A Multi-Commodity Flow Model for Optimal Routing in Wireless MESH Networks

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***Abstract*— We propose a mathematical programming model of the routing problem in multi-hop wireless networks that takes into account quality of service requirements considering bandwidth constraints. The proposed approach is suitable for Wireless MESH Networks (WMN) where topology is almost fixed and routes can be optimized based on global objectives. We then consider and solve the scheduling problem, illustrating how routing and scheduling models can be combined to route flows with guaranteed bandwidth. As an interesting application of the proposed approach, we present some numerical examples that show how our model can be used to estimate the impact of transmission range on network capacity.**

***Index Terms* - Wireless MESH Networks, Multi-hop Wireless, QoS Routing, Multi-commodity flows.**

I. INTRODUCTION

The problem of routing in multi-hop wireless networks has been deeply investigated in the research area of Mobile Ad-hoc NETworks (MANET). Wireless MESH Networks (WMN) are multi-hop wireless networks like MANETs, but their characteristics make the routing problem quite different [1], [2]. WMNs have almost static topologies that change mainly due to node failures, which are relatively infrequent. Therefore, the distribution of network state information is not much costlier than in wired networks, and even a centralized control of route selection can be adopted [3]–[7]. Moreover, energy consumption is usually not a key issue for network nodes.

In this letter we propose a new model for the Quality of Service (QoS) routing problem in multi-hop wireless networks with bandwidth constraints. The model is an extension of the well known multi-commodity flow problem where link capacity constraints are replaced with new ones that take into account interference constraints among different radio links. Differently from previous work on QoS routing in ad hoc networks [8], [9], our formulation allows to decouple the routing and the scheduling problems that can be solved in different steps.

In Section II we present the new multi-commodity flow model. In Section III we model and solve the scheduling problems showing how routing and scheduling solutions can be combined to route flows with guaranteed bandwidth. In Section IV we present some numerical examples that show how the proposed approach can be used to estimate the network capacity with different values of the transmission range. Final remarks are given in Section V.

II. MULTI-COMMODITY FLOW MODEL

Let us consider a directed graph $G = (V, E)$ where nodes represent the wireless stations and directed arcs (i, j) connect stations within transmission range. Since WMNs are characterized by infrequent topology changes and rare node failures, we assume that nodes are fixed [10]. For sake of simplicity we assume isotropic propagation in all directions and ideal channel conditions, and the transmission range of each node is assumed equal to the interference range.

A set of K connections is offered to the network. Each connection is represented by the notation (s^k, t^k, d^k), for $k = 1, \dots, K$, where s^k, t^k and d^k represent the connections source node, destination node and required bandwidth, respectively.

Let $f^k_{(i,j)}$ be the decision variable representing the amount of flow of the $k-th$ commodity routed on link (i, j). $F_{(i,j)} = \sum_{k=1}^{K} f^k_{(i,j)}$ represents hence the total flow on arc (i, j).

For each node $n \in V$, we define $A(n) = \{i \in V | (n, i) \in E\}$ as the set of nodes that can be reached from node n, and $B(n) = \{i \in V | (i, n) \in E\}$ as the set of nodes that can transmit to node n. Let $N(n) = A(n) \cup B(n)$ be the set of nodes adjacent to n.

We assume that when a transmission occurs on a link, nodes adjacent to the transmitting and the receiving nodes are prevented to transmit or receive. Such an assumption assures that the hidden terminal problem does not occur both for the considered transmission and that of the acknowledgement packet, like for IEEE 802.11 systems.

To model capacity and interference constraints, we define for each node n the set $S^1_n = \{(n, i) \in E\} \cup \{(i, n) \in E\}$ that contains all the links that are adjacent to node n, and for each node $m \in N(n)$, the set $S^2_{(n,m)} = S^1_m - \{S^1_m \cap S^1_n\}$ including all links that have at least one end in m but do not belong to S^1_n.

Based on these definitions, it is possible to define the set $G_n = S^1_n \cup \{\bigcup_{m \in N(n)} S^2_{(n,m)}\}$, formed by all the links that are within two hops from node n. It is possible to associate to each set G_n a capacity C_n, that depends on the transmission rate and the protocol overheads, and a constraint on the maximum aggregate flow that can be routed over the links belonging to G_n. Since transmissions on links in S_n^1 cannot occur simultaneously, all flows routed on these links contribute in consuming common resources associated to the set G_n. Similarly, transmissions on links in S_n^1 and in any set $S_{(n,m)}^2$ cannot occur simultaneously and therefore also flows routed on links in $S_{(n,m)}^2$ consume common resources. However, since simultaneous transmissions are allowed on links in different sets $S_{(n,h)}^2$ and $S_{(n,l)}^2$, $h \neq l$, only the most loaded set is to be considered in the capacity constraint that can therefore be written as:

$$\sum_{(i,j)\in S_n^1} F_{(i,j)} + \max_{m\in N(n)} \sum_{(i,j)\in S_{(n,m)}^2} F_{(i,j)} \leq C_n, \qquad \forall n \in V \quad (1)$$

We can now illustrate the *Optimum Wireless MESH Routing* (OWMR) problem formulation. We consider a variation of the concurrent flow problem: find the maximum α, α^*, such that for each commodity we can route αd^k flow units from s^k to t^k. Thus we maximize the fraction of offered traffic admitted in the network considering the following LP (Linear Programming) problem:

$$Maximize \sum_{k\in K} \alpha d^k \quad (2)$$

$$s.t. \sum_{i\in A(n)} f_{(n,i)}^k - \sum_{i\in B(n)} f_{(i,n)}^k = \begin{cases} \alpha d^k & \text{if } n = s^k \\ -\alpha d^k & \text{if } n = t^k \\ 0 & \text{otherwise} \end{cases}$$

$$\forall n \in V, k \in K \quad (3)$$

$$\sum_{(i,j)\in S_n^1} F_{(i,j)} + \sum_{(i,j)\in S_{(n,m)}^2} F_{(i,j)} \leq C_n \qquad \forall n \in V, m \in N(n) \quad (4)$$

$$\alpha \geq 0 \quad (5)$$

$$f_{(i,j)}^k \geq 0 \qquad \forall (i,j) \in E, k \in K \quad (6)$$

The objective function (2) is the total amount of flow accepted in the network.

Constraint (3) represents the flow balance equations expressed for each node of the graph. Note that this constraint defines one or multiple paths for each commodity between its source and destination nodes.

Constraints (4) are completely equivalent to the non-linear constraint (1), where the max operator has been eliminated by extending the constraint to all sets $S_{(n,m)}^2$ related to node n, comprising in this way also the heaviest loaded one that gives the most restrictive condition.

Note that we can consider alternate formulations to the OWMR problem. For example, the objective function (2) can be changed as follows:

$$Maximize \sum_{k\in K} \alpha^k d^k \quad (7)$$

where $\alpha^k \geq 0, \forall k \in K$. Constraint (3) is modified accordingly, substituting α with α^k. With such formulation, the total load accepted in the network can be greater than that obtained using the objective function (2). However, this can lead to an unfair allocation of network capacity. For this reason, in the following we consider the OWMR formulation (2)-(6) that maximizes the total load accepted into the network maintaining for each connection k the proportionality to its bandwidth demand, expressed through the d^k parameter.

III. TRAFFIC SCHEDULING

The LP model we have proposed is based on a fluidic approach and it does not guarantee that a scheduling scheme able to transmit routed flows exists. In fact, only integer solutions of the scheduling problem can be actually implemented in a TDMA (Time Division Multiple Access) scheme, which is the MAC scheme we assume throughout this paper to obtain numerical results.

For this reason we define a procedure to evaluate the gap between the traffic admitted based on our routing model and the maximum amount of flow that can be actually routed and scheduled in the network.

We implemented a variation of the classic graph multi-coloring problem, where we assume that transmissions of all nodes in the network are scheduled to avoid collisions. Time is divided in slots, all having the same duration, organized in frames. The bandwidth that can be allocated in each time slot is equal to the capacity of the physical radio link divided by the number of time slots contained in a frame, N_S. We assume, for simplicity, that the same network technology is used on each node, and therefore we set $C_n = C$, $\forall n \in V$.

Solving the OWMR problem, we determine the amount of flow $F_{(i,j)}$ that must be allocated on each link (i,j); therefore, we can determine the number of required time slots (i.e. colors) as $\lceil \frac{F_{(i,j)}}{C} N_S \rceil$, the closest higher integer obtained dividing $F_{(i,j)}$ by $\frac{C}{N_S}$.

Obviously, N_S determines the granularity with which bandwidth is allocated. If we increase N_S we reduce the effect of bandwidth quantization but this increases in turn the number of integer variables and consequently the computation time needed to solve the scheduling problem.

We consider two different interference constraints: in the first set (DATA-Only), if node s transmits to node t in time slot k, interference occurs only if t receives more than one transmission in slot k, modelling a scenario where data packets are not acknowledged.

The second set of constraints is more restrictive since we assume an acknowledged exchange (DATA-ACK) in which a transmission from s to t can occur in slot k only if all neighbors of both s and t do neither transmit nor receive in the same slot.

We formulated the graph multi-coloring problem, considering the number of required colors on each link (i,j) and the constraints specified above (i.e. DATA-Only and DATA-ACK) using the AMPL language [11]. The problem was solved using CPLEX [12], a state-of-the-art ILP (Integer LP) commercial solver, determining the minimum number of slots, N_{Min}, required to schedule the traffic routed by our LP formulation. If $N_{Min} \leq N_S$ the flow is fully schedulable; on the contrary, if $N_{Min} > N_S$, we consider a scaled capacity

$C\frac{N_S}{N_{Min}}$ in constraint (4) for every group G_n, and we re-solve our OWMR model obtaining fully schedulable traffic routes. This two-steps approach allows to find optimal solutions to the problem of routing flows with guaranteed bandwidth in a multi-hop wireless network. Such a problem is basically the same problem commonly considered for QoS routing in ad hoc networks that is usually solved using heuristics [8], [9].

From a practical point of view, the proposed routing scheme can be applied to wireless MESH networks based on traffic scheduling, such as IEEE 802.16, since such standard uses a slotted TDMA protocol scheduled by the base station to allocate capacity to subscribers in a point-to-multipoint network topology [13].

However, it could also be considered for IEEE 802.11 MESH networks taking an additional margin on the radio link capacity to account for conflicts (i.e. frame collisions) on the channel.

To show an application of the proposed approach, we first considered Manhattan topologies, starting from a basic grid of 3×3 nodes, illustrated in Fig. 1 (the leftmost network). Each of the 9 nodes offers the same amount of traffic (d^k) to all the other 8 nodes, for a total of $K = 72$ connections offered to the network. Then we added between every couple of directly connected nodes a number n_R of additional relays that have only the task of forwarding packets. n_R ranges between 0 (the basic grid at the left of Fig. 1) and 4. Fig. 1 shows a particular of the topologies with $n_R = 1$ and $n_R = 2$; the big circles represent the nodes that generate and receive traffic, while the small circles represent the additional relays.

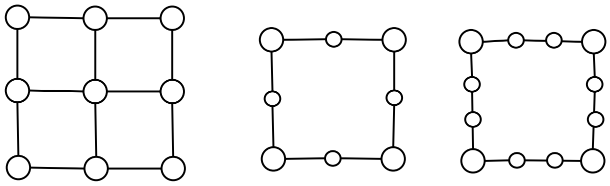

Figure 1. Manhattan basic grid ($n_R = 0$) and particular for $n_R = 1$ and $n_R = 2$ topologies

Fig. 2 shows the performance obtained in all the different Manhattan topologies, measured by the total load accepted in the network, without considering the traffic relayed by intermediate nodes.

The upper line represents the total traffic accepted in the network by our OWMR model (i.e. $\sum_{k\in K} \alpha^* d^k$), setting $C_n = C = 10$ Mbit/s, $\forall n \in V$. Using the DATA-Only set of interference constraints and setting $N_S = 100$ we found that these flows are fully schedulable in all the considered topologies.

The middle line shows the accepted and scheduled load that results from the DATA-ACK set of constraints applying the two-steps approach described above, since with this set of constraints N_{Min} is greater than N_S. We observe that the gap between the two sets of constraints decreases when the number of relaying nodes increases.

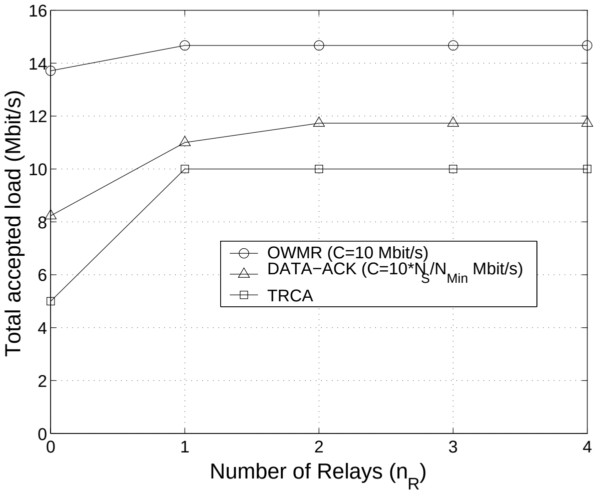

Figure 2. Accepted and scheduled load as a function of the number of relays in the Manhattan Topologies

Finally, we report for comparison the results obtained using the Transmitter-Receiver Conflict Avoidance (TRCA) model (the lower curve) proposed in [4]. Such interference model has been implemented by replacing constraint (4) in our formulation with the following expression:

$$\sum_{(i,j)\in G_n} F_{(i,j)} \leq C_n, \forall n \in V \qquad (8)$$

As our formulation models more closely the interference imposed in a real 802.11 system, these results show that such systems can accommodate larger traffic than that predicted in [4]. This is more evident for the basic 3x3 Manhattan topology.

IV. NUMERICAL EXAMPLES

In the previous Section we defined a procedure that allows us to find flow allocations that are fully schedulable, by scaling the radio-link capacity based on the solution of the multi-coloring problem. We validated our model through extensive simulation by applying it to several network scenarios to evaluate the effect of the transmission range on network capacity.

The OWMR formulation illustrated in Section II was implemented using the AMPL language [11], and solved using CPLEX [12]. Network topologies were generated using custom C code.

A. Grid Network

We first considered a 6×6 grid network, illustrated in Fig. 3, where nodes are 100 meters apart. We picked at random 36 couples of nodes. In each couple, one node acts like sender, the other like receiver and all the senders offer to the network the same amount of traffic, d^k. We set $C_n = C = 10$ Mbit/s for all network nodes. We varied the transmission range of the nodes and we measured the total load accepted by the network, averaged on 10 random extractions of the $K = 36$ connections. The results are shown in Fig. 4. In this scenario the accepted traffic increases as the transmission range increases until a maximum value is reached. Then we observe a very small decrease for long transmission ranges.

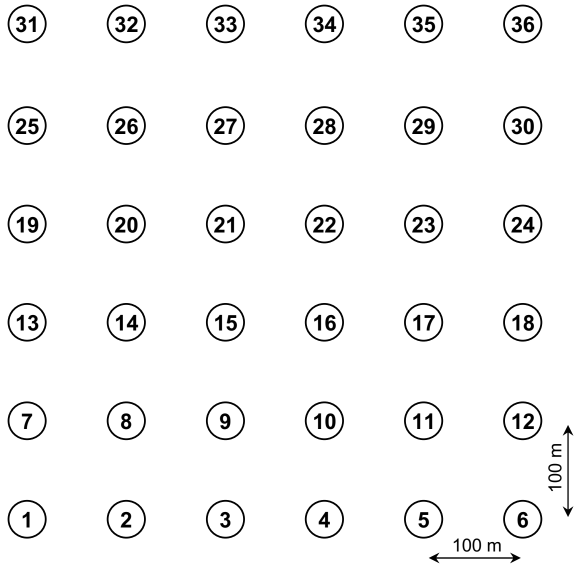

Figure 3. Grid Network Topology

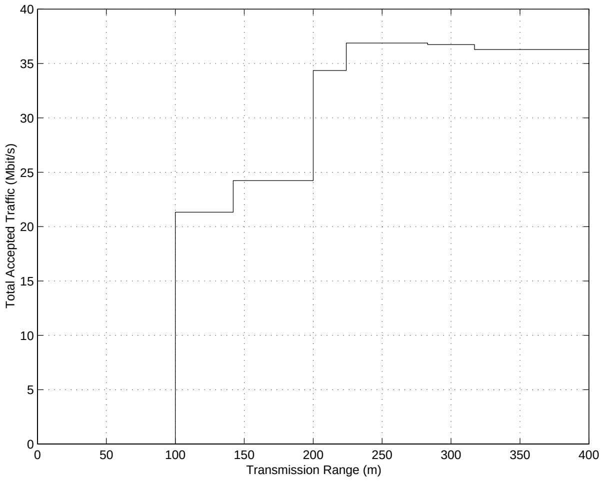

Figure 4. Results in the Grid Network Topology

B. Mesh Network

We then considered a mesh network scenario in which multiple users are interconnected and access the Internet through a multi-hop wireless network. We considered a 1000m × 1000m square area, where 25 nodes are distributed uniformly at random. The area has been divided into 4 equal 500m × 500m square sectors. The 4 nodes that are located closest to the center of each of these sectors act as concentrators and collect the traffic of the neighbor nodes. All the other 21 nodes send their traffic to the closest concentrator. Fig. 5 illustrates an example of the resulting topology, indicating the concentrator nodes as squares in the 4 sectors. In this example, all the transmission ranges of the nodes are equal to the minimum value (215m) necessary to obtain a connected network.

Given this scenario, we considered the following situations:

- Each one of the 4 concentrators sends traffic to all the others, thus modelling local traffic exchanges typical of a campus network.
- One concentrator, randomly chosen, acts as Internet Access Point for the whole network, while the other 3 concentrators send the collected traffic to it.

We suppose that all the nodes offer to the network the same amount of traffic, d^k. We averaged the results over 10 random node locations, varying the transmission ranges of the nodes. The radio link capacity C_n was set equal to 10 Mbit/s for all network nodes. The results are shown in Fig. 6.

In both scenarios the accepted traffic increases up to a maximum value and then decreases down to a value corresponding to the fully-connected network. The difference between these two values is remarkable. Such a behavior is due to two opposite effects: if we increase the transmission range, we reduce the number of hops/transmissions needed to reach destination, which is equal to one when the network is fully connected; on the other hand, a higher transmission range increases interference and therefore limits the number of possible parallel transmissions (resource reuse). In some scenarios only one effect prevails, while in the considered

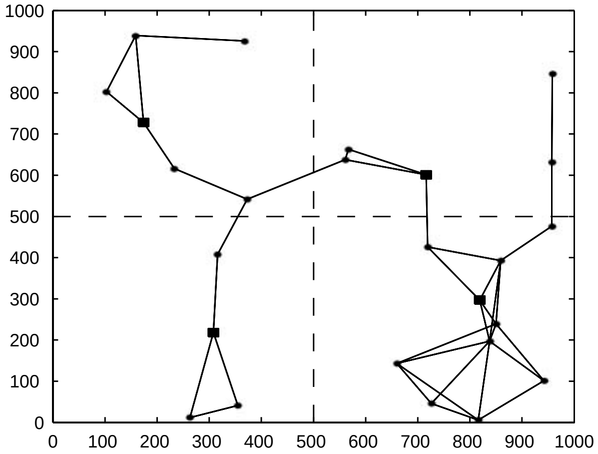

Figure 5. Random Topology with 4 concentrators

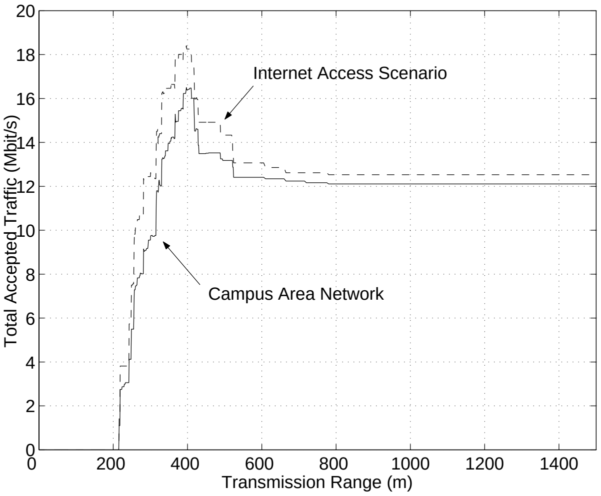

Figure 6. Results in the Random Network scenarios

random networks both effects are relevant and as a result there is an optimum value for the transmission range. Such result and the proposed model can be used as a support for topology control techniques [14].

Finally it is worth noting that, even if our formulation allows flows to be split over multiple paths, we verified that in all the network scenarios considered all flows were routed on a single path. We do not argue that this happens in general, however we think that the particular expression of the capacity constraints (4) greatly reduces the benefit of splitting flows which on the contrary is not negligible in classical multi-commodity flow problems. Such behavior of our formulation is particularly remarkable as flow splitting requires packet reordering, which is not always tolerated by connection oriented transport layers like TCP (Transmission Control Protocol).

V. CONCLUSION

We proposed a mathematical model of the routing problem in wireless mesh networks that extends classical multi-commodity flow models. To find solutions that can be scheduled considering a TDMA scheme, we also modelled and solved the conflict-free scheduling problem. We showed how fully schedulable solutions can be obtained with a two-steps approach. As an interesting application of the proposed approach we presented numerical results on the impact of the transmission range on network capacity.

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