_2 \rightarrow ... \rightarrow t_{i-1} \rightarrow p_i$,

  \[ (p_j \in P, j = 1, 2, ..., i; t_k \in T, k = 1, 2, ..., i - 1) \]

- **Single-Link road III**: $p_1 \rightarrow t_1 \rightarrow p_2 \rightarrow t_2 \rightarrow ... \rightarrow t_{i-1} \rightarrow p_i \rightarrow t_i$,

  \[ (p_j \in P, j = 1, 2, ..., i; t_k \in T, k = 1, 2, ..., i) \]

- **Single-Link road IV**: $t_1 \rightarrow p_1 \rightarrow t_2 \rightarrow p_2 \rightarrow ... \rightarrow p_{i-1} \rightarrow t_i \rightarrow p_i$,

  \[ (p_j \in P, j = 1, 2, ..., i; t_k \in T, k = 1, 2, ..., i) \]

**Definition 2.7** If transitions of a Single-Link road are replaced by T-type subnets, we get a Single-Link subnet.

Note that, let T-type subnet replace $t_x, t_y$ of Single-Link roads I, II, III, and IV, and we get Single-Link subnet I, II, III, and IV, respectively.

Single-Link subnet I:

$$N_{T_1} \rightarrow p_1 \rightarrow N_{T_2} \rightarrow p_2 \rightarrow ... \rightarrow p_{i-1} \rightarrow N_{T_i}$$

($N_{T_k}$ is the T-type subnet of $N$, $k = 1, 2, ..., i$)

Single-Link subnet II:

$$p_1 \rightarrow N_{T_1} \rightarrow p_2 \rightarrow N_{T_2} \rightarrow ... \rightarrow N_{T_{i-1}} \rightarrow p_i$$

($N_{T_k}$ is the T-type subnet of $N$, $k = 1, 2, ..., i - 1$)

Single-Link subnet III:

$$p_1 \rightarrow N_{T_1} \rightarrow p_2 \rightarrow N_{T_2} \rightarrow ... \rightarrow N_{T_{i-1}} \rightarrow p_i \rightarrow N_{T_i}$$

($N_{T_k}$ is the T-type subnet of $N$, $k = 1, 2, ..., i$)

**Definition 2.8** Let $N_i = (P_i, T_i; F_i, W_i)(i = 1, 2)$ be two nets, if $N = (P, T; F, W)$ satisfy:

1. $P_0 = P_1 \cap P_2 \neq \phi, T_0 \cap T_2 \neq \phi$,
2. $P = P_1 \cup P_2, T = T_1 \cup T_2, F = F_1 \cup F_2$,
3. $\forall x, y \in P \cup T: W(x, y) = W_1(x, y) \cup W_2(x, y)$,
4. $N_1$ and $N_2$ shared a Single-Link subnet,

then $N$ is said to be a synthesis net of $N_1$ and $N_2$ which shared Single-Link subnet.

**Definition 2.9** Let $\sum_i = (N_i, M_0_i)(i = 1, 2)$ be two net systems, if $\sum = (N, M_0)$ satisfy:

$N$ is a synthesis net of $N_i(i = 1, 2)$ which shared Single-Link subnet, and

$$M_0 = \begin{pmatrix} M_{10} & \forall p \in P_1 - P_0 \\ M_{20} & \forall p \in P_2 - P_0 \end{pmatrix}$$

then $\sum$ is said to be a synthesis net system of $\sum_i(i = 1, 2)$ which shared Single-Link subnet system.

### III. Structural Liveness Preservation of Ordinary Petri Synthesis Net

In this section, we investigate structural property preservation of the synthesis net.

**Theorem 3.1** Suppose that $N_1$ and $N_2$ are two structural live Petri nets, $N$ is the synthesis net of $N_1$ and $N_2$, which shared Single-Link subnet $I$, $N_0$. Then $N$ is structural live.

**Proof.** Since $N_1$ and $N_2$ are structural live, then there exists $M_{10}, M_{10}[t_1 >$, such that $\sum_1 = (N_1, M_{10})$ is live; There exists $M_{20}, M_{20}[t_1 >$, such that $\sum_2 = (N_2, M_{20})$ is live. Since $N_1$ and $N_2$ shared a Single-Link subnet $I$, by the characteristic of Single-Link subnet $I$, $\sum_1$ and $\sum_2$ have the same occurrence transition sequence about $T_0$. Suppose that the transition sequence is $t_1, t_2, ..., t_k$. By Definition 2.9, in
If in Single-Link subnet 1, except \( t_1, t_k \), every node of the subnet does not connect nodes outside the subnet, then the resource of \( P_0 \) can be hold after \( t_1 \) fires, that is, in \( \Sigma \), \( t_2 \) can be enabled after \( t_1 \) fires. Then in \( \Sigma \),

\[
\forall \sigma \in (T - \{ t_2 \})^*, \quad M_0[\sigma]\sim M_1, \quad \exists M_2 \in R(M_1), \quad M_2[t_2] >, \quad \ldots, \quad \forall \sigma' \in (T - \{ t_k \})^*, \quad M_0[\sigma']\sim M_p, \quad \exists M_q \in R(M_p), \quad M_q[t_k] >, \quad \text{that is,} \quad \forall t \in T_0, t \text{ is live.}
\]

Suppose that \( \Sigma \) is not live, then \( \exists t \in T - T_0 \),

\[
\exists M_1 \in R(M_0), \quad \forall M_2 \in R(M_1), \quad -M_2[t_2] >.
\]

Suppose that \( t_1 \in T_0 - T_0 \), since \( \forall t \in T_0, t \) is live, then \( \exists t, M_3 \in R(M_1),M_3[t_1] > \), and if \( p \in P_1 - P_0 \),

\[
M_i(p) = M_i, \quad \text{if } p \in P_0, \quad M_i(p) = M_i(p) + M_2(p), \quad \text{where} \quad M_1 \in R(M_0), \quad M_2 \in R(M_2) .
\]

Then \( \forall M \in R(M_0), \quad -M[t_1] > \), that is, in \( \Sigma \),

\[
\forall M_2 \in R(M_1), \quad M_1 \in R(M_1), \quad -M_2[t_2] >.
\]

\( \Sigma \) is not live, this contradict with the fact that \( \Sigma \) is live.

**Theorem 3.2** Suppose that \( N_1 \) and \( N_2 \) are two structural live Petri nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared Single-Link subnet II \( N_0 \). If

\[
\forall t \in \{ t \mid t \in p_2^*, p_2 \text{ is the last node of } N_0 \}, \quad \Sigma = (N, M_0), \quad t \text{ is live, then } N \text{ is structural live.}
\]

**Theorem 3.3** Suppose that \( N_1 \) and \( N_2 \) are two structural live Petri nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared Single-Link subnet III \( N_0 \). Then \( N \) is structural live.

**Theorem 3.4** Suppose that \( N_1 \) and \( N_2 \) are two structural live Petri nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared Single-Link subnet IV \( N_0 \). If

\[
\forall t \in \{ t \mid t \in p_2^*, p_2 \text{ is the last node of } N_0 \}, \quad \Sigma = (N, M_0), \quad t \text{ is live, then } N \text{ is structural live.}
\]

**IV. Property Preservation of Synthesis FC (AC) Net**

**Theorem 4.1** Suppose that \( N_1 \) and \( N_2 \) are FC nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared Single-Link subnet I \( N_0 \). If

(1) in \( N_j(j = 1, 2) \), \( (t_1)^* = \{ t_1 \} \), where \( t_1 \) is the first node of \( N_0 \).

(2) in \( N_{Tk} \) (\( k = 1, 2, \ldots, i \)), \( \forall p \), in \( N_1, N_2 \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \),

then \( N \) is a FC net.

**Proof.** By conditions of the theorem, \( N_1 \) and \( N_2 \) shared Single-Link subnet I \( N_0 \). By condition (1), \( t_1 \) is the first node of \( N_0 \). In \( N_1, N_2 \), \( (t_1)^* = \{ t_1 \} \), then in \( N_j \) \( (j = 1, 2) \), \( \forall p_1 \in t_1, p_1^* = \{ t_1 \} \), that is \( | p_1 | \leq 1, i = 1, 2 \). In \( N \), \( p_1^* = p_2^* = \{ t_1 \} \), that is \( \forall p \in t_1, | p^* | \leq 1 \). By condition (2), in \( N_{Tk} \), \( \forall p \), in \( N_1, N_2 \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \), then in \( N \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \). According to \( p_1 \), by Definition 2.6, \( | p_1^* | \leq 1 \). The case of \( N_{Tk} \) is similar to that of \( N_{Tk} \). The case of \( p_2 \) is similar to that of \( p_1 \), ..., the case of \( N_{Tk} \) is similar to that of \( N_{Tk} \), that is, in \( N_0 \), \( \forall p \), if in \( N_1, N_2 \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \), then in \( N \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \). Because \( N_1 \) and \( N_2 \) are FC nets, \( \forall p \in P - P_0 \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \). Then \( N \) is a FC net.

**Theorem 4.2** Suppose that \( N_1 \) and \( N_2 \) are AC nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared Single-Link subnet I \( N_0 \). If

(1) in \( N_j(j = 1, 2) \), \( (t_1)^* = \{ t_1 \} \), where \( t_1 \) is the first node of \( N_0 \).

(2) in \( N_{Tk} \) \( (k = 1, 2, \ldots, i) \), \( \forall p_1, p_2 \), in \( N_1, N_2 \), \( p_1^* \cap p_2^* \neq \phi \Rightarrow p_1^* \subseteq p_2^* \) or \( p_2^* \subseteq p_1^* \),

then \( N \) is an AC net.

V. Structural Liveness Preservation of Synthesis FC (AC) Net

By Theorem 3.1 and Theorem 4.1-4.2, we get the following properties.

**Theorem 5.1** Suppose that \( N_1 \) and \( N_2 \) are two structural live FC nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \) which shared a Single-Link subnet I \( N_0 \). If

(1) in \( N_j(j = 1, 2) \), \( (t_1)^* = \{ t_1 \} \), where \( t_1 \) is the first node of \( N_0 \).

(2) in \( N_{Tk} \) \( (k = 1, 2, \ldots, i) \), \( \forall p \), in \( N_1, N_2 \), \( | p^* | \leq 1 \) or \( *(p^* ) = \{ p \} \),

then \( N \) is an AC net.
then \( N \) is a structural live FC net.

**Theorem 5.2** Suppose that \( N_1 \) and \( N_2 \) are two structural live AC nets, \( N \) is the synthesis net of \( N_1 \) and \( N_2 \), which shared a Single-Link subnet \( N_0 \). If

1. in \( N_j \) (\( j = 1, 2 \)), \( (t_i^*)^* = \{t_i\} \), where \( t_i \) is the first node of \( N_0 \),
2. in \( N_{TR} \) (\( k = 1, 2, \ldots, i \)), \( \forall p_1, p_2 \), in \( N_1, N_2 \),

\[ p_1^* \cap p_2^* \neq \emptyset \Rightarrow p_1^* \subseteq p_2^* \text{ or } p_2^* \subseteq p_1^*, \]

then \( N \) is a structural live AC net.

**VI. APPLICATIONS**

In this section we will use the synthesis method to model the system that enterprise_1 and enterprise_2 rent a same plant to produce some product. Preparing raw and processed materials, enterprise_1 and enterprise_2 rent a plant to produce some product. Finished products are transferred to enterprise_1 and enterprise_2, respectively.

The model of \( N_1 \) and \( N_2 \) is described in Fig.2 and Fig.3.

![Fig. 2 Net N1](image1)

![Fig. 3 Net N2](image2)

![Fig. 4 Synthesis Net N](image3)

The synthesis net \( N \) is described in Fig.4. According to Fig.2 and Fig.3, it is easy to see that \( N_1 \) and \( N_2 \) are two structural live FC nets. By Theorem 5.1, the synthesis net \( N \) is a structural live FC net.

**VI. CONCLUSIONS**

In this paper we investigate structural property preservations of Petri synthesis net. A Petri net synthesis method is proposed, which is the key method to ensure the synthesis net preserving well behaved properties.

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