Modeling of IEEE 802.11 DCF for Transient State Conditions

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Abstract—**Performance of IEEE 802.11 distributed coordination function (DCF) has been studied by several authors under steady-state conditions. Behavior of DCF under transient state conditions and its modeling is important for analyzing several scenarios, such as, initialization of nodes in a wireless ad hoc network. In this paper, we present a mathematical model for IEEE 802.11 DCF to capture its behavior under transient as well as steady state conditions. Numerical results obtained from our model have been shown to be in agreement with results published in [5] under steady-state conditions.**

Index Terms—**IEEE 802.11 DCF, transient state, steady state, contention/backoff window**

I. INTRODUCTION

IEEE 802.11 has been standardized and widely adopted for WLANs in the past few years [1]. In the specification, it specifies two fundamental access mechanisms, i.e., point coordination function (PCF) and distributed coordination function (DCF). DCF mechanism supports ad hoc networking configuration and has been widely adopted in wireless networks. DCF mechanism is based on the carrier sense multiple access with collision avoidance (CSMA/CA) protocol. In DCF, data frames are transmitted via two mechanisms, i.e., basic access mechanism and request-to-send (RTS)/clear-to-send (CTS) mechanism. The basic access mechanism is illustrated in Fig.1. As per this protocol, a node must initially listen to the channel before transmitting its packet. If there is no transmission ongoing for a short interval (i.e., DIFS), the node may transmit its packet. If the medium is busy, a node has to wait till the end of current transmission. It will then wait for an additional DIFS time, and then generate a random delay before transmitting its packet. This delay is uniformly chosen in the range (0, W-1), where W is the length of minimum contention window. If there is no other transmission before this time period expires, a node will transmit its packet. If there are transmissions from other nodes during this time period, a node will freeze its backoff counter until the end of current transmission. Then the station resumes decrementing its counter value only after the channel is found to be idle for DIFS time. Whenever the backoff value of a node reaches 0, it transmits its packet immediately. If the data packet is not transmitted successfully, the length of contention window W is doubled until its maximum value $W_{\max.}$ The backoff window is reset to $W_{\min}$ whenever a data packet is transmitted successfully. We focus our analysis only on basic access mechanism.

The performance analysis of IEEE 802.11 DCF has been studied by several authors through simulation [9-10] or through mathematical modeling [2-5]. In [9-10], authors proposed a modified backoff algorithm for IEEE 802.11 DCF in a mobile ad hoc network (MANET). They observed through extensive simulation studies that their algorithm performs better than algorithms proposed in [1], [11], [12] with node mobility, in terms of packet delivery ratio (PDR) and average end-to-end delay (EED). In [2-5], authors have analyzed performance of IEEE 802.11 DCF under steady-state conditions in terms of throughput, delay, etc.

Initialization of nodes in a wireless ad hoc network is an important task in the absence of fixed infrastructure like dynamic host configuration protocol (DHCP) server and access point (AP) [6-8]. It is a process that involves assigning each of the wireless nodes a unique ID in a distributed manner and occurs when network is being setup. Hence, mathematical models presented in [2-5] cannot be used to study this transient phenomenon. Therefore, we felt the need and importance to develop a mathematical model to study the behavior of IEEE 802.11 DCF for transient state conditions.

The paper is organized as follows. Section II describes the modeling of DCF. Numerical results followed by discussions are presented in Section III. In Section IV, we present our conclusions.

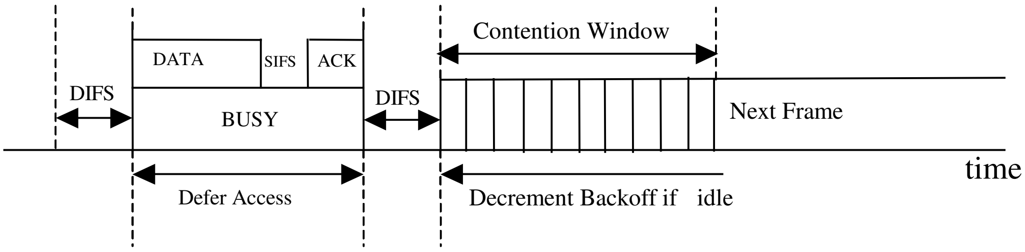

Fig.1. The IEEE 802.11 basic access method

II. MODELING OF IEEE 802.11 DCF

In our work, we consider a two-state channel model as shown in Fig.2. We assume that the channel goes to idle state with probability one immediately after busy state of the channel. This assumption is justified because none of the nodes will have a zero backoff value immediately after packet transmission by any other node. This assumption also ensures fairness amongst nodes contending for channel access. We also assume that the channel is error-free and backoff window length (*W*) is constant (i.e., only one backoff stage). We assume that there are *N* active nodes in the network.

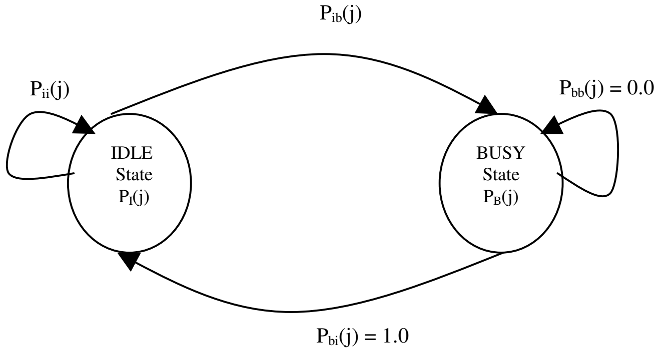

Fig. 2. Two-state Markov chain model for the channel

Let $P_I(j)$ and $P_B(j)$ denote the probabilities that the channel is in idle state and busy state respectively at any j^{th} slot. Let $P_{ii}(j)$, $P_{ib}(j)$, $P_{bb}(j)$ and $P_{bi}(j)$ denote state transition probabilities.

Let X_n denote a random variable with a range $0 \le x \le W-1$. Let us consider *J* trials of a statistical experiment. Let $X_n = x(j)$ is an outcome of the j^{th} trial. Now the outcome X_n in the j^{th} trial can be one of the following as described by (1) to (3).

$$\Pr[X_n = x(j) = x(j-1)] = P_B(j-1)\Pr[x(j-1) \neq 0] \quad (1)$$

Equation (1) gives us the probability that a node freezes its backoff value at any j^{th} slot as the channel status found to be busy at $(j-1)^{th}$ slot.

$$\Pr[X_n = x(j) = x(j-1)-1] = P_I(j-1)\Pr[x(j-1) \neq 0] \quad (2)$$

Equation (2) is the probability that a node decrements its backoff value by one at any j^{th} slot as the channel status found to be idle at $(j-1)^{th}$ slot.

$$\Pr\left[X_n = x(j) = x'\right] = \Pr\left[x(j-1) = 0\right]\left(\frac{1}{W-1}\right),$$

where $1 \le x' \le W-1$ (3)

Equation (3) gives the probability that a node transmits at $(j-1)^{th}$ slot and chooses a new backoff value x'. The range of x' ensures that the node which has transmitted recently will not transmit immediately by choosing a 0 backoff value. This helps to maintain fairness among nodes contending for channel access.

At any slot *j*, channel has to be either in idle or in busy state. Hence, the sum of channel state probabilities has to be one, that is,

$P_I(j) + P_B(j) = 1.0$, for all *j* and $1 \le n \le N$ (4)

We know the probability that a node chooses a 0 backoff value at j = 0 is $\Pr[x(0) = 0] = \frac{1}{W}$ and hence the probability that a node does not choose 0 backoff value at *j* = 0 is $\Pr[x(0) \neq 0] = \frac{W-1}{W}$. This is true for all $1 \le n \le N$. And also,

$$P_I(0) = \left(\frac{W-1}{W}\right)^N \text{ and } P_B(0) = 1 - \left(\frac{W-1}{W}\right)^N \quad (5)$$

where $P_I(0)$ is the probability that the channel is idle during 0^{th} slot (i.e., probability that none of the nodes chooses 0 backoff value at j = 0) and $P_B(0)$ is the probability that the channel is busy during 0^{th} slot. We know the probability that a node chooses any backoff value during 0^{th} slot as,

$$\Pr[x(0)=x]=\frac{1}{W},\ 0\le x\le W-1 \text{ and } 1\le n\le N \tag{6}$$

Further, the sum of the probabilities that a node is in any one of the backoff states is,

$$\sum_{x''=0}^{W-1}\Pr[x(j)=x'']=1.0\ ,\ \text{for all j} \tag{7}$$

We are interested to find, $\Pr[X_n = x(j) \neq 0]$ in the j-th trial of the experiment while *N* such identical statistical experiments have been conducted up to the (j-1)th trial. Let us now consider *N* identical statistical experiments and define the following probabilities during the (j-1)th trial of all these experiments,

Probability that the channel is idle in the (j-1)th slot,

$P_I(j-1)=$ Probability that none of the nodes have 0 backoff value at the beginning of (j-1)th slot $= \prod_{n=1}^{N}\Pr[X_n = x(j-1) \neq 0]$ and

$$P_B(j-1)=1-P_I(j-1)=1-\prod_{n=1}^{N}\Pr[X_n = x(j-1)\neq 0] \tag{8}$$

And also, we derive the probability that a node has a 0 backoff value at j = 1 by using (5) and (6) as,

$$\Pr[X_n = x(1)=0]=P_I(0)\Pr[X_n = x(0)=1]=\left(\frac{W-1}{W}\right)^N\left(\frac{1}{W}\right) \tag{9}$$

Hence, $\Pr[X_n = x(1)\neq 0]=1-\left(\frac{W-1}{W}\right)^N\left(\frac{1}{W}\right)$ (10)

Now, we can calculate the probability that the channel is idle at j = 1 as

$$P_I(1)=\prod_{n=1}^{N}\Pr[X_n = x(1)\neq 0]=\prod_{n=1}^{N}\left(1-\left(\frac{W-1}{W}\right)^N\left(\frac{1}{W}\right)\right) \tag{11}$$

Hence,

$$P_B(1)=1-P_I(1)=1-\prod_{n=1}^{N}\Pr[X_n = x(1)\neq 0]=1-\prod_{n=1}^{N}\left(1-\left(\frac{W-1}{W}\right)^N\left(\frac{1}{W}\right)\right)$$

Now, for any *j*, the probability that a node is in any one of the backoff states can be obtained recursively as,

$$\tau(j)=\Pr[X_n = x(j)=0]=P_I(j-1)\Pr[X_n = x(j-1)=1]$$
$$j\ge 1,\ 1\le n\le N \tag{12}$$

Equation (12) gives us the probability that a node transmits in any jth slot (i.e., probability that a node has 0 backoff value in jth slot).

$$\Pr[X_n = x(j)=k]=P_I(j-1)\Pr[X_n = x(j-1)=k+1]+P_B(j-1)\Pr[X_n = x(j-1)=k]+$$
$$\Pr[X_n = x(j-1)=0]\left(\frac{1}{W-1}\right),\ j\ge 1\, and\, 1\le k\le W-2 \tag{13}$$

And when k = W-1,

$$\Pr[X_n = x(j)=k]=P_B(j-1)\Pr[X_n = x(j-1)=k]+\Pr[X_n = x(j-1)=0]\left(\frac{1}{W-1}\right)$$
$$j\ge 1 \text{ and } 1\le n\le N \tag{14}$$

Now, we can calculate the channel state probabilities for any *j* using,

$$P_I(j)=\prod_{n=1}^{N}\Pr[X_n = x(j)\neq 0] \text{ and} \tag{15}$$

$$P_B(j)=1-P_I(j)=1-\prod_{n=1}^{N}\Pr[X_n = x(j)\neq 0,\ j\ge 1 \tag{16}$$

Once we know channel state probabilities, we can also compute state transition probabilities using the following equations,

$P_{ii}(j)=$ Prob. that the channel was in idle state in (*j*-1)th slot and channel is being idle in *j* th slot $= P_I(j-1)P_I(j)$ and (17)

$P_{ib}(j)=$ Prob. that the channel was in idle state in (*j*-1)th slot and the channel is busy in *j* th slot $=$ $1-P_{ii}(j)=P_I(j-1)P_B(j)$ (18)

Using (15)-(18), we can calculate P_I(j), P_B(j), P_{ii}(j) and P_{ib}(j) for a given *N* and *W*.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical results obtained from our model to study the behavior of DCF for transient state conditions in terms of node transmission probability, channel idle/busy state probabilities, channel state transition probabilities and packet success probability. We also compare our model with the model given in [5] under steady state conditions in terms of probability of successful transmission of a packet.

Fig. 3 indicates the variation in the probability that a node transmits with increase in number of slots, as obtained from (12). It shows the effect of increasing the number of nodes (*N*) in a network on node transmission probability with variation in number of slots. In steady state (for example after 100th slot), we found that the probability of transmission of a node decreases with increase in number of slots and with increase in number of nodes. This observation is in agreement with the results found in [2-5]. This is obvious because increasing the number of nodes in the network increases contention for channel access. Mathematical models in [2-5] could not capture the behavior of the network under transient state conditions, whereas our model does.

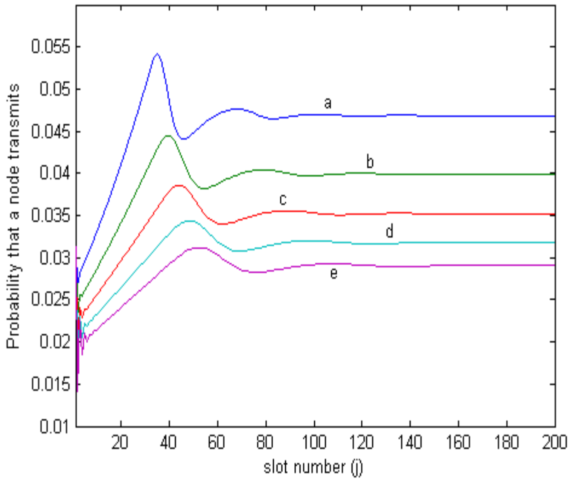

Fig. 3. Probability of packet transmission: a) N = 5; b) N = 10; c) N = 15; d) N = 20; e) N = 25

Fig. 4 gives the probability that the channel is idle/busy at any slot j, which is obtained from (15) and (16). In steady state, we found that the channel idle (busy) probability decreases (increases) with increase in number of slots and with increase in number of nodes in the network. This is expected as the number of nodes contending for channel access in a network increases.

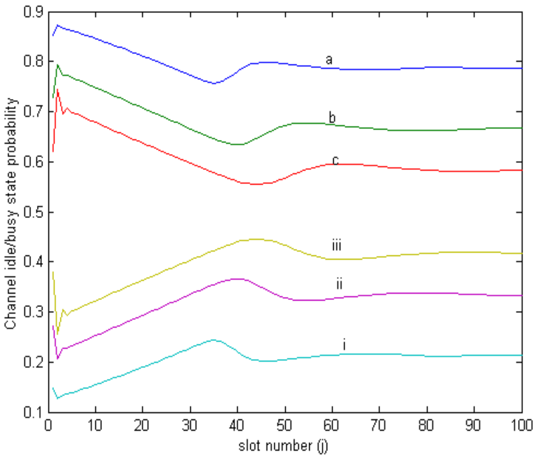

Fig. 4. Channel state probabilities: idle state probability a) N = 5; b) N = 10; c) N = 15; busy state probability i) N = 5; ii) N = 10; iii) N = 15

Fig. 5 is obtained using equations (15) to (18). This gives the effect of increasing *N* on various steady state probabilities. We observe that the probability the channel is in idle state decreases and the probability that the channel is in busy state increases with increasing number of nodes in the network. And also we notice the probability that the channel is in idle state and remains in idle state decreases as we increase *N* and the probability that the channel goes from idle to busy state increases with increasing *N*.

From our model, we can also calculate the probability of successful transmission ($P_s(j)$) in any jth slot using,

$$P_s(j) = N\tau(j)[1-\tau(j)]^{N-1} \tag{19}$$

where $\tau(j)$ can be obtained using (12).

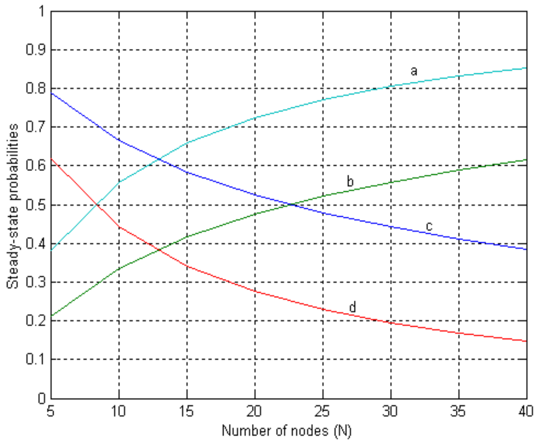

Fig. 5. Steady-state probabilities of the channel vs. number of nodes: a) P_{ib} b) P_B c) P_I d) P_{ii}

Hence, the probability of collision at any jth slot can be calculated using,

$$P_c(j) = 1 - P_s(j) - P_I(j) \tag{20}$$

where $P_I(j)$ can be obtained from (15) and $P_s(j)$ from (19) .

Fig. 6 shows effect of number of nodes in a network on probability of successful transmissions, probability of packets being lost in collision and probability that the channel is idle. This is obtained using (19), (20) and (15) respectively. We observe the probability that the channel is in idle state decreases and probability of collisions increases with increasing *N* under steady-state conditions. This is expected as the number of active nodes in a network increases.

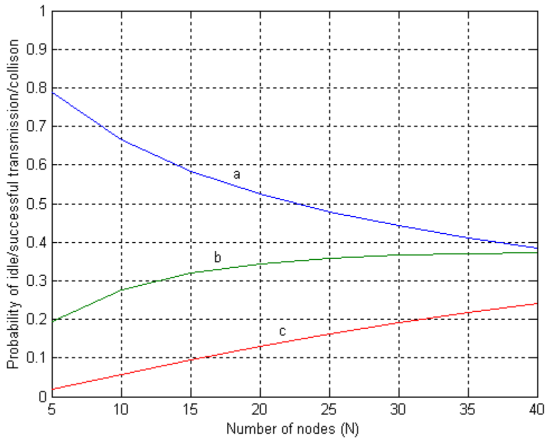

Fig. 6. Channel state probabilities (idle/success/collision) vs. number of nodes: a) P_I b) P_S c) P_C

We now compare the performance of our model with that of Xiao's [5]. Xiao's model is chosen for comparison as it takes care of drawbacks present in [2-4]. In this model, probability of successful transmission in a slot (P_s) in steady-state is calculated using,

$$P_s = N\tau(1-\tau)^{N-1} \tag{21}$$

where τ is the probability that a node transmits in a slot (i.e., probability that a node has 0 backoff value). In our model, we can calculate τ using (12). For Xiao's model, $\tau = \dfrac{2-2P_B}{1-2P_B+W}$ (for constant backoff window length), where $P_B = 1-(1-\tau)^N$ is the probability that the channel is busy due to at least one transmission [5].

Fig. 7 shows the effect of number of nodes on probability of successful transmission of packets. We observe that the probability of successful transmission calculated from our model closely follows Xiao's model under steady-state conditions. Moreover, we have also modeled the behavior of DCF under transient state conditions, which is not captured in [5].

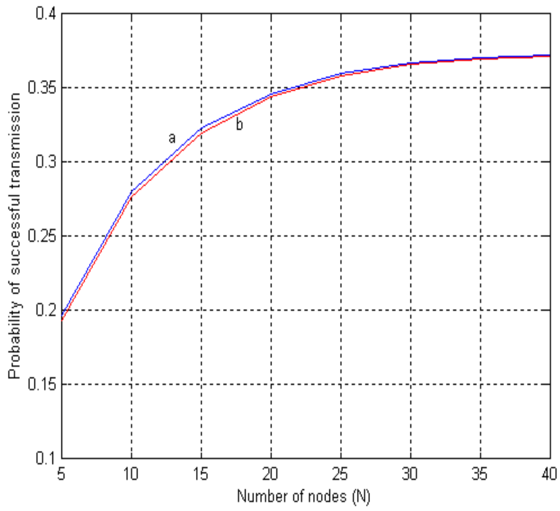

Fig. 7. Probability of successful transmission vs. number of nodes:
a) Xiao's model b) Our Model

IV. CONCLUSIONS

In this paper, we have presented a mathematical model for IEEE 802.11 DCF under transient state conditions and analyzed its performance. From our model, we could find various parameters, such as, node's packet transmission probability, probability of successful packet transmission, etc. We had also verified that results obtained from our model are in agreement with the results published in [5] under steady-state conditions. This work is highly useful in scenarios, such as, developing mathematical models for initialization of nodes in a wireless ad hoc network based on IEEE 802.11 DCF. Our future work will be in this direction.

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