Bhattacharyya Coefficient in Correlation of Gray-Scale Objects

M. Sohail Khalid
College of Electrical and Mechanical Engineering, National University of Sciences and Technology, Pakistan
Email: soh_78@yahoo.com

M. Umar Ilyas
College of Telecommunication Engineering, National University of Sciences and Technology, Pakistan
Email: muilyas@wol.net.pk

M. Saquib Sarfaraz
Technische Universitat Berlin, Germany

M. Asim Ajaz
College of Electrical and Mechanical Engineering, National University of Sciences and Technology, Pakistan

Abstract— **Bhattacharyya coefficient is a popular method that uses color histograms to correlate images. Bhattacharyya Coefficient is believed to be the absolute similarity measure for frequency coded data and it needs no bias correction. In this paper, we show that when this method is applied to gray scale images, it produces biased results. Correlation based on this measure is not adequate for common gray scale images, as the color in grayscale is not a sufficient feature. The biased ness is explored and demonstrated through numerous experiments with different kinds of non-rigid maneuvering objects in cluttered and less cluttered environments, in context to the object tracking. The spectral performance of the Bhattacharyya curve is compared with the spatial matching criterion i.e. Mean Square Difference.**

Index Terms—**bhattacharyya coefficient, correlation, mean square difference, object tracking**

I. Introduction

In pattern recognition applications we often need to find the similarity between two images or two sections of images. This may be done by correlating the spatial information or by matching their spectral features. Bhattacharyya coefficient [5] is one of the criteria, which gives a measure of similarity between the probability density functions (spectral information) of two images. It is a divergence-type measure [6] which has a straightforward geometric interpretation. N. A. Thacker et. al. [10] showed that Bhattacharyya Coefficient is an absolute similarity measure and needs no bias correction. Many researchers used this measure to find similarity in images or sections of images. For example in object tracking applications, D. Comaniciu et. al. [1] used this measure and mean shift procedure for optimization. They used weighted object model densities through a kernel profile to yield a differentiable similarity function using Bhattacharyya coefficient, so that the efficient gradient based optimization algorithms can be applied. Y. Rui and Y. Chen [2] used particle filtering which found its basis in conditional density propagation (CONDENSATION) [3]. They also used Bhattacharyya coefficient as a similarity measure. H. Chen and T. Liu [4] used Kullback Leiber information theoretic criterion as the similarity measure. They compared Bhattacharyya coefficient with different optimization techniques. They emphasized on the better performance of trust region optimization over mean shift. While considering the Bhattacharyya coefficient as a similarity measure, all of the above authors used the histograms of the colored images as an estimate to their densities. Performance of this measure is not yet discussed in literature. In this paper we consider the gray scale images and explore the performance of Bhattacharyya coefficient. Experiments show that estimate of target position using Bhattacharyya coefficient is biased. We can say that the color information present in grayscale is not enough for a similarity measure like Bhattacharyya Coefficient to work properly, as the gray color will contribute in the construction of histogram in such a way that the measure will give bias. In the experiments, actual location of the object is ascertained by mean square difference (MSD), which is a high accuracy method due to its pixel wise spatial correlation. However, certain factors like lack of robustness prohibit its use in unsupervised real-time tracking applications. We have shown that MSD gives far better results than Bhattacharyya Coefficient when similarity is sought between pairs of images.

Based on "Biased Nature of Bhattacharyya Coefficient in Correlation of Gray-Scale Objects" by M. Sohail Khalid, M. Bilal Malik, which appeared in the Proceedings of 4th International Symposium on Image and Signal Processing and Analysis ISPA 2005, Zaghreb, Croatia, September 2005.

We begin with a discussion of object representation based on the image histogram. Section 3 shows the representation of weighted densities and construction of image histogram. The definition of Bhattacharyya coefficient and MSD appears in section 4 and 5 respectively and finally section 6 presents the experiments showing the biased behaviour of the curve.

II. Object Representation

To characterize the object, first a feature space is chosen. The object is represented by its probability density function (pdf). The pdf can be estimated by *m*-bin histogram of object, where *m* is the number of colors. The histogram is not the best nonparametric density estimate [7], but it is good enough for most pattern recognition applications. Other discrete density estimates can also be employed. The reference object is the one to be searched in the same image or may be in next image of a video sequence or in any image where a similar object may be found. The candidate objects are tested against the reference object to check the similarity between them. Both the reference and the candidate objects are represented by *m*-bin histograms as an estimate to their pdf's. Both the pdf's are to be estimated from the data.

$$\hat{\mathrm{r}} = \{\hat{r}_u\}_{u=1\ldots m} \qquad \hat{\mathrm{c}} = \{\hat{c}_u(y)\}_{u=1\ldots m} \tag{1}$$

where $\hat{\mathrm{r}}$ and $\hat{\mathrm{c}}$ represent the *m*-bin histograms of reference object and the candidate object at location y, respectively.

III. Weighted Histogram Using Kernel

An isotropic kernel is used, with a convex and monotonically decreasing kernel profile which assigns smaller weights to the pixels away from the center. These weights increase the robustness of estimation of the probability density function, as the pixels farther from the center are often affected by clutter or interference from the background. Epanechnicov Kernel [1, 7] is one example that can be used for this purpose. Now a histogram based on the kernel can be constructed.

Let $\{x_i\}_{i=1\ldots n}$ be the pixel locations in the region defined as the target object. The function $b: R^2 \rightarrow \{1\ldots m\}$ associates the pixel at position x_i to the index $b(x_i)$ of its bin. The target object histogram can be constructed by computing the probability of the feature $u = 1\ldots m$ as

$$\hat{r}_u = N\sum_{i=1}^{n} k(x_i)d[b(x_i) - u] \tag{2}$$

where d is Kronecker delta function and $k(x_i)$ is the kernel, spatially weighting the pixels, giving higher weights towards the center and less weights along the edges of the object. The normalization constant N is derived by imposing the condition $\sum_{u=1}^{m} r_u = 1$, which results in

$$N = \frac{1}{\sum_{i=1}^{n} k(x_i)} \tag{3}$$

Using the same notation the histogram $\hat{c}_u(y)$ for the candidate object can be computed as

$$\hat{c}_u(y) = N\sum_{i=1}^{n} k(y - x_i)d[b(x_i) - u] \tag{4}$$

where

$$N = \frac{1}{\sum_{i=1}^{n} k(y - x_i)} \tag{5}$$

$\hat{c}_u$ is the function of the pixel position y. The correlation between $\hat{r}_u$ and different instances of $\hat{c}_u$ can be computed by a similarity function.

IV. Bhattacharyya Coefficient

Bhattacharyya coefficient is the similarity measure we used. It defines a normalized distance among target histograms and histograms of candidates. The sample estimate [8] of Bhattacharyya coefficient between *c* and *r* is defined as

$$\hat{r}(y) \equiv r[\hat{\mathrm{c}}_u(y), \hat{\mathrm{r}}_u] = \sum_{u=1}^{m} \sqrt{\hat{c}_u(y)\hat{r}_u} \tag{6}$$

The similarity function inherits the properties of the kernel profile when the target and candidate histograms are represented according to *p* and *q*. A differentiable kernel profile yields a smooth differentiable similarity function.

A. Maximum of Bhattacharyya Coefficient

It is expected that the maximum of this function should be at the position of the moved object or the similar object in the subsequent frame or image. Smoothness of the function makes it possible to search the maximum using any gradient based search algorithm, but here we are not concerned with the methodology or efficiency of automatic search. We are in fact interested in the accuracy of finding the position of the object. In other words we are exploring the question that how well the peak of the function represents the coordinates of the object which is to be searched?

V. Target Localization

By target localization we mean finding the spatial coordinates of the object in the image or frame of interest. These coordinates can be found using some similarity measure. The estimate of target location is the maximum value of this similarity measure.

A. Mean Square Difference (MSD)

MSD is an accurate matching criterion because of its spatial nature. Its problem is the lack of robustness due to various reasons; a brief account of which follows. MSD may not give good results with significant changes in illumination of the object. It also experiences difficulties if the size or orientation of the object is rapidly changing. Finally, MSD may completely breakdown under occlusions. Due to these reasons, MSD is not a good practical solution. Its narrow peak and numerous local maxima make it difficult for gradient based search methods to be used to find the maximum. However, here we are not concerned with the efficient automatic search, so full exhaustive search may be used. Nevertheless, we can still use it to assess the performance of other criteria because the maximum of this function indicates high similarity based on the gray level of pixel intensities. In doing so, we will have to make sure that we avoid the cases that are not handled well with MSD. The expression for the MSD is given as

$$\mathrm{MSD} = \frac{1}{n^2}\sum_{i=1}^{n}(X_i - Y_i)^2 \tag{7}$$

where X_i and Y_i are the corresponding pixels of the adjacent object windows.

B. Comparison of MSD and Bhattacharyya Coefficient

The sharp peak of MSD gives exact coordinates of slightly moved or transformed object. Sharpness of the peak is not adequate for the application of gradient based optimization methods. Bhattacharyya coefficient, through a differentiable kernel, yields a fairly smooth function, but target localization by this curve is problematic due to its biased nature. In the experiments we compare the peaks of Mean Square Difference and Bhattacharyya coefficient functions and observe that there is a fairly large difference between the two.

VI. Experiments

Experiments are performed using pairs of images from many video sequences, with different sizes of objects and different kind of movements i.e., slow moving, fast moving, rotating bodies etc, in cluttered and less cluttered environments.

In the two adjacent frames of a video sequence, a rectangular window containing the object is taken from the first frame and then the similarity coefficient is calculated by correlating the same size of windows in the subsequent frame. A 3D plot of the Mean Square Difference (MSD) is plotted against the pixel positions. Maximum of the plot shows the pixel position where best match occurs. The sharp peak of MSD shows the high matching with the target.

The Bhattacharyya coefficient plots are obtained using the same scheme. Then these plots are compared with the MSD plots to observe the difference of the maxima of the two in terms of the pixel positions.

A. The Car Sequence

Following is the experiment performed on the car sequence, the first frame and the object is specified in Figure 1. This sequence is the example of the object with relatively cluttered background. The MSD and Bhattacharyya coefficient plots are shown below respectively.

Figure 1. The first frame of car sequence the white rectangle shows the reference object which is to be searched in the subsequent frame.

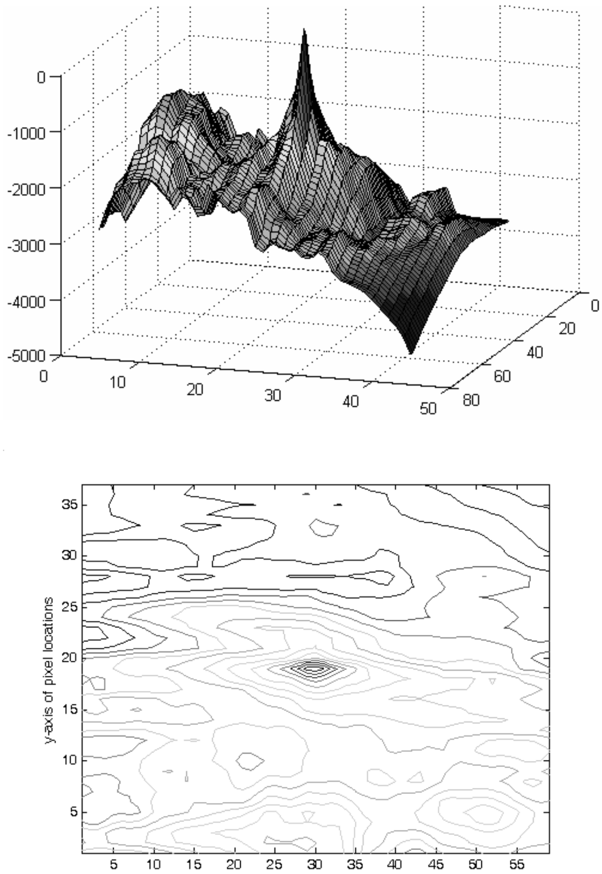

Figure 2. Mean Square Difference curve between the reference object and the candidate objects in frame number 1 & 2 of car sequence. The peak shows the location where the most similar candidate exists. The contour plot illustrates the sharpness of the peak.

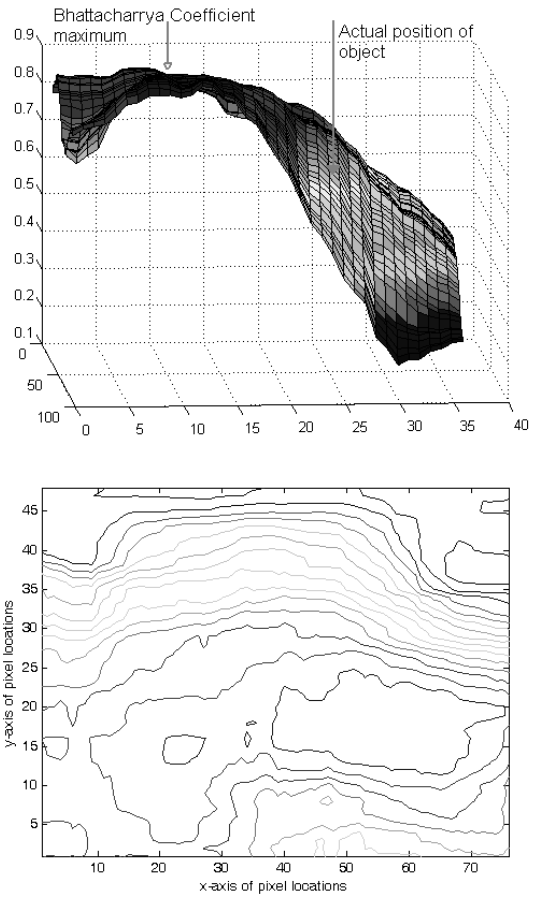

Figure 3. Bhattacharya coefficient Curve between frame 1 & 2 of car sequence, left arrow shows the maximum of Bhattacharyya coefficient, the right arrow shows the actual maximum or the actual location of the object as determined by the Mean Square Difference. The contour graph shows the wide and smooth peak.

B. Tracking of car as object

The comparison of the Bhattacharyya coefficient and the MSD peak or the location of the actual object is observed through a sequence of frames. The car in the 1st frame of car sequence is taken as the first reference object. It is searched in the 2nd frame, and then the object found in 2nd frame by MSD is searched in 3rd frame and so on. The car is tracked through out for the next 15 frames.

The original location in 2nd frame is established through exhaustive search using MSD. An error plot between the coordinates of actual location and Bhattacharyya coefficient Curve maximum between each pair of frames is plotted. The error is calculated according to the relation

$$e = \sqrt{(x - x_i)^2 + (y - y_i)^2} \tag{8}$$

where x, y are the coordinates of maximum of MSD curve and x_i, y_i are the coordinates of the maximum value of the Bhattacharyya coefficient.

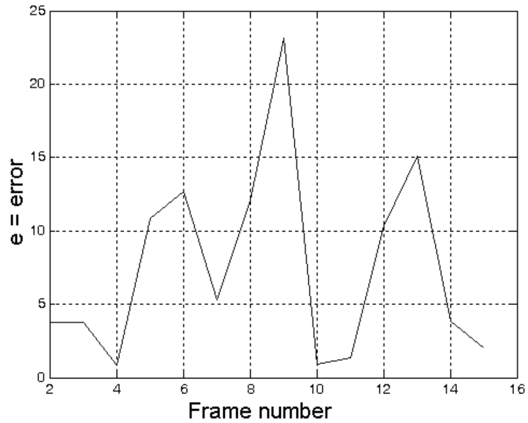

Figure 4. The error shows a measure of the drift of the peak of Bhattacharyya coefficient from the actual location of the object through the frames of video sequence.

The object in the above sequence is found to be present in a relatively comp lex background. The MSD curve is quite sharp establishing the correct position of the object, while Bhattacharyya coefficient curve is relatively smoother but showing a drift. In tracking experiment, Bhattacharyya curve shows an average drift of around 10 pixels showing its poor performance.

C. The Bottle Sequence

Another experiment with bottle sequence is given as below. This example shows the case of a rotating body with relatively clear background. The bottle in figure 5 shows distinct features on a very clear and uniform background. The drift of the peak of the Bhattacharyya coefficient even in this simple case establishes the erroneous behavior of the curve.

D. Tracking of bottle as object

The bottle in the 15th frame of bottle5 sequence is taken as the first reference object, then it is searched in the 16th frame, then the object found in 16th frame by MSD is searched in 17th frame and so on. The bottle is tracked trough out for the next 10 frames. The error graph for this is shown in Fig. 8.

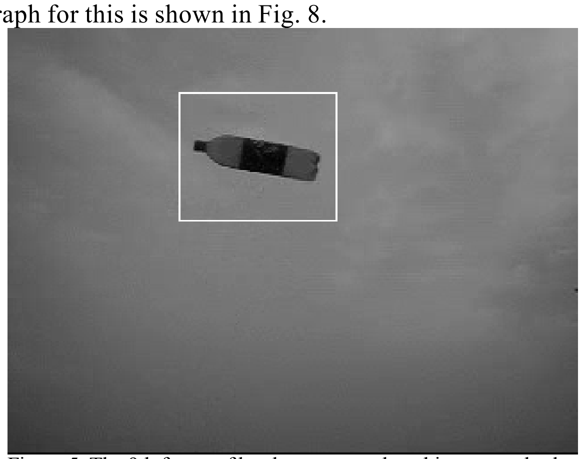

Figure 5. The 9th frame of bottle sequence the white rectangle shows the reference object which is to be searched in the subsequent frame.

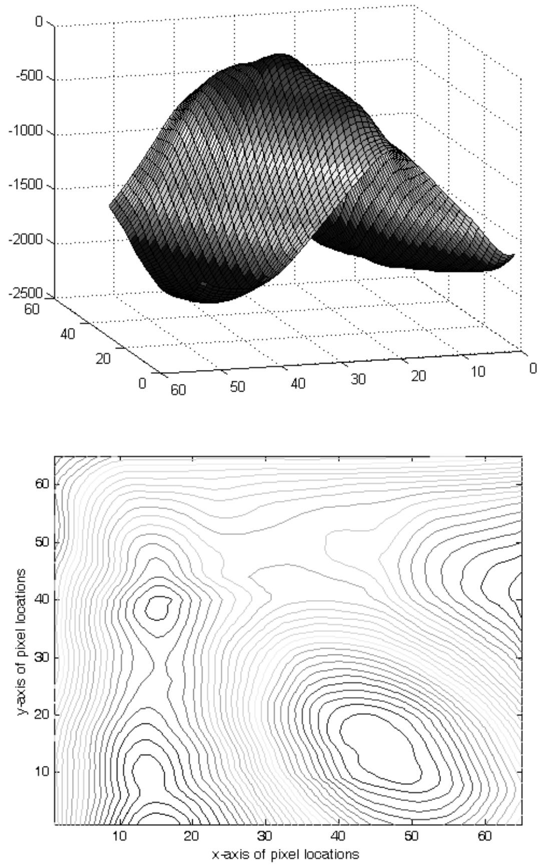

Figure 6. Mean Square Difference curve between the reference object and the candidate objects in frame number 9 & 10 of bottle sequence. The peak shows the location where the most similar candidate exists. As the object here is uniform with very clear background, so the resulting MSD surface is relatively smooth.

The object in the bottle sequence is relatively uniform and there are fewer details in the background. The MSD curve in this case is smoother. It was even expected more that the peak of the Bhattacharyya coefficient would be much closer to the MSD, but the error plot as in Figure 8, shows the error magnitude of around 11 and 22 pixels in some frames. This shows poor correlation of Bhattacharyya coefficient.

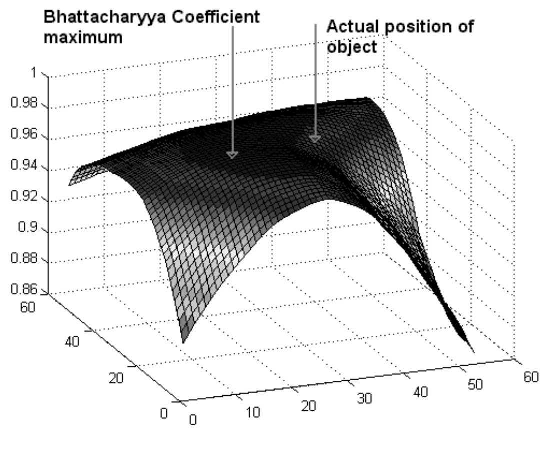

Figure 7a. Bhattacharya coefficient curve

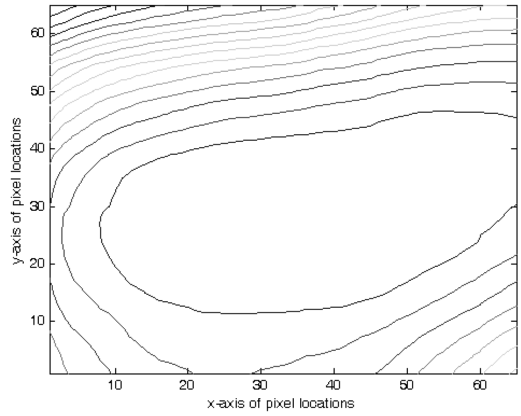

Figure 7. Bhattacharya coefficient curve between frame 9 & 10 of bottle sequence along with the contour plot.

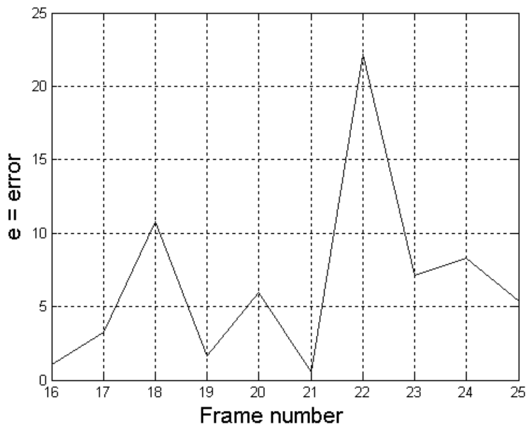

Figure 8. The error shows a measure of the drift of the peak of Bhattacharyya coefficient from the actual location of the object through the frames of video sequence.

E. Other Experiments

Instead of showing the detailed description of experiments along with the graphs, some results are summarized in tabular form. Table 1 shows the drift of the maximum of Bhattacharyya curve from actual maximum in different sequences. First column is the simple difference of coordinates of the maximum values of Bhattacharyya coefficient and MSD curve. Second column shows the drift in terms of the distance $=\sqrt{(x-x_i)^2+(y-y_i)^2}$ where x, y and x_i, y_i are the coordinates of the maximum values of Bhattacharyya coefficient and MSD respectively. The table includes other experiments with different video sequences.

VII. CONCLUSION

The limitation of Bhattacharyya coefficient in correlating gray-scale objects is verified through a series of experiments. A common observation in these experiments is the biased behaviour of estimate of the actual target position. Since Bhattacharya coefficient is a pure spectral method, we may conclude that the spectral information is not adequate in applications like correlation of gray-scale images or sections of images, or applications like tracking of gray-scale objects.

Incorporating some spatial information in Bhattacharyya coefficient can help improve accuracy of target localization. This aspect is planned to appear in future work.

TABLE I.

ERRORS IN TERMS OF POSITION AND ABSOLUTE VALUE

Sequence	Drift in terms of pixel position	Absolute distance
Car_1: frame 1 &2	11,23	25.5
Car_1: frame 30 &31	21,19	28.3
Car_2: frame 1 &2	0,32	32
Car_2: frame 40 &41	7,15	16.5
plane: frame 1 &2	2,2	2.8
$Bottle_5$: frame 71 & 72	22,5	22.5
$Bottle_6$: frame 9 & 10	13,10	16.4

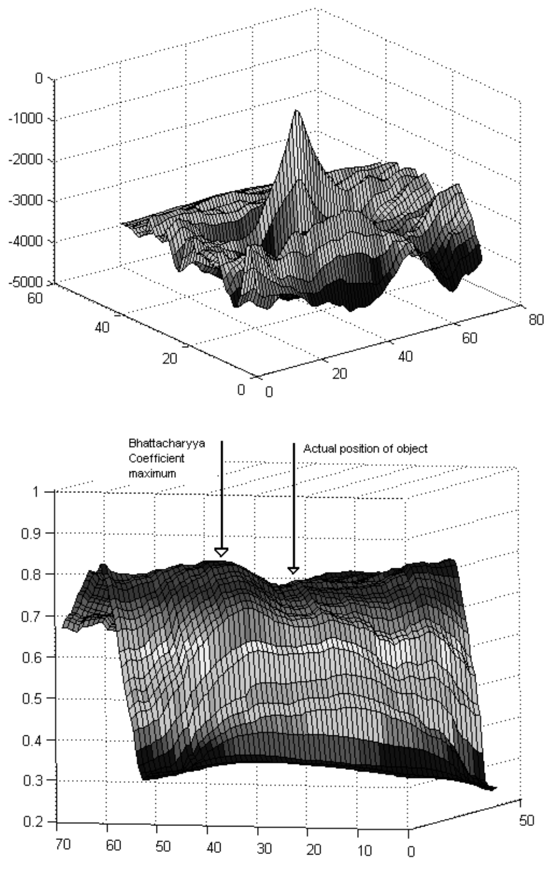

Figure 9. MSD curve and Bhattacharya coefficient curve between frame 1 & 2 of car_2 sequence.

ACKNOWLEDGMENT

The author wishes to thank all of his co authors, administration of College of E&ME and all the teachers for their endless support.

REFERENCES

[1] D. Comaniciu, V. Ramesh, P. Meer, “Real-Time Tracking of Non-Rigid Objects using Mean Shift, IEEE Conf. on Comp. Vis. and Pat. Rec., Hilton Head Island, South Carolina, 2000.

[2] Y. Rui and Y. Chen, “Better proposal distributions: Object tracking using unscented particle filter,” in Proc. IEEE Conf. on Computer Vision and Pattern Recognition, Kauai, Hawaii, volume II, 2001, pp. 786–793.

[3] M. Isard, A. Blake, Contour Tracking by Stochastic Propagation of Conditional Density, European Conference on Computer Vision (1996) 343–356.

[4] H. Chen and T. Liu, “Trust-region methods for real-time tracking,” in Proc. 8th Intl. Conf. on Computer Vision, Vancouver, Canada, volume II, 2001, pp. 717–722.

[5] K. Fukunaga, Introduction to Statistical Pattern Recognition. Academic Press, second edition, 1990.

[6] J. Lin, “Divergence measures based on the Shannon entropy,” IEEE Trans. Information Theory, vol. 37, 1991, pp. 145–151.

[7] D. W. Scott, Multivariate Density Estimation. Wiley, 1992.

[8] T. Kailath, “The divergence and Bhattacharyya distance measures in signal selection,” IEEE Trans. Commun. Tech., vol. 15, 1967, pp. 52–60.

[9] N. A. Thacker, F. J. Aherne and P. I. Rockett, TIPR'97, Prague 9-11 June, 1997

[10] N. A. Thacker, F. J. Aherne and P. I. Rockett, “The Bhattacharyya Metric as an Absolute SimilarityMeasure for Frequency Coded Data” Kybernetika, 34, 4, 363-368, 1997.

M. Sohail Khalid earned the Master’s of Electrical Engineering degree from College of Electrical and Mechanical Engineering, National University of Sciences and Technology, Rawalpindi, Pakistan, in the field of communications and signal processing in 2005.

He worked as research associate in College of E&ME from 2001 to Feb. 2006. Now he is affiliated with Air University, Islamabad, Pakistan as Lecturer.

M. Umar Ilyas earned the Master’s of Computer Engineering degree from College of Electrical and Mechanical Engineering, National University of Sciences and Technology, Rawalpindi, Pakistan, in the field of communications and signal processing in 2005. He has also earned the Master’s degree in information security from College of Telecommunication Engineering National University of Sciences and Technology in 2006.

He has worked on many research projects and giving consultancy to different universities and technological organizations.