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Property Preservation and Application of a kind of Petri net Synthesis

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***Abstract*—A Petri net synthesis method is proposed, which is the key method to ensure the synthesis net preserving well behaved properties. Conditions of structural liveness preservation of ordinary Petri synthesis net are proposed.**

***Index Terms*—Petri nets; property preservation; synthesis; structural liveness; system modeling**

I. INTRODUCTION

Petri nets are well known for their graphical and analytical capabilities in specification and verification, especially for concurrent systems. Many properties can be analytically defined and many techniques are available for development and verification. In particular, the approach based on property-preserving transformations will be described in more detail in this paper.

Usually, a design may be subject to many transformations, such as synthesis, refinement, reduction, etc. A transformation may be used for system generation or system verification. It is important that a transformation should not destroy or create those properties under investigation.

Petri net synthesis is an important transformation. In the literature, there exist many methods to solve net synthesis problem. For example, Agerwala et al. [1] presented a synthesis rule for concurrent systems and proved the preservation of P-invariants under the 1-way merge. An effective solution to the net synthesis problem for path-automatic specifications is presented in [2]. A set of module nets was presented in [3]. The module net composed of some kinds of small nets which got synchrony by sharing transitions. A kind of ST-net was defined in [4], under some conditions, these ST-nets can be used to model some systems. Franceschinis [5] presented the application of a compositional modeling methodology to the re-engineering of stochastic Well Formed Net (SWN) models of a contact center, the advantages are that this approach, based on the definition of classes and instances of submodels, can provide to the application of SWN to complex case studies. A Petri net based approach was presented in [6] to solve the digital system's high-level synthesis problem. The refinement and abstract representation method of Petri net is proposed [7], which is the key method to ensure the synthesis net preserving the well-behaved properties. Xia [8] investigated property preservations of a kind of synthesis Petri net. With some additional constrains, liveness and boundedness are preserved after merging some sets of subnets.

This paper investigates one type of transformations and its property-preserving approach for verification. A kind of sharing Single-Link subnet synthesis method is proposed. Conditions of structural liveness preservation of ordinary Petri synthesis net are proposed.

The remainder of this paper is organized as below. Section Ⅱ presents basic definitions. Section Ⅲ investigates structural liveness preservation of the ordinary synthesis net. Sufficient conditions of synthesis FC (AC) net is FC (AC) net are presented in Section Ⅳ. Section Ⅴ studies property preservation of the synthesis FC (AC) net. Section Ⅵ gives an application example. Section Ⅶ concludes this paper.

II. BASIC DEFINITIONS

In this section we will quickly review key definitions.

Definition 2.1 Let $N=(P,T;F,W)$ be a net, $\sum=(N,M_0)$ be a net system,

(1) Transition $t \in T$ is said to be enabled under M, iff $\forall p \in^{\bullet} t, M(p) \geq W(p,t)$, namely $M[t>$.

(2) If t is enabled under M, t can be fired. M can be transformed to M'.

Definition 2.2 Let $\sum=(N,M_0)$ be a net system.

(1) Transition $t \in T$ is live, if $\forall M \in R(M_0)$, $\exists M' \in R(M)$, $M'[t>$.

(2) $\sum$ is live, if $\forall t \in T$, t is live.

(3) $\sum$ is structural live, if there exists M_0, such that (N,M_0) is live.

Definition 2.3 Let N be a net,

(1) N is said to be a free choice (FC) net, if $\forall p \in P, |p^{\bullet}| \leq 1$ or ${}^{\bullet}(p^{\bullet})=\{p\}$.

(2) N is a asymmetric choice (AC) net, if $\forall p_1,p_2 \in P, p_1^{\bullet} \cap p_2^{\bullet} \neq \phi \Rightarrow p_1^{\bullet} \subseteq p_2^{\bullet}$ or $p_2^{\bullet} \subseteq p_1^{\bullet}$.

Definition 2.4 Let $N=(P,T;F,W)$ and $N_0=(P_0,T_0;F_0,W_0)$ be two nets. If

(1) $P_0 \subset P, T_0 \subset T$ and $P_0 \neq \phi, T_0 \neq \phi$,

(2) $F_0 = F \cap ((P_0 \times T_0) \cup (T_0 \times P_0))$,

then N_0 is said to be a subnet of N.

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Here, we present a kind of T-type subnet (Fig.1).

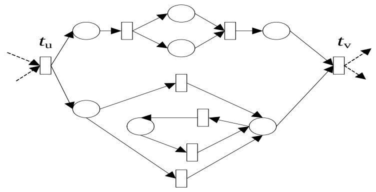

Fig.1 An example of T-type subnet

Note that, except t_u, t_v, every node of the T-type subnet dose not connect nodes outside the subnet.

Definition 2.5 Let $N=(P,T;F,W)$ be a net, $N_0=(P_0,T_0;F_0,W_0)$ be a subnet of N. If

(1) ${}^{\bullet}P_0 \cup P_0^{\bullet} \subseteq T_0$,

(2) Let $T_0' = T_0 - \{t_u, t_v\}, {}^{\bullet}T_0' \cup T_0'^{\bullet} \subseteq P_0$ and ${}^{\bullet}t_u \in P - P_0$, $t_v^{\bullet} \in P - P_0, t_u \neq \phi, t_v \neq \phi$,

then N_0 is said to be a T-type subnet of N.

Definition 2.6 Let $N=(P,T;F,W)$ be a net, if $\exists x_v, y_v \in P \cup T, v = 1,2,\ldots,n$ such that

$x_1^{\bullet} = \{y_1\}, {}^{\bullet}y_1 = \{x_1\}, y_1^{\bullet} = \{x_2\}, {}^{\bullet}x_2 = \{y_1\}$,

$x_2^{\bullet} = \{y_2\}, \ldots, x_{i-1}^{\bullet} = \{y_i\}, {}^{\bullet}y_i = \{x_{i-1}\}$,

then $x_1 \to y_1 \to x_2 \to \ldots \to y_i \, (1 \le i \le n)$ is said to be a Single-Link road.

There are 4 kinds of Single-Link roads.

Single-Link road Ⅰ:

$t_1 \to p_1 \to t_2 \to p_2 \to \ldots \to p_{i-1} \to t_i$

$(p_j \in P, j = 1,2,\ldots,i-1; t_k \in T, k = 1,2,\ldots,i)$

Single-Link road Ⅱ:

$p_1 \to t_1 \to p_2 \to t_2 \to \ldots \to t_{i-1} \to p_i$

$(p_j \in P, j = 1,2,\ldots,i; t_k \in T, k = 1,2,\ldots,i-1)$

Single-Link road Ⅲ:

$p_1 \to t_1 \to p_2 \to t_2 \to \ldots \to t_{i-1} \to p_i \to t_i$

$(p_j \in P, j = 1,2,\ldots,i; t_k \in T, k = 1,2,\ldots,i)$

Single-Link road Ⅳ:

$t_1 \to p_1 \to t_2 \to p_2 \to \ldots \to p_{i-1} \to t_i \to p_i$

$(p_j \in P, j = 1,2,\ldots,i; t_k \in T, k = 1,2,\ldots,i)$

Definition 2.7 If transitions of a Single-Link road are replaced by T-type subnets, we get a Single-Link subnet.

Note that, let T-type subnet replace t_k of Single-Link roads Ⅰ, Ⅱ, Ⅲ and Ⅳ, and we get Single-Link subnet Ⅰ, Ⅱ, Ⅲ and Ⅳ, respectively.

Single-Link subnet Ⅰ:

$N_{T1} \to p_1 \to N_{T2} \to p_2 \to \ldots \to p_{i-1} \to N_{Ti}$

(N_{Tk} is the T-type subnet of N, $k = 1,2,\ldots,i$)

Single-Link subnet Ⅱ:

$p_1 \to N_{T1} \to p_2 \to N_{T2} \to \ldots \to N_{Ti-1} \to p_i$

(N_{Tk} is the T-type subnet of N, $k = 1,2,\ldots,i-1$)

Single-Link subnet Ⅲ:

$p_1 \to N_{T1} \to p_2 \to N_{T2} \to \ldots \to N_{Ti-1} \to p_i \to N_{Ti}$

(N_{Tk} is the T-type subnet of N, $k = 1,2,\ldots,i$)

Single-Link subnet Ⅳ:

$N_{T1} \to p_1 \to N_{T2} \to p_2 \to \ldots \to p_{i-1} \to N_{Ti} \to p_i$

(N_{Tk} is the T-type subnet of N, $k = 1,2,\ldots,i$)

Definition 2.8 Let $N_i = (P_i, T_i; F_i, W_i)(i=1,2)$ be two nets, if $N=(P,T;F,W)$ satisfy:

(1) $P_0 = P_1 \cap P_2 \neq \phi, T_0 = T_1 \cap T_2 \neq \phi$,

(2) $P = P_1 \cup P_2, T = T_1 \cup T_2, F = F_1 \cup F_2$,

(3) $\forall x, y \in P \cup T : W(x,y) = W_1(x,y) + W_2(x,y)$,

(4) N_1 and N_2 shared a Single-Link subnet,

then N is said to be a synthesis net of N_1 and N_2 which shared Single-Link subnet.

Definition 2.9 Let $\sum_i = (N_i, M_{i0})(i=1,2)$ be two net systems, if $\sum = (N, M_0)$ satisfy:

N is a synthesis net of $N_i (i=1,2)$ which shared Single-Link subnet, and

$$M_0(p) = \begin{cases} M_{10}(p) & \forall p \in P_1 - P_0 \\ M_{20}(p) & \forall p \in P_2 - P_0 \\ M_{10}(p) + M_{20}(p) & \forall p \in P_0 \end{cases},$$

then $\sum$ is said to be a synthesis net system of $\sum_i (i=1,2)$ which shared Single-Link subnet system.

III. STRUCTURAL LIVENESS PRESERVATION OF ORDINARY PETRI SYNTHESIS NET

In this section, we investigate structural property preservation of the synthesis net.

Theorem 3.1 Suppose that N_1 and N_2 are two structural live Petri nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅰ N_0. Then N is structural live.

Proof. Since N_1 and N_2 are structural live, then there exists $M_{10}, M_{10}[t_1 >$, such that $\sum_1 = (N_1, M_{10})$ is live; There exists M_{20}, $M_{20}[t_1 >$, such that $\sum_2 = (N_2, M_{20})$ is live. Since N_1 and N_2 shared a Single-Link subnet Ⅰ, by the characteristic of Single-Link subnet Ⅰ, $\sum_1$ and $\sum_2$ have the same occurrence transition sequence about T_0. Suppose that the transition sequence is $t_1, t_2, \ldots, t_k$. By Definition 2.9, in

$\Sigma = (N, M_0)$, $M_0[t_1 >$, in Σ_1 , $M_{10}[t_1 > M_{11}$, $M_{11}[t_2 >$, in Σ_2, $M_{20}[t_1 > M_{21}$, $M_{21}[t_2 >$. Since in Single-Link subnet Ⅰ, except t_1, t_k, every node of the subnet dose not connect nodes outside the subnet, then the resource of P_0 can be hold after t_1 fires, that is, in Σ , t_2 can be enabled after t_1 fires. Then in Σ, $\forall \sigma \in (T - \{t_2\})^*$, $M_0[\sigma > M_1$, $\exists M_2 \in R(M_1)$, $M_2[t_2 >$, …, $\forall \sigma' \in (T - \{t_k\})^*$, $M_0[\sigma' > M_p$, $\exists M_q \in R(M_p)$, $M_q[t_k >$, that is, $\forall t \in T_0, t$ is live.

Suppose that Σ is not live, then $\exists t_u \in T - T_0$, $\exists M_1 \in R(M_0), \forall M_2 \in R(M_1), \neg M_2[t_u >$. Suppose that $t_u \in T_1 - T_0$, since $\forall t \in T_0, t$ is live, then $\exists t_v \in T_0, \exists M_3 \in R(M_1), M_3[t_v >$, and if $p \in P_1 - P_0$, $M_3(p) = M_{11}(p)$. If $p \in P_0$, $M_3(p) = M_{11}(p) + M_{21}(p)$, where $M_{11} \in R(M_{10})$, $M_{21} \in R(M_{20})$. Then $\forall M \in R(M_3), \neg M[t_u >$, that is, in Σ_1 , $\forall M_{12} \in R(M_{11})$, $M_{11} \in R(M_{10})$, $\neg M_{12}[t_u >$. Then Σ_1 is not live, this contradict with the fact that Σ_1 is live.

Theorem 3.2 Suppose that N_1 and N_2 are two structural live Petri nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅱ N_0 . If $\forall t \in \{t \mid t \in p_y^\bullet, p_y$ is the last node of N_0 }, in $\Sigma = (N, M_0)$, t is live, then N is structural live.

Theorem 3.3 Suppose that N_1 and N_2 are two structural live Petri nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅲ N_0. Then N is structural live.

Theorem 3.4 Suppose that N_1 and N_2 are two structural live Petri nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅳ N_0 . If $\forall t \in \{t \mid t \in p_y^\bullet, p_y$ is the last node of N_0 }, in $\Sigma = (N, M_0)$, t is live, then N is structural live.

Ⅳ. Property Preservation of Synthesis FC (AC) Net

Theorem 4.1 Suppose that N_1 and N_2 are FC nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅰ N_0. If

(1) in $N_j (j = 1,2)$, $({}^\bullet t_1)^\bullet = \{t_1\}$, where t_1 is the first node of N_0,

(2) in N_{Tk} ($k = 1,2,...,i$), $\forall p$, in N_1, N_2, $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$,

then N is a FC net.

Proof. By conditions of the theorem, N_1 and N_2 shared Single-Link subnet Ⅰ N_0. By condition (1), t_1 is the first node of N_0 . In N_1, N_2 , $({}^\bullet t_1)^\bullet = \{t_1\}$, then in $N_i (i = 1,2)$, $\forall p_i \in {}^\bullet t_1, p_i^\bullet = \{t_1\}$, that is $| p_i^\bullet | \le 1, i = 1,2$. In N , $p_1^\bullet = p_2^\bullet = \{t_1\}$, that is $\forall p \in {}^\bullet t_1$, $| p^\bullet | \le 1$. By condition (2), in N_{T1}, $\forall p$, in N_1, N_2 , $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$, then in N , $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$. According to p_1 , by Definition 2.6, $| p_1^\bullet | \le 1$. The case of N_{T2} is similar to that of N_{T1}. The case of p_2 is similar to that of p_1, …, the case of N_{Ti} is similar to that of N_{T1}, that is, in N_0, $\forall p$, if in N_1, N_2, $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$, then in N , $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$. Because N_1 and N_2 are FC nets, $\forall p \in P - P_0, | p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$. Then N is a FC net.

Theorem 4.2 Suppose that N_1 and N_2 are AC nets, N is the synthesis net of N_1 and N_2 which shared Single-Link subnet Ⅰ N_0. If

(1) in $N_j (j = 1,2)$, $({}^\bullet t_1)^\bullet = \{t_1\}$, where t_1 is the first node of N_0,

(2) in N_{Tk} ($k = 1,2,...,i$), $\forall p_1, p_2$, in N_1, N_2 , $p_1^\bullet \cap p_2^\bullet \ne \phi \Rightarrow p_1^\bullet \subseteq p_2^\bullet$ or $p_2^\bullet \subseteq p_1^\bullet$,

then N is an AC net.

Ⅴ. Structural Liveness Preservation of Synthesis FC (AC) Net

By Theorem 3.1 and Theorem 4.1-4.2, we get the following properties.

Theorem 5.1 Suppose that N_1 and N_2 are two structural live FC nets, N is the synthesis net of N_1 and N_2 which shared a Single-Link subnet Ⅰ N_0. If

(1) in $N_j (j = 1,2)$, $({}^\bullet t_1)^\bullet = \{t_1\}$, where t_1 is the first node of N_0,

(2) in N_{Tk} ($k = 1,2,...,i$), $\forall p$, in N_1, N_2, $| p^\bullet | \le 1$ or ${}^\bullet(p^\bullet) = \{p\}$,

then N is a structural live FC net.

Theorem 5.2 Suppose that N_1 and N_2 are two structural live AC nets, N is the synthesis net of N_1 and N_2 which shared a Single-Link subnet Ⅰ N_0. If

(1) in $N_j\,(j=1,2)$, $({}^\bullet t_1)^\bullet = \{t_1\}$, where t_1 is the first node of N_0,

(2) in N_{Tk} ($k=1,2,...,i$), $\forall p_1, p_2$, in N_1, N_2 , $p_1^\bullet \cap p_2^\bullet \neq \phi \Rightarrow p_1^\bullet \subseteq p_2^\bullet$ or $p_2^\bullet \subseteq p_1^\bullet$,

then N is a structural live AC net.

Ⅵ. APPLICATIONS

In this section we will use the synthesis method to model the system that enterprise_1 and enterprise_2 rent a same plant to produce some product. Preparing raw and processed materials, enterprise_1 and enterprise_2 rent a plant to produce some product. Finished products are transferred to enterprise_1 and enterprise_2, respectively.

The model of N_1 and N_2 is described in Fig.2 and Fig.3.

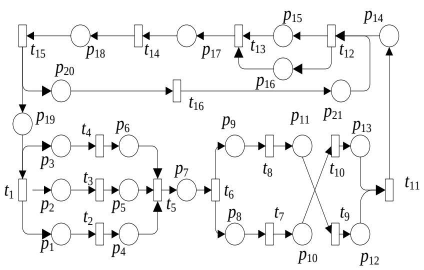

Fig. 2 Net N_1

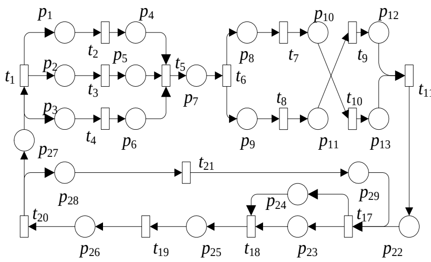

Fig. 3 Net N_2

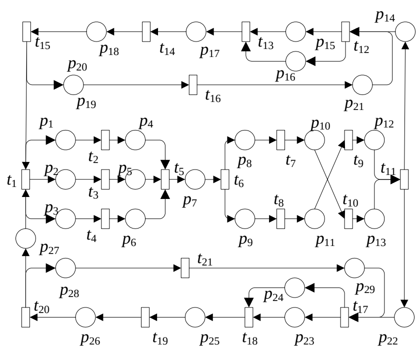

Fig. 4 Synthesis Net N

The synthesis net N is described in Fig.4. According to Fig.2 and Fig.3, it is easy to see that N_1 and N_2 are two structural live FC nets. By Theorem 5.1, the synthesis net N is a structural live FC net.

Ⅵ. CONCLUSIONS

In this paper we investigate structural property preservations of Petri synthesis net. A Petri net synthesis method is proposed, which is the key method to ensure the synthesis net preserving well behaved properties.

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