An Innovative Pricing Method for Telecommunication Services Pricing through American Options

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Abstract— **With the evolution of telecommunication networks and of their services the role of service provider is changed, so nowadays there is a coexistence of Network Operators and Virtual Operators. The difference between these players is not in the way they offer a service but primarily in their economic objectives and risk attitudes. Essentially, Network Operators own their infrastructures and typically have to sustain both fixed costs (CAPEX) and recurrent costs (OPEX), while Virtual Operators may have a simpler cost structure, mainly consisting of OPEX for the hire of network resources. Since these two operators can provide the same service in two different markets, their objectives differ substantially: on one hand, Network Operators aim to recover their costs with traditional pricing schemes (flat or time based), while the goal of Virtual Operators is to stay in the market, trying to attract as more users as they can through innovative pricing schemes (time varying, congestion pricing) providing, at the same time, QoS levels comparable to the corresponding ones of Network Operators. From above considerations we can understand the importance for Virtual Operators and for Regulatory Agencies to develop new pricing schemes. To develop a new pricing scheme it is necessary to study a model that shows vs. time the occupancy of network resources and moreover we present a method to estimate the final price of the service offered from Virtual Operator, for a given occupancy state of network resources, which is evaluated by American Options Pricing Scheme. This manner it is possible to address the problems of co-existence in the same communication market of these two service providers and the one of prevention of possible arbitrage, driven by different pricing schemes.**

Index Terms— **American option, Ornstein-Uhlenbeck process, Virtual Operator, congestion pricing.**

I. Introduction

In the present telecommunication market there two different types of operators: Network Operators which are the classical telecommunication operators, and Virtual Operators which are a newer type of operators. These two types of operators, which offer substantially the same service, coexist in the telecommunication market and are competing between them playing with two items: Service price and QoS. These two elements have a strong impact on the customers' decisions but they have also a totally different impact on the cost structure of Network and Virtual operators, because Network operators own their network and their infrastructure and have to support the costs deriving from the universal service, whereas Virtual operators don't own any network but hire it from Network operators and only own a small quantity of infrastructures typically gateways. So Network operators have to sustain fixed costs (CAPEX) and recurrent costs (OPEX) but they defined standard methods to determine the unitary service cost, whereas Virtual operators have a much simpler cost structure mainly consisting of OPEX but haven't today any efficient method, to determine the impact of costs on the unitary service. Another type of partners, that play an important role in the aforementioned scenario, are the Regulatory Agencies which establish some rules to be respected and fix the price of some elements. From what precede it is clear the importance of establishing a price structure, particularly for Virtual operators and for Regulatory Agencies. The framework presented in this paper is based on the consideration that the telecommunication market, together its related services, is similar to the financial market in which some assets can be bought as American Options, so with some important differences and cautions, telecommunication services can be priced using methods derived from those of financial analysis.

As is well known, an American option is a financial instrument [1] primarily used in the stocks' market, that can be of the two types call or put, but only the call options are useful for our aim and will be considered. A Call American option is a contract between two parties: the option buyer who, paying the option price, buys the right (but not the obligation) to purchase before a limit date a specified asset at a specified price K (strike price) and the option writer (the seller) who undertakes the obligation to sell him this asset at the price K.

A Call American Option is a typical risk hedging instrument that matches two different opinions on an asset: if the writer, who is assumed to own the asset at time t=0, thinks that the price of that asset will be upper bounded by K in the next time interval [0,T] he can sell a purchase option at that price to an option buyer, who on the contrary bets on the existence of (at least) one time instant t (0,T] when the price of the asset will be higher than the strike price K.

Option contracts are exchanged between the parties at the expense of money paid by the buyer to the writer.

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Finding the exact price of an American option contract has revealed one of the challenges in modern simulation and financial engineering [2]–[5].

To this purpose, the paper is organized as follows: in section II is presented an overview on the problems of the telecommunication market and of the present and near future pricing methodologies for telecommunication services; in section III is presented a technical background in which are highlighted: the mathematical basis for modelling the network resources through a fluid model, the motivation for which telecommunication services can be priced through the use of corresponding Call American options; in section IV it is completely outlined the theoretical development and the relative algorithm; in section V are presented the numerical results obtained using the standard classical methods, and in section VI are presented the conclusions with the foreseen future developments.

II. Overview

As it is written in the Introduction, nowadays in telecommunication there are two types of operators: Network operators which are the classical operators and Virtual operators which are a newer type of operators that don't own any network and hire the network resources from the first type of operators. Some other issues regard the increasing number of networks which are governed by economic concerns rather than by technology and we examine these issues in what follows. About the condition of two types of operators we can observe that this condition has brought to a strong competition between these operators that is played with the instruments of price and QoS, but the limited pricing instruments (flat, time based, and congestion based) and the small differences in the QoSs produced by the different technologies has brought to very similar prices. As a consequence the typical customer has a big degree of difficult in the selection of an operator, so there is a strong stickiness that obstacles the change of operator. This situation can be overcome only with a stronger difference in the prices for the offered services, so there is a strong request for new pricing instruments which permit to make a choice between the different players over the different networks. To name just a few of them we can mention:

- Network neutrality: according to Next Generation Networks paradigm, services can be provided by means of a plurality of access platforms, owned by different operators, which connect to a common IP backbone and interact through gateway functionalities, and they should be charged irrespectively of the underlying access network. This way any difference in pricing should be motivated not merely by the adoption of a particular technology platform, but by other perceivable parameters.
- Seamless QoS: bandwidth availability has greatly reduced the early concerns on QoS over packet networks. Today, users can experience practically the same QoS regardless the access platforms, the network backbone technology, with the only limitations due to the terminal capabilities and CoS they bought.
- Interconnection: the variety of access platforms and backbones is affecting the concept of network interconnection. Emerging concepts like competitive termination or VoIP peering [6] essentially express the same underlying idea: users can be reached via the less expensive path from origin to destination, and network nodes must be able to perform such routing decisions in real time, according to market rules rather then technical rules.
- Terminal evolution: software capabilities have evolved dramatically. Terminals can be equipped with software agents capable of taking complex decisions on behalf of the users in response to signals (congestion, pricing levels) coming from the network.

The new and aforementioned pricing instruments can the obtained, by and the adoption, with the suitable variations, of well known financial instruments. This brings us to a natural analogy between financial market and telecommunication market with a correspondence between financial assets and telecommunication services. Particularly the structure of requests for telecommunication services is quite similar to that of a financial derivative known as Call American option. To clarify this analogy it is evident that a telecommunication service customer can use it at every moment and he should desire to choice every time the best price for him, whereas the Virtual operator, who has a recurrent cost structure, because he hired a set of network resources, should desire to adapt his prices to the congestion situation of his network, maximizing the relative income and optimizing the investment. This situation has a precise correspondence in the financial market which is given by the Call American options. This financial instrument permits also to avoid the arbitrages which damage the Virtual operator because in an environment in which many operators compete offering the same services, a third party can buy a service a low price in a market and sell it a higher price in another market. With this analogy between telecommunication and financial markets we can say that the differences between Network operators and Virtual operators reside in the manner they stay in the market, in their economic objectives and in their pricing strategies. To simplify the matter, we assume that the same service (from user's perception perspective) is provided by two service providers S_A and S_B: S_A is a Network operator who owns his network, sustaining both investments (CAPEX) and operational costs (OPEX) to develop its business; S_B is a Virtual operator which provides his services using a leased network and whose costs mainly arise from the hire of a virtual network. The economic objectives of S_A and S_B deeply differ: S_A is a risk averse player whose primary objective is to recover its fixed and variable costs with a minor concern on the optimization of its (often abundant) resources; S_B has the primary goal of staying in the market, trying to attract the highest number of users to its virtual network, and this makes of S_B a more risky player. The different attitudes towards risks may reflect in the pricing strategies of S_A and S_B: in fact, while S_A will presumably offer its services at a flat rate or at a constant resource usage rate, S_B may be attracted by evolutionary pricing schemes (congestion pricing) which encourage the use of network resources during low-load periods and discourage new arrivals when the network is close to congestion. We further assume that minutes of traffic can be freely traded from the markets represented by providers S_A and S_B, which we define respectively risk less and risky markets. In the risk-less market, minutes can be purchased at a predictable price, on the contrary in the risky market minutes are purchased at a time varying price, following a demand-offer law, represented by the level of congestion: the higher the number of occupied resources, the higher their scarcity and price. If the perceived QoS from the two providers is indistinguishable from user's perspective, the situation is formally identical to what happens in the financial market where risk less assets (money drawn from one's bank account) can be exchanged for risky assets (equities traded in the stocks' market). In the telecommunication context, Call American options can be sold in the risky market to upper bound at every time the price per minute of traffic at a strike price K, even if the congestion price can fluctuate above that level.

A. Network Type vs. Right Price

As was said in advance another important problem to be considered is the increasing number of networks which are governed by economic concerns rather than by technology. In particular one of the hot topics in the everyday life of telecommunication operators is the selection of the proper price to associate to each offered service. This task is normally accomplished by the marketing divisions before the launch a new service, by means of surveys on user behaviors willingness to pay and traffic forecasting to determine the ”right” price.

Lately, it has been argued that pricing is in active part of the network planning process, and it should be dealt as a mean to control traffic load. In the last decade many efforts have emerged to formalize this new aspect, and to provide engineering tools to the network designer's community.

Some approaches have emerged. For example auctions have been proposed as a mean to set the appropriate price of a service. Auctions are considered an efficient mechanism to set prices, since they require little a priori knowledge of the potential market's response to the offer of the items or services for sale. Since the value of the item to the bidders is signaled by the bids themselves (when the auction mechanism is correctly defined in order to get a so-called truthful auction), auctions represent an implementation of value pricing: the final price is consistent with the winning bidders' valuations (meaning that it is lower or equal to them). In addition, the auctioneer, gathering the whole set of bids, has an immediate picture of the present state of the demand function, which is what every a priori mechanism for setting prices would require. Operationally, set the price by an auctions means that each packet in a session should carry a signal on the maximum willingness to pay expressed by the end user. The service provider (the auctioneer) can run a slotted auction at the beginning of every interval, admitting to the network only those packets whose bids are beyond a threshold or up to saturate the capacity of the network [15]. A great number of proposals have been set forth regarding the mechanisms to allocate resources and to determine the prices; the most noticeable one is the Vickrey auction, which assigns the item to the k most generous bidders at the price offered by the k +1-st bidder (i.e. the largest among the rejected bids). This mechanism, also known as the single-price auction since the price is equal for all the winning bidders, has the advantage of being perceived as fair and satisfies the important condition of truthfulness, i.e. of inducing the bidders to declare the true value of the item (which varies from bidder to bidder). It is worth noticing the fact that under this pricing mechanism, price levels are variable, and the price evolution at each slot can be expressed as a stochastic process. Auction based traffic enables a fair mechanism of traffic control: in fact, only the most valuable traffic (in terms of user willingness to pay) is admitted and the remaining traffic is discarded. Despite the formal elegance and the low computational burden associated to auction mechanisms, and to Vickrey's in particular, this scheme has not been widely adopted in the telecommunications arena. This is due to the fact, as highlighted by [13], that truth revelation, even if it is a desirable property at the individual level since it enables the extraction of the maximum surplus, is very costly at the aggregate level. Pricing every participant at the bid of the k+1 (the first excluded) bidder wastes a great quantity of value and pushes the total revenue of the auction to a very low level, depending also on the distribution of the values attributed to the items by the end users [10].

Other dynamic pricing schemes are intended as a mean to strengthen the traffic control capabilities of the TCP protocol. In fact, when a link of the network becomes overloaded, one or more packets are dropped; and the loss of a packet is taken as an indication of congestion. In such cases, the destination informs the source, and the source rate is decreased. The TCP protocol then smoothly increases its rate until it again receives an indication of congestion. This cycle of increase and decrease serves to optimize the available bandwidth is available, and to share it between flows.

This approach is widely adopted, but recently some holes have been discovered, namely the incentive for the most aggressive users to modify the TCP protocol in order to get a larger share of the available bandwidth, or even to avoid triggering any form of congestion control. One mechanism that blocks this free-riding opportunities for malicious users and enforces TCP to work properly is pricing. In many works in literature, price is regarded as the missing feedback signal from the network to the end-users that plain TCP does not implement. In fact, if the congestion effects of users' actions can be made known to all end-nodes, then the end-nodes themselves are capable to determine what should be their demands upon network resources. Many pricing alternatives exist to signal the congestion effect of load increase back to the users. To name just the most relevant contributions, [14] has defined an approach to telecommunications service pricing, called Paris Metro Pricing, consisting in a logical separation of the network in two sub-networks the two networks are identical, but each packet traversing the first class network is charged more than a packet transmitted over the second class: in this way each user may decide for each packet which price to pay. By this trick, both the network provider and end users have incentives to use network resource properly, and any malicious attempt of free riding is discouraged both by the high price level per packet in the first class network and by the counter-measures and complains made by the users who require high quality levels. Another pricing scheme has been proposed by [12]. Their approach termed proportionally fair pricing is based on the concept of shadow pricing. This quantity is defined as the marginal increment in expected cost at the resource for a marginal increment in load. Essentially, it is the cost of congestion. It is shown that in case of congestion, the price to associate to each packet can be expressed as the product of two quantities: the individual load and the shadow price, proportional to the probability that congestion occurs. So, two effects are captured by this pricing scheme: the individual contribution to congestion and the externality imposed by all other users to congestion. This approach is further developed by [7] who explicates the individual contribution to congestion as a function of the duration of the presence of a user in the network. The longer the duration the higher the probability to reach a congestion during a session.

All such pricing schemes have shown some difficulties to obtain widespread adoption, this is due, in the writers' opinion to the scarce network literacy of telecommunications users. The terms bandwidth and traffic volumes are very popular only among the new generation of TLC users and have poor impact on the choices of more aged users. Moreover, under a dynamic pricing scheme the users loses the control of his expenditures, due to he fact that the instantaneous price level depends on some factors (other users' demand) which are out of his control. Users are very concerned about their expenditures and are more sensitive to price variations rather than to quality improvements. This statement is justified by what normally happens in mobile voice telephony. Despite the presence of a high voice quality technology (UMTS), the number of old generation (GSM) mobile users is still very high. It is also true that users are not totally averse to risks. Referring again to mobile telephony, everyone correctly recognizes the positive effects of pre-paid tariff plans, against post-paid, as a leverage to control expenditures and to balance the traffic. But it is rarely highlighted the fact that users pay their fees before receiving the service, and in current 2G networks the risk of cal interruption or poor voice quality is still high. These drawbacks have no or very limited impact on users choice, who highly appreciate GSM service, whose revenues are still increasing. We have to argue that most users prefer to run the risk of receiving poor quality, provided that they are allowed to control their expenditures.

The effort of this work is to translate the technical terms of bandwidth and quality of service in terms of economic risks, and to propose a new pricing regime, where dynamic pricing can co-exist with flat or time based pricing. Users in this way can choose between a time varying pricing or a constant pricing according to their risk attitudes. Moreover, even in the case of selection of a dynamic tariff plan, a user can purchase an option contract from the service provider, which bounds the maximum level of price oscillation, thus allowing the optimization of user's expenditures. We believe that this pricing policy can be well suited for virtual operators, because these new players are more aggressive in the market than traditional operators, and also because their cost structure gives them the chance of a more risky attitude.

III. Technical Background

In this section we expose the technical background of the main points of our pricing process that are: fluid model of the use of network resources and the price of relative service, and the methods for the pricing of Call American options.

A. Used Resource Network Model

As first thing we study the fluid models associated to the spot price, we largely refer to [7], [9]. We assume that the risk averse provider S_A owns a CAC based link, where service requests arrive according to a Poisson process with frequency λ, mean duration $\frac{1}{\mu}$ and constant bandwidth r_1=1; a new arrival is admitted if at least one circuit is available. The number n of circuits can be dimensioned according to the Erlang-B formula for M/M/n/n queues with ϵ_1 blocking probability. Provider S_A charges the resource usage at a constant rate per minute p €cent/min (flat rate can be reduced to a resource usage per minute rate) which is independent of the number of currently occupied circuits. The average instantaneous revenue rate P_1 from the system (neglecting for simplicity the blocked arrivals) is $p\frac{\lambda}{\mu}$ €cent/min, where $\frac{\lambda}{\mu}$ is intensity of traffic into the network. Provider S_B may choose to offer the same service over a virtual link of capacity C on a best effort basis. Arrival and departure rates are the same as provider S_A, unitary bandwidth is $r_2 \leq r_2$. Capacity C is dimensioned according to an M/M/∞ service system with the following rules [8], based on the stationary probability of the state i of the queue

$$C = \left(\frac{\lambda}{\mu} + \zeta\sqrt{\frac{\lambda}{\mu}}\right) r_2$$

and

$$P_r\left(i \geq N^* = \frac{C}{r_2}\right) \leq \epsilon_2.$$

According to the best effort paradigm, any new arrival is always accepted into the system and ϵ_2 represents the fraction of time that the number of concurrent calls is higher than $\frac{C}{r_2}$ and all users experience a service downgrading "on the fly". The probability threshold ϵ_2 can be engineered to give users the same perceived quality as the blocking probability ϵ_1.

A diffusion approximation for the stochastic process of the state i_t of the M/M/∞ queue [7] is

$$di_t = -\mu\left(i_t - \frac{\lambda}{\mu}\right)dt + \sqrt{2\lambda}dW_t \tag{1}$$

where W_t is a standard Brownian motion [4]. The stochastic differential equation (SDE) (1) is a mean reverting Ornstein-Uhlenbeck process, with mean reversion speed parameter μ, the inverse of average call duration, and stationary mean $\frac{\lambda}{\mu}$. At steady state, the state i_∞ of the queue is Gaussian with mean $\frac{\lambda}{\mu}$ and variance $\frac{\lambda}{\mu}$.

Following [12], a scheme which provides a practical and reasonable application of fair congestion pricing is to set the price in proportion to the stationary probability of exceeding the threshold N^*, provided that a new user, arriving at time t, finds the system in state $i_t \leq N^*$; if the new user arrives when the system is overloaded $i_t > N^*$, the price is made insensitive on the initial state. We have

$$p_t = \psi(i_t) = \begin{cases} \gamma Pr\{i_\infty \geq N^* | i_\infty > i_t\} & i_t \leq N^* \\ \gamma & i_t > N^* \end{cases}$$
$$= \begin{cases} \gamma \dfrac{1-\Phi(\frac{N^*-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}})}{1-\Phi(\frac{i_t-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}})} & i_t \leq N^* \\ \gamma & i_t > N^* \end{cases} \tag{2}$$

where Φ is the standard normal distribution function. The parameter γ in this pricing formula is a degree of freedom which can be determined by equalling the mean revenue rate P_1 of the network operator S_A and that of the virtual operator S_B, as follows [1].

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T i_t\psi(i_t)dt = \frac{\lambda}{\mu}p \tag{3}$$

[1]Alternatively, γ can be interpreted as the difference between the highest and the lowest instantaneous price offered by the virtual operator. In this case, the price formulation for S_B becomes

$$p_t = \psi(i_t) = \begin{cases} p_{min} + (p_{max} - p_{min}) \dfrac{1-\Phi\left(\frac{N^*-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}}\right)}{1-\Phi\left(\frac{i_t-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}}\right)} & i_t \leq N^* \\ p_{max} & i_t > N^* \end{cases}$$

If the maximum price p_{max} is fixed, then the degree of freedom is the minimum price p_{min}, which can be determined by solving equation (3) in p_{min}.

B. American Call Option Formulation

Now, we explain the theory of American Call Option that will be used to calculate the final price of the service.

An American option can be exercised at any time within time interval [0,T], and we, without essential loss of generality, may suppose that the option can be exercised only at $T+1$ times 0, 1, 2, ..., T. We assume that the underlying security prices are denoted by $(p_t \in R^d : t = 0, 1, ..., T)$. Note that in this case the underlying security price is the service price. In addition, the description of the state p_t may include variables such as the values of the stochastic interest rates and volatilities, and supplementary path dependent information, so that the resulting process is a Markovian one. Therefore, each p_t may take values in a more general space denoted by P. The value of the option at time t if exercised at that time is denoted by $g_t : P \to R^+$ (i.e., its exercise value or intrinsic value). Let T_k denote the set of stopping times taking value in k, k +1,..., T (recall that is a stopping time w.r.t. $\{p_k\}$ if $\{\tau = k\}$ is determined by observing $p_1,...,p_k$). Note that each stopping time represents an exercise strategy by the owner of the option at time period k. Let

$$J_k(l) = \sup_{\tau\in T_k} E[g_t(p_t) \mid p_k = l], l \in P \tag{4}$$

where, the expectation is taken under the risk-neutral measure. Then $J_k(l)$ is the value of the option at time k given that the option is not exercised before the time k. The initial state p_0 is fixed and known. So, our pricing problem is to evaluate $J_0(p_0)$. This formulation is sufficiently general to include discounted payoffs through appropriate definition of the $\{p_k\}$ and $\{g_k\}$ [4], and hence these are not explicitly stated. It can be shown that there exists an optimal exercise policy specified by a collection of increasing stopping times $(\tau_k^* : k \leq T)$ where

$$\tau_k^* = inf\{m \geq k : g_m(p_m) \geq J_m(p_m)\}$$

then

$$J_k(l) = E[g_{t_k^*}(p_{t_k^*}) \mid p_k = l], l \in P$$

a.s. Note that the knowledge of the optimal policy corresponds to the knowledge at each time of the state if that time is optimal to exercise the option or to hold on to it. Suppose that the p.d.f. of p_{k+1} conditioned on p_k = l evaluated at y is given by $f_k(l, y)$ under the risk-neutral measure. Let $Q_k(l)$ denote the conditional expectation

$$E[J_{k+1}(p_{k+1}) \mid p_k = l] = \int_P J_{k+1}(y)f(l,y)dy \tag{5}$$

We refer to Q = ($Q_k(l)$: l $\in$ P, k $\leq$ T-1) as the continuation value function because $Q_k(l)$ denotes the value of the option at time k and state l if it was not exercised at time k. It is not feasible to evaluate $J_k(l)$ by evaluating expectation $E[g_\tau(p_\tau)|p_k = l]$ for each stopping time τ in (4).

Fortunately, it can be shown that the value functions J = $(J_k(l) : l \in P, k \leq T)$ satisfy the following intuitively plausible backward recursions [4]:

$$J_T(l) = g_T(l)$$

$$J_T(l) = max\{g_k(l); Q_k(l)\}$$

The evaluation of the value functions using these recursions requires the discretization of the state space and the successive recursive approximate solution. However, when the dimension of the underlying process is large, even this becomes computationally unviable due to state-space blow-up.

The aforementioned solution of these backward recursions bring s us to the estimation of the value function and of the associated optimal exercise policy $(\tau_k^*, k \leq T)$. Often, this procedure is realized with a two-step procedure. First, the optimal policy is learnt approximately. Normally, this policy is evaluated using the standard Monte Carlo procedure, but there exists a new and improved method based on Cross-Entropy [11]. Note that the value function corresponding to the policy learning is the lower bound of the option price. In particular, we see that a zero variance importance sampling estimator always exists although its determination requires a-priori knowledge of the value functions [4]. Though not implementable, the form of zero variance estimator is useful as, once we develop approximations for the value functions, these in turn provide an implementable approximate zero variance estimator.

IV. American Call Option Pricing Formulation

In this section will be showed as it is possible to calculate the final price of the service using the spot price (III) and using the American Call Option theory (III), again, we show as it is possible by the Option to avoid the arbitrage problem, which makes not fair the market.

A. The Arbitrage Problem

As written previously, in the telecommunication market we can find Network Operators and Virtual Operators, so it is possible to buy the same service (asset) in two different markets. As it is well known in the financial market this situation can allows a third part to make an "arbitrage".

An arbitrage is an opportunity to buy an asset at lower price in a market and to resell the same asset at a higher price in another market.

In telecommunications it is a well know concept, arising particularly in the international traffic market, where direct traffic from one nation A to another B can be routed through a third nation C, if the following relation between interconnection tariffs holds $P_{AB} > P_{AC} + P_{CB}$. This situation is know as geographical arbitrage, and it has been very common due to the long duration of interconnection contracts and the bilateral nature of such agreements.

This kind of arbitrage is progressively vanishing due to the advances in technology which have reduced the time scale of routing decisions and to the presence of many alternatives to terminate a call (competitive termination). We now exemplify how in a free market of traffic with two providers, a risk averse Network Operator and a risky Virtual Operator, another type of arbitrage can arise. Let's assume to observe the free market in two different time instants (t=0 and t=1 for simplicity).

The market at time t=0 consists of two portfolios: the first one made of one option, with unitary price J, to purchase a minute of traffic at strike price K and u minutes of traffic from the riskless market at unitary price E, the second one made of v minutes of traffic from the risky operator at unitary price S. At time t=1 two outcomes are allowed. If the virtual network is congested, the price of the "risky minutes" will go up from S to S^+ (with $S^+ > K$) and the option can be exercised providing a net value (pay-off) S^+-K. Without loss of generality we can assume that the price of traffic in the riskless market will be E at t=1.

In this case the market is worth

$$\begin{cases} \text{portfolio1:} \quad (S^+ - K) + uE \\ \\ \text{portfolio2:} \quad vS^+ \end{cases}$$

In case of underload at t=1, the price level of risky minutes will go down from S to S^- (with $S^- < K$) and the option will not be exercised. In this case the market is worth

$$\begin{cases} \text{portfolio1:} \quad uE \\ \\ \text{portfolio2:} \quad vS^- \end{cases}$$

In order to prevent arbitrages the value of the two portfolios must be the same at t=0 and *t*=1, which is the case if we set

$$v = \tfrac{S^+ - K}{S^+ - S^-} \ ; \ u = \tfrac{S^-}{E} \tfrac{S^+ - K}{S^+ - S^-} \ .$$

The value J of the option at time t=0 (the time when the option contract is underwritten) is

$$J = \frac{S^+ - K}{S^+ - S^-}(S - S^-) = J(1)\left(\frac{S - S^-}{S^+ - S^-}\right) \quad (6)$$

if we indicate with $J(1)$ the value of the option at *t*=1.

The equality in equation (6) defines a distribution on $J(1)$ as follows

$$J(1) = \begin{cases} S^+ - K & \text{with probability} \quad \frac{S - S^-}{S^+ - S^-} \\ \\ 0 & \text{with probability} \quad \frac{S^+ - S}{S^+ - S^-} \end{cases} \quad (7)$$

which is independent of the actual probability of the outcomes S^+ and S^- (the so called real world probabilities, which do not enter the option pricing formula), but only depends on a new probability measure which is a function of the value of the outcomes S^+ and S^- (the so called risk neutralized probability). A fundamental result for asset pricing in a complete market (the reader may refer to [4] for a thorough explanation of these terms) is that the price of the option at time t=0 is the risk neutralized expected value of the option at its exercise time. In formulas, $J=E^*(J(1))$, by indicating with $E^*(\ )$ the expected value under the distribution(7).

If the value of the two portfolios is not the same at t=0, a third party arbitrager could buy the low price portfolio and sell the high price portfolio at a net positive profit with zero initial investment.

Back to the telecommunication counterpart of this example, the arbitrage opportunity is represented in figure 1, where an arbitrager, provided that the option is not properly priced, can alternatively buy/sell risky minutes and options and sell/buy risky minutes, routing the traffic where it is more convenient and making a net positive profit at no expenses.

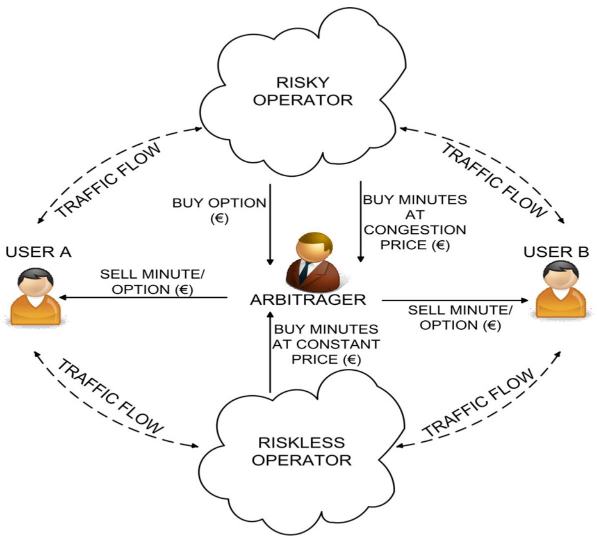

Figure 1. An Example of Arbitrage Opportunity

In this toy deterministic model only one exercise opportunity is allowed, and the option is both european and american. In more realistic situations the option can be exercised at any time before it expires, and price evolves according to a more complex stochastic process, depending on the current load on a congestible resource.

In the next sections we will develop the stochastic process that puts together services demand, resource allocation and spot price, and we will give our proposal for an american option formulation of service pricing in a virtual network.

B. The Risk-Neutral Model

As showed above the arbitrages bring to a no equilibrate market and this condition is not a good one for operators and customers, so it is very important to avoid the arbitrage. For the above reason, when we use financial options, some considerations about probability space have to be taken.

The dynamics of the spot price is governed by the SDE (1) and the price formula (2). This is the "real world" spot price stochastic process, regulated by the state of the M/M/∞ queue. In order to develop an option pricing formulation for S_B, we need to subtract the risk from the instantaneous price, which can be accomplished as in [9] by introducing a proper bias on the pricing formula (2). In fact, by lowering the mean $\frac{\lambda}{\mu}$ of the Ornstetin-Uhlenbeck process in equation (1) to the quantity $(1-\delta)\frac{\lambda}{\mu}$, with $\delta > 0$, the effect is to reduce the instantaneous price to the quantity $\hat{p}_t$

$$\hat{p}_t = \hat{\psi}(i_t) = \psi(i_t - \frac{\delta\lambda}{\mu}) \tag{8}$$

The amount of bias δ to introduce in equation (8) to fully determine the risk neutralized price $\hat{p}_t$ can be determined by solving the following equation for δ

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T i_t\psi(i_t - \frac{\delta\lambda}{\mu})dt = \alpha(1+\beta)\frac{\lambda}{\mu}p \tag{9}$$

where α is the ratio between the costs and the revenues of the virtual operator and β is the riskless interest rate. The quantity $p^* = (1+\beta)\alpha p$ has a straightforward economic interpretation: for each 1€cent/min that the virtual operator earns on average, α €cent/min is the amount of money that he spends to hire resources, and $(1+\beta)\alpha$ €cent/min is the expected return if α €cents/min are spent on a riskless activity.

Also, $\frac{\lambda}{\mu}p^*$ is the so called market price of the risk, or the average riskless rate that the provider S_B requires to enter the risky market with a forecast revenue rate of $\frac{\lambda}{\mu}p$. It is a degree of freedom which depends on its risk attitude (through the interest rate β) and economic efficiency (through the cost-revenue ratio α). The riskier the provider S_B, the lower market price of risk. Roughly speaking, an operator is said to be risky if he is available to move part of his revenues to a risky outcome, provided that this choice gives him a better chance to get a higher market share.

C. Service Price Formulations

The price of an american option contract, indicated with the notation $J_0(T,i_0,K)$ underwritten at time t=0, when a new service request arrives, for an exercise interval [0,T], strike price K €cent/min and initial state i_0 is given by

$$\begin{aligned} J_0(T,i_0,K) = \max_{\tau\in(0,T]} E^*[(\hat{p}_\tau - K)^+|i_0] = \\ \max_{\tau\in(0,T]} E^*[max(\hat{p}_\tau - K, 0)|i_0] \end{aligned} \tag{10}$$

where E^* is the expectation of $\hat{p}_t$ under the risk neutralized probability measure, and τ is a stopping time [4] not exceeding T . $J_0(T,i_0,K)$ has the dimensions of €cent/min and it can be regarded as a bonus charge that gives a user the right to purchase traffic at K rather then at p_t any time that p_t >K.

Usually, the occurrence of the event p_t >K and its frequency during a call are out of the user's control. This fact could not be perceived as fair by users. Thus we propose the following two different pricing formulations, based on the elementary building block of the american option contract J_0(T,i_0,K), in the form of a response charge paid in advance at the beginning of a call, which is a familiar pricing feature to most users in today's tariff plans:

Formulation 1)

1 At t=0 a new service request arrives, when the number of already accepted users is i_0, declaring its duration T (this value can be estimated from the time series of a user) and its maximum price per minute K

2 The network evaluates and notifies back to the user the quantity O_0(T,i_0, K given by

$$O_0(T, i_0, K) = J_0(T, i_0, K)E\{\int_0^T I[(p_t > K)|i_0]dt\} \quad (11)$$

where I() is the indicator function giving 1 when the argument is true; O_0(T,i_0,K) can be interpreted as a response charge in €cent, consisting in the product of J_0(T,i_0,K) in €cent/min and $E\{\int_0^T I[(p_t > K|i_0)]dt\}$ which is the average time during the call that price p_t will be higher than the strike K. Paying once and in advance O_0(T,i_0,K) the user is assured that the instantaneous price of the call will never exceed K.

3 A software agent in the terminal is programmed to react to O_0(T,i_0,K) accepting or refusing the option contract after comparing this value with a threshold in €cent predefined by the user.

Formulation 2)

Formulation 1 is valid on a call by call basis, depending on the declared service duration T and the initial state i_0 of the system. If this information is not available, but a distribution is available for T and i_0, the virtual operator can still develop an american option pricing scheme by observing that O_0(T,i_0,K) is a conditional expectation, depending on the declared duration T of the call and on the initial state i_0 of the system. By averaging O_0(T, i_0,K) as follows

$$O_0(K) = \int\int O_0(T, i_0, K)f(T)g(i_0)dTdi_0 \quad (12)$$

where f(T) and g(i_0) are the probability density functions of T and i_0, we obtain an option contract O_0(K) which can be applied to any call irrespective of its duration and of the initial state i_0 of the system at t=0. In this alternative, pricing formulation for the virtual operator simplifies further. In fact, the virtual operator can enter the market offering couples (O_0(K),K) of strike prices K and response charges O_0(K) for each selected strike price. If the user accepts to pay in advance the response charge O_0(K) at the beginning of each call, he is assured that during the call (no matter how long this will last) his maximum price per minute will be always upper bounded by K.

D. Discretization Of The Problem

In the III and in this section a formulation to calculate the price for a telecommunication service it has been given. Following this methodology to obtain the final price of the service, all formulas require a closed form solution to calculate the necessary parameters. But finding a closed form for equation (3) and (9) is a hard challenge. Instead, it is possible to estimate those parameters by the use of a numerical method, but to do this a discrete form for above equation must be written. In the first step the occupancy of network resources it must be found. As it is well known [17] the solution of equation (1) is a Gaussian random variable with a mean value M and a variance V. It is possible to write this solution by a standard normal variable N(0,1) [16] an in this case we have:

$$i_t = \sqrt{V}N(0, 1) + M. \quad (13)$$

To calculate M we Know that

$$i_t = i_0e^{-\mu t} + \frac{\lambda}{\mu}(1 - e^{-\mu t}) + \sqrt{2\lambda}\int_0^t e^{-\mu(t-s)}dW_s$$

[17] the mean value of i_t E[i_t] is

$$E[i_t] = E[i_0]e^{-\mu t} + \frac{\lambda}{\mu}(1 - e^{-\mu t})$$

in fact $\sqrt{2\lambda}\int_0^t e^{-\mu(t-s)}dW_s$ is 0 because is the sum of Gaussian variables with 0 mean value. Following the same reasoning for the variance we obtain:

$$E[i_t - E[i_t]]^2 = \frac{\lambda}{\mu}(1 - e^{-2\mu t})$$

from above equation we can verify that in stationary regime M=V=$\frac{\lambda}{\mu}$. Then, with those values of M and V it is possible rewrite the equation (13) in this manner:

$$i_t = \sqrt{\frac{\lambda}{\mu}(1 - e^{-2\mu t})}N(0, 1) + E[i_0]e^{-\mu t} + \frac{\lambda}{\mu}(1 - e^{-\mu t})$$

this equation can be discretized, so using Euler discretization formula [17] we obtain:

$$i_t = \sqrt{\frac{\lambda}{\mu}(1 - e^{-2\mu Dt})}N(0, 1)+i_{t-1}]e^{-\mu Dt}+\frac{\lambda}{\mu}(1-e^{-\mu Dt}) \quad (14)$$

where Dt is the discretization interval. Since, if i_t is known it is possible to estimate the first parameter γ of equation (2) from equation (3), if we rewrite ψ(i_t) as γA(i_t) where A(i_t) is

$$A_t = \begin{cases} \dfrac{1-\Phi(\frac{N^*-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}})}{1-\Phi(\frac{i_t-\frac{\lambda}{\mu}}{\sqrt{\frac{\lambda}{\mu}}})} & i_t \leq N^* \\ 1 & i_t > N^* \end{cases}$$

then

$$\gamma = \frac{\lambda}{\mu} p \frac{1}{\frac{1}{T}\int_0^T i_t A(i_t) dt}$$

the integral can be calculated with a approximation formula, (for example with Gauss formulas, using the samples of it obtained with (14). To obtain an accurate value of γ it is possible to calculate many values of γ , to calculate the occurrences of each value and to choose that with the major occurrence. When the value of γ is know it is possible to resolve the equation (9), with Raphson-Newton method for example, to obtain δ parameter. Parameters δ and γ are used to calculate the price of equation (8), the set of these prices in a time interval [0,T] is used to evaluate the option price (10) for the service as showed in [11]. The final price that a Virtual Operator requests to customers can be calculated using equation (11) or (12), where the integral can be calculated using again approximated formulas.

In the following section will present some numerical results that we obtained following the showed algorithms.

V. Numerical Results

In addition to the development of the framework for the adoption of an American option pricing scheme for a virtual operator, we have also run Monte Carlo simulations of the Ornstein-Uhlenbeck diffusion approximation process of an M/M/∞ queue and of the spot price under both natural distribution and risk neutralized distribution to assert the validity of our proposal. The selected simulation parameters are: mean arrival rate λ=0.05[1/sec], mean departure rate μ=0.0055 [1/sec], cost/revenue ratio α=75 %, riskless interest rate β=10 %, QoS threshold N*=17, reference price of network operator p=15 €cent/min. The resulting market price of the risk is $\frac{\lambda}{\mu}p$=112.5 €cent/min. After Monte Carlo simulation the estimated values of parameters γ and δ of the model are γ =282.3 and δ =3.95 %. Figure 2 shows a typical plot of the instantaneous state i_t of the M/M/∞ queue and the corresponding price p_t and under natural and risk neutralized distributions. Option values J(T,i_0,K), estimated as in equation (10) for i_0= 4, 8, 12, T= 3min and T= 2min are represented in figure 3 for strike prices ranging from 25 % to 100 % of the reference circuit-like price p. Figure 4 shows the response charges O_0(K) when the initial state is drawn from a Gaussian distribution with equal mean and variance $\frac{\lambda}{\mu}$= 9,09 and service duration is taken from an exponential distribution with mean 3 min; it is worth notice the fact that due to the trade off between low strike prices and high option prices, users have no incentive to cheat declaring low strike prices, but instead they can learn their resource usage and the typical load on the system during their presence and choose the couple (O_0(K),K) that best fits their resource usage patterns.

VI. Conclusions

We have proposed an American option based pricing scheme for telecommunication services provided by Virtual operators. This scheme is valid under time variable

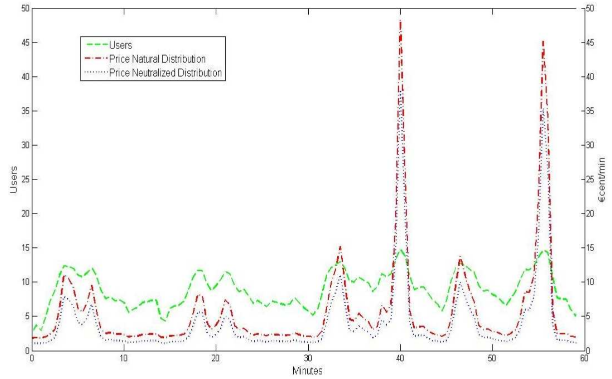

Figure 2. users and price (natural and risk neutralized distributions) vs. time λ=0,05 [1/sec], μ=0.0055 [1/sec], δ=3.95%, γ=282.3

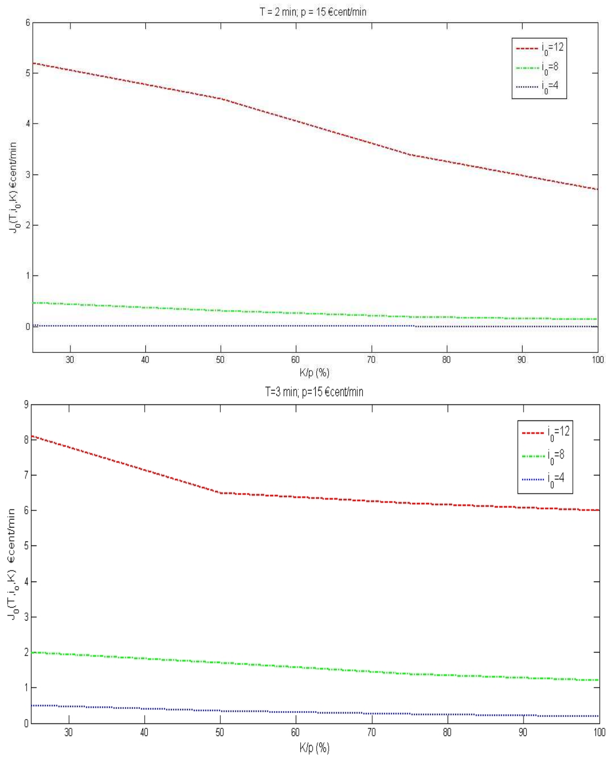

Figure 3. Option Values $J_0(\mathrm{T}, i_0, K)$

pricing regimes which encourage the use of resources when the network is lightly loaded and discourage new arrivals during heavy-loaded periods. Option pricing is the elementary building block for a fair sharing of risk between users and operators, giving rise to very simple tariff plans based on response charges and maximum rate per minute, similar to those currently adopted by most operators worldwide. All the parameters needed to accomplish the pricing of American options have been estimated by Monte Carlo simulations. For American options pricing in presence of high capacities of the system or a high number of option exercise times, this approach can become computationally too heavy. To overcome these difficulties we developed a new method based on enhanced financial engineering [11] and vari-

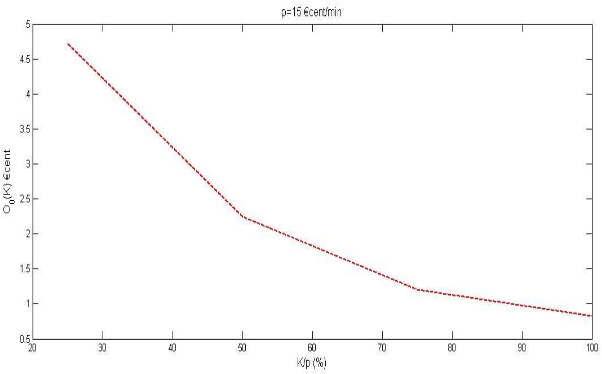

Figure 4. Response Charge $O_0(K)$

ance reduction techniques (Cross Entropy Importance Sampling) [10], [11] to reduce the computational costs associated with Monte Carlo simulations and to obtain fast and accurate estimates of the American option contracts for a wide range of system parameters. We are also extending this new algorithm for the p.d.f.s associated with telecommunication services.

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