H_∞ Sample-Data Control of Fuzzy Systems with Input Delay

Jun Yoneyama
Department of Electrical Engineering and Electronics, Aoyama Gakuin University, Kanagawa, Japan
Email: yoneyama@ee.aoyama.ac.jp

***Abstract*— This paper is concerned with H_∞ sampled-data control for uncertain fuzzy systems. A new approach to sampled-data control is introduced. The system is modelled as a continuous-time fuzzy system, while in practice, the control input has a piecewise-continuous delay. Sufficient conditions for the closed-loop system with a sampled-data state feedback controller to achieve a prescribed H_∞ disturbance attenuation level are given in terms of linear matrix inequalities(LMIs). We derive such conditions via descriptor approach to fuzzy time-delay systems under the assumption that sampling-time is not greater than some prescribed number. As such a prescribed number goes to zero, our conditions coincide with sufficient ones for continuous-time H_∞ state feedback control for fuzzy systems. We also propose a design method of sampled-data state feedback controller for uncertain fuzzy systems. Numerical examples are given to illustrate our sampled-data state feedback control.**

***Index Terms*— Takagi-Sugeno fuzzy systems, sampled-data control, time-delay systems, uncertain systems, linear matrix inequality**

I. INTRODUCTION

Takagi-Sugeno fuzzy system([12], [14]) is one of the most effective systems to treat a wide class of nonlinear systems. Since the work by [14], there has been considerable research on the stability and stabilization of such a fuzzy system. Relaxed stability conditions and stabilization methods were obtained in [15], [17] and references therein. Moreover, H_∞ control for fuzzy systems was considered in [2], [6], [8], [9], [13], [19], [20] where H_∞ controllers were obtained.

These results are mainly for continuous-time fuzzy systems. It is, however, difficult to implement a continuous-time controller directly to practical nonlinear systems by using digital devices. To overcome such difficulty, we look for sampled-data control. A class of sampled-data fuzzy systems has been considered in [10] where an approach based on a jump system, which is the form of continuous- and discrete-time model, has been employed. An H_∞ control problem for fuzzy sampled-data systems was solved via a similar approach in [11]. Jump system approach may lead to difficult conditions to be solved for fuzzy systems. Sampled-data stabilization of fuzzy systems was also considered in [3] and [21] where sufficient stability conditions for a fuzzy system with sampled-data inputs were obtained and a design method of a stabilizing sampled-data state feedback controller was proposed. The approach taken in [3] and [21] was closely related to a time-varying delay system approach because a fuzzy system with zero-order sampled-data control input results in the closed-loop system with time-varying state delays. Recently, stability analysis and control design for fuzzy systems with time-varying delays are active. Sufficient stability conditions for systems with time-varying delays were obtained in terms of LMIs(for example, [1]). Robust stability and robust H_∞ control were investigated in [2], [5], [7], and [16] where sufficient robust stability conditions and robust H_∞ control methods were proposed for uncertain fuzzy time-delay systems.

In this paper, we consider the H_∞ sampled-data control for uncertain Takagi-Sugeno fuzzy systems. We take an input delay approach. When we consider the delayed control to a fuzzy system, the closed-loop system becomes a fuzzy system with time varying delay. A descriptor system representation is used to obtain solvable conditions for the closed-loop system to achieve a prescribed H_∞ disturbance attenuation level. A design method of an H_∞ sampled-data state feedback controller of a fuzzy system is proposed by delay-dependent conditions that are given in terms of linear matrix inequalities. As a sampling-time tends to zero, our delay-dependent conditions become sufficient H_∞ conditions for continuous-time counterpart. We also extend our result to the H_∞ disturbance attenuation for uncertain fuzzy systems. Numerical examples are given to illustrate a design method of H_∞ sampled-data state feedback controllers for uncertain fuzzy systems.

II. TIME-DELAY SYSTEMS

In this section, we introduce Takagi-Sugeno fuzzy systems. Consider the Takagi-Sugeno fuzzy model, described by the following IF-THEN rules:

$$\begin{aligned}
\textbf{IF} \quad & \xi_1 \text{ is } M_{i1} \text{ and } \cdots \text{ and } \xi_p \text{ is } M_{ip}, \\
\textbf{THEN} \quad & \dot{x}(t) = (A_i + \Delta A_i)x(t) + B_{1i}w(t) \\
& \qquad +(B_{2i} + \Delta B_{2i})u(t), \\
& z(t) = C_i x(t) + D_i u(t), \quad i = 1, \cdots, r \\
& x(\theta) = 0, \; \theta \in [-h_M, \; 0]
\end{aligned}$$

where $x(t) \in \Re^n$ is the state, $w(t) \in \Re^{m_1}$ is the disturbance, $u(t) \in \Re^{m_2}$ is the control input, and $z(t) \in$

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$\Re^q$ is the controlled output. h_M will be described later. The matrices $A_i,\ B_{1i},\ B_{2i},\ C_i$ and D_i are constant matrices of appropriate dimensions. r is the number of IF-THEN rules. M_{ij} are fuzzy sets and $\xi_1,\ \cdots,\ \xi_p$ are premise variables. We set $\xi = [\,\xi_1 \quad \cdots \quad \xi_p\,]^T$ and $\xi(t)$ is assumed to be given or to be a measurable function. The time varying uncertainties are of the form

$$[\,\Delta A_i \quad \Delta B_{2i}\,] = H_i F_i(t)\,[\,E_{1i} \quad E_{2i}\,]$$

where $F_i(t) \in \Re^{l\times j}$ is an unknown time varying matrix satisfying

$$F_i^T(t)F_i(t) \le I,\ i = 1, \cdots, r,$$

and $H_i,\ E_{1i}$ and E_{2i} are known constant matrices of appropriate dimensions.

The state equation and the controlled output are described by

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{r} \lambda_i(\xi(t))\{(A_i + H_iF_i(t)E_{1i})x(t) \\ &+B_{1i}w(t) + (B_{2i} + H_iF_i(t)E_{2i})u(t)\}, \\ z(t) = &\sum_{i=1}^{r} \lambda_i(\xi(t))\{C_ix(t) + D_iu(t)\} \end{aligned} \quad (1)$$

where

$$\lambda_i(\xi) = \frac{\beta_i(\xi)}{\sum_{i=1}^{r}\beta_i(\xi)},\quad \beta_i(\xi) = \prod_{j=1}^{p} M_{ij}(\xi_j)$$

and $M_{ij}(\cdot)$ is the grade of the membership function of M_{ij}. We assume

$$\beta_i(\xi(t)) \ge 0,\ i = 1, \cdots, r,\ \sum_{i=1}^{r}\beta_i(\xi(t)) > 0$$

for any $\xi(t)$. Hence $\lambda_i(\xi(t))$ satisfy

$$\lambda_i(\xi(t)) \ge 0,\ i = 1, \cdots, r,\ \sum_{i=1}^{r}\lambda_i(\xi(t)) = 1$$

for any $\xi(t)$.

We consider the sampled-data control input. It may be represented as delayed control as follows;

$$\begin{aligned} u(t) = u_d(t_k) = u_d(t - (t - t_k)) = u_d(t - h(t)), \\ t_k \le t \le t_{k+1},\ h(t) = t - t_k \end{aligned} \quad (2)$$

where u_d is a discrete-time control signal and the time varying delay $h(t) = t - t_k$ is piecewise linear with the derivative

$$\dot{h}(t) = 1 \quad \text{for} \quad t \ne t_k.$$

t_k is the sampling instant satisfying $0 < t_1 < t_2 < \cdots < t_k < \cdots$. Define the maximum sampling interval $h_M = t_{k+1} - t_k$. Then, we have $h(t) \le t_{k+1} - t_k = h_M$ for all t.

Our problem is to find an H$_\infty$ sampled-data state feedback controller that stabilizes the system (1) with $w(t) = 0$ and makes it satisfy

$$J = \int_0^\infty (z^T(t)z(t) - \gamma^2 w^T(t)w(t))dt < 0. \quad (3)$$

for all $w(t) \ne 0$ and prescribed $\gamma > 0$. We consider the following rules for a controller:

IF $\quad \xi_1$ is M_{i1} and $\cdots$ and ξ_p is M_{ip},
THEN $\quad u(t) = K_ix(t_k),\quad i = 1, \cdots, r$

where K_i are to be determined. Then the natural choice of a controller is given by

$$u(t) = \sum_{i=1}^{r}\lambda_i(\xi(t_k))K_ix(t_k). \quad (4)$$

We represent a piecewise control law as a continuous-time control with a time varying piecewise continuous(continuous from the right) delay $h(t) = t - t_k$ as given in (2). Thus we look for a state feedback controller of the form

$$u(t) = \sum_{i=1}^{r}\lambda_i(\xi(t_k))K_ix(t - h(t)). \quad (5)$$

The closed-loop system (1) with (5) is given by

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\ &\times\{(A_i + H_iF_i(t)E_{1i})x(t) \\ &+(B_{2i} + H_iF_i(t)E_{2i})K_jx(t - h(t)) + B_{1i}w(t)\}, \\ z(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\ &\times\{C_ix(t) + D_iK_jx(t - h(t))\}. \end{aligned} \quad (6)$$

If the closed-loop system (6) is stable with satisfying (3), we say (6) achieves the H$_\infty$ disturbance attenuation γ.

When we consider the nominal case, the closed-loop system becomes

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\ &\times\{A_ix(t) + B_{2i}K_jx(t - h(t)) + B_{1i}w(t)\}, \\ z(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\ &\times\{C_ix(t) + D_iK_jx(t - h(t))\}. \end{aligned} \quad (7)$$

The following lemmas are useful to prove our main results.

Lemma 2.1: ([4]) For any given matrices $X_i,\ Y_i,\ i = 1, \cdots, r$ and $S > 0$ with appropriate dimensions, we have

$$\begin{aligned} &2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi)\lambda_j(\xi)X_i^TSY_j \\ &\qquad \le \sum_{i=1}^{r}\lambda_i(\xi)(X_i^TSX_i + Y_i^TSY_i), \\ &2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r}\lambda_i(\xi)\lambda_j(\xi)\lambda_k(\xi)\lambda_l(\xi)X_{ij}^TSY_{kl} \\ &\qquad \le \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi)\lambda_j(\xi)(X_{ij}^TSX_{ij} + Y_{ij}^TSY_{ij}). \end{aligned}$$

Lemma 2.2: ([18]) Given matrices $Q = Q^T,\ H,\ E$ and $R = R^T > 0$ with appropriate dimensions

$$Q + HF(t)E + E^TF^T(t)H^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq R$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \frac{1}{\varepsilon}HH^T + \varepsilon E^T R E < 0.$$

III. H$_\infty$ Sampled-Data Control

Here we consider the H$_\infty$ sampled-data control of an uncertain fuzzy system. We first give sufficient conditions for a nominal closed-loop system to achieve the H$_\infty$ disturbance attenuation γ. Then we propose a design method of sampled-data state feedback controller for a nominal fuzzy system. Finally, we extend our result to a uncertain fuzzy system.

A. H$_\infty$ Performance Analysis

First, we make an H$_\infty$ performance analysis of the nominal closed-loop system (7).

Theorem 1: Given control gain matrices $K_i,\ i = 1, \cdots, r$, the system (7) achieves the H$_\infty$ disturbance attenuation if there exist $P_{11} > 0,\ P_{21},\ P_{22},\ R > 0$ such that

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12ij} & \Phi_{13i} & \Phi_{14ij} \\ * & -h_M R & 0 & \Phi_{24ij} \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad i, j = 1, \cdots, r, \tag{8}$$

where

$$\begin{aligned} P &= \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}, \\ \Phi_{11ij} &= \bar{\Phi}_{11ij} + \begin{bmatrix} 0 & 0 \\ 0 & h_M R \end{bmatrix}, \\ \bar{\Phi}_{11ij} &= P^T \begin{bmatrix} 0 & I \\ A_i + B_{2i}K_j & -I \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & I \\ A_i + B_{2i}K_j & -I \end{bmatrix}^T P, \\ \Phi_{12ij} &= -h_M P^T \begin{bmatrix} 0 \\ B_{2i}K_j \end{bmatrix}, \\ \Phi_{13i} &= P^T \begin{bmatrix} 0 \\ B_{1i} \end{bmatrix}, \\ \Phi_{14ij} &= \begin{bmatrix} (C_i + D_i K_j)^T \\ 0 \end{bmatrix}, \\ \Phi_{23ij} &= -h_M K_j^T D_i^T. \end{aligned}$$

Proof: Proof is based on the following descriptor representation of (7). Let $y(t) = \dot{x}(t)$. Then we have

$$\begin{aligned} \dot{x}(t) &= y(t), \\ 0 &= -y(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\{A_i x(t) \\ &\quad + B_{2i}K_j x(t - h(t)) + B_{1i}w(t)\}, \\ z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\{C_i x(t) \\ &\quad + D_i K_j x(t - h(t))\}. \end{aligned} \tag{9}$$

(9) can be written in a compact form:

$$\begin{aligned} E\dot{x}_c(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\{A_{cij}x_c(t) \\ &\quad - \begin{bmatrix} 0 \\ B_{2i}K_j \end{bmatrix} \int_{t-h(t)}^{t} y(s)ds + \begin{bmatrix} 0 \\ B_{1i} \end{bmatrix} w(t)\}, \\ z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k))\{C_{cij}x_c(t) \\ &\quad - D_i K_j \int_{t-h(t)}^{t} y(s)ds\}, \end{aligned}$$

where $x_c = [x^T \quad y^T]^T$ and

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix},\ A_{cij} = \begin{bmatrix} 0 & I \\ A_i + B_{2i}K_j & -I \end{bmatrix},$$

$$C_{cij} = [\, C_i + D_i K_j \quad 0 \,].$$

It is clear from the above description that the descriptor system (9) has no impulsive solutions(a coefficient matrix of $y(t)$ is identity matrix and it is invertible.).

Now, we consider the following Lyapunov functional:

$$V(x_c) = V_1(x_c) + V_2(y)$$

where

$$\begin{aligned} V_1(x_c) &= x_c^T(t) E P x_c(t), \\ V_2(y) &= \int_{t-h_M}^{t}\int_{\theta}^{t} y^T(s) R y(s) ds d\theta, \end{aligned}$$

and $P,\ R > 0$ are to be determined and P satisfies

$$EP = P^T E. \tag{10}$$

From (10), we note that P must be of the form

$$P = \begin{bmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{bmatrix}$$

where $P_{11} \in \Re^{n\times n},\ P_{21} \in \Re^{n\times n},\ P_{22} \in \Re^{n\times n}$. Then, we take the derivative of $V(x_c)$ with respect to t and calculate the following:

$$\begin{aligned} &\frac{d}{dt}V(x_c) + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \\ = &\ 2\dot{x}_c^T(t) E P x_c(t) + h_M y^T(t) R y(t) \\ &- \int_{t-h_M}^{t} y^T(s) R y(s) ds \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\ &\lambda_k(\xi(t))\lambda_l(\xi(t_k))(C_{cij}x(t) \\ &- D_i K_j \int_{t-h(t)}^{t} y(s)ds)^T \\ &\times (C_{ckl}x(t) - D_k K_l \int_{t-h(t)}^{t} y(s)ds) \\ &- \gamma^2 w^T(t)w(t) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))x_c^T(t)\Gamma_{ij}x_c(t) \\
&\quad -\int_{t-h(t)}^{t} y^T(s)Ry(s)ds \\
&\quad -\int_{t-h_M}^{t-h(t)} y^T(s)Ry(s)ds \\
&\quad -2\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\
&\quad \times\int_{t-h(t)}^{t} x_c^T(t)P^T\begin{bmatrix}0\\ B_{2i}K_j\end{bmatrix}y(s)ds \\
&\quad -2\sum_{i=1}^{r}\lambda_i(\xi(t))x_c^T(t)P^T\begin{bmatrix}0\\ B_{1i}\end{bmatrix}w(t) \\
&\quad +\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))(C_{cij}x(t) \\
&\quad -D_iK_j\int_{t-h(t)}^{t} y(s)ds)^T \\
&\quad \times(C_{cij}x(t)-D_iK_j\int_{t-h(t)}^{t} y(s)ds) \\
&\quad -\gamma^2w^T(t)w(t) \\
&\leq \frac{1}{h^2(t)}\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k)) \\
&\quad \times\int_{t-h(t)}^{t}\int_{t-h(t)}^{t}\zeta^T(t,s)\bar{\Phi}_{ij}(t)\zeta(t,v)dsdv \\
&\quad \text{if } h(t)\neq 0,
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{d}{dt}V(x_c)+z^T(t)z(t)-\gamma^2w^T(t)w(t) \\
&= 2\dot{x}_c^T(t)EPx_c(t)+h_My^T(t)Ry(t) \\
&\quad -\int_{t-h_M}^{t} y^T(s)Ry(s)ds \\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t_k))x_c^T(t) \\
&\quad \times(\Gamma_{ij}+C_{cij}^TC_{cij})x_c(t), \quad \text{if } h(t)=0
\end{aligned} \tag{11}
$$

where $\zeta(t,s)=[\,x_c^T(t) \quad h(t)y^T(s) \quad w^T(t)\,]^T$ and

$$
\begin{aligned}
\bar{\Phi}_{ij}(t) = &\begin{bmatrix}\Phi_{11ij} & h_M^{-1}\Phi_{12ij} & \Phi_{13i}\\ * & -h^{-1}(t)R & 0\\ * & * & -\gamma^2I\end{bmatrix} \\
&+\begin{bmatrix}\Phi_{14ij}\\ h_M^{-1}\Phi_{23ij}\\ 0\end{bmatrix}[\,\Phi_{14ij}^T \quad h_M^{-1}\Phi_{23ij}^T \quad 0\,].
\end{aligned}
$$

If conditions (8) are satisfied, then Schur complement formula and simple mathematical manipulation show

$$
\begin{aligned}
&\begin{bmatrix}\Phi_{11ij} & h_M^{-1}\Phi_{12ij} & \Phi_{13i}\\ * & -h_M^{-1}R & 0\\ * & * & -\gamma^2I\end{bmatrix} \\
&+\begin{bmatrix}\Phi_{14ij}\\ h_M^{-1}\Phi_{23ij}\\ 0\end{bmatrix}[\,\Phi_{14ij}^T \quad h_M^{-1}\Phi_{23ij}^T \quad 0\,]<0, \\
&\qquad\qquad i,j=1,\cdots,r.
\end{aligned}
$$

These conditions suffice to show

$$\frac{d}{dt}V(x_c)+z^T(t)z(t)-\gamma^2w^T(t)w(t)<0$$

in both cases where $h(t)\neq 0$ and $h(t)=0$. Integrating both sides of the above inequality from $t=0$ to ∞, we obtain

$$V(x_c(\infty))-V(x_c(0))+J<0.$$

Since $V(x_c(\infty))>$ and $V(x_c(0))=0$, we have $J<0$. This proves that the closed-loop system (9) satisfies (3). The stability of the closed-loop system (9) with $w(t)=0$ can be proven in a similar way. Differentiating $V(x_c)$ and showing $\dot{V}(x_c)<0$ when conditions (8) are satisfied, we prove the stability of the closed-loop system (9).

Remark 3.1: In the derivation of conditions in Theorem 1, we employ a descriptor representation that is known to reduce conservatism by introducing free weighting matrices to (11). In addition, unlike [21], we do not involve an inequality such as $-2a^Tb\leq a^TRa+b^TR^{-1}b$ to give an upper bound of a cross term $x_c^T(t)P^T\begin{bmatrix}0\\ B_{2i}K_j\end{bmatrix}y(s)$ in (11). This also reduces conservatism in Φ_{ij}.

Let us now consider the continuous-time state feedback control

$$u(t)=\sum_{i=1}^{r}\lambda_i(\xi(t))K_ix(t) \tag{12}$$

to (1) with $\Delta A_i=0$ and $\Delta B_{2i}=0$, we obtain the closed-loop system

$$
\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))\{(A_i+B_{2i}K_j)x(t) \\
&\quad +B_{1i}w(t)\}, \\
z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))\{(C_i+D_iK_j)x(t)\}
\end{aligned}
$$

which can be written in the descriptor system:

$$
\begin{aligned}
\dot{x}(t) &= y(t), \\
0 &= -y(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t)) \\
&\quad \times\{(A_i+B_{2i}K_j)x(t)+B_{1i}w(t)\}, \\
z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))\{(C_i+D_iK_j)x(t)\},
\end{aligned} \tag{13}
$$

which coincides with (9) with $h(t)=0$. Yoneyama and Ichikawa showed in [19] that if

$$\begin{bmatrix}\bar{\Phi}_{11ij} & \Phi_{13i} & \Phi_{14ij}\\ * & -\gamma^2I & 0\\ * & * & -I\end{bmatrix}<0, \quad i,j=1,\cdots,r,$$

then (13) is admissible(i.e., regular, impulse-free, stable) with satisfying (3). If P satisfies the above conditions, then the conditions (8) hold for sufficiently small h_M. Therefore we have the following result.

Corollary 1: If the continuous-time state feedback controller (12) makes the nominal fuzzy system (1) with $\Delta A_i=0$ and $\Delta B_{2i}=0$ achieve the H_∞ disturbance attenuation γ, then the sampled-data state feedback controller (4) with the same control gain matrices K_i makes the system (1) with $\Delta A_i=0$ and $\Delta B_{2i}=0$ achieve the

H_∞ disturbance attenuation γ for all sufficiently small h_M.

B. State Feedback Control

In this section, we seek a design method of the sampled-data control for fuzzy systems. Unfortunately, Theorem 1 does not give feasible LMI conditions for obtaining state feedback control gain matrices K_i. Hence, we must look for another set of LMI conditions. To this end, we employ P^{-1} to obtain feasible LMI conditions and a design method of a sampled-data state feedback controller.

Theorem 2: The sampled-data controller (4) makes the nominal fuzzy system (1) with $\Delta A_i = 0$ and $\Delta B_{2i} = 0$ achieve the H_∞ disturbance attenuation γ if there exist $Q_{11} > 0,\ Q_{21},\ Q_{22},\ L_i,\ i = 1, \cdots, r$ such that for some $\mu > 0$

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11ij} & \Xi_{12ij} & \Xi_{13i} & \Xi_{14ij} & \Xi_{15} \\ * & \Xi_{22} & 0 & \Xi_{24ij} & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Xi_{25} \end{bmatrix} < 0, \quad i,j = 1,\cdots,r \tag{14}$$

where

$$\begin{aligned} \Xi_{11ij} &= \begin{bmatrix} Q_{21} + Q_{21}^T & \bar{\Xi}_{12ij} \\ \hat{\Xi}_{12ij}^T & -Q_{22} - Q_{22}^T \end{bmatrix}, \\ \hat{\Xi}_{12ij} &= Q_{11}A_i^T + L_j^T B_{2i}^T - Q_{21}^T + Q_{22}, \\ \Xi_{12ij} &= -h_M \begin{bmatrix} 0 \\ B_{2i}L_j \end{bmatrix}, \\ \Xi_{13i} &= \begin{bmatrix} 0 \\ B_{1i} \end{bmatrix}, \\ \Xi_{14ij} &= \begin{bmatrix} Q_{11}C_i^T + L_j^T D_i^T \\ 0 \end{bmatrix}, \\ \Xi_{15} &= h_M \begin{bmatrix} Q_{21}^T \\ Q_{22}^T \end{bmatrix}, \\ \Xi_{22} &= -\frac{h_M}{\mu} Q_{11}, \\ \Xi_{24ij} &= -h_M L_j^T D_i^T, \\ \Xi_{25} &= -\mu h_M Q_{11}. \end{aligned}$$

In this case, state feedback control gains in (4) are given by

$$K_i = L_i Q_{11}^{-1}. \tag{15}$$

Proof: If (8) holds, then we have $P_{11} > 0$. It also follows from $(2,2)$-block of Φ_{ij} that $-(P_{22} + P_{22}^T)$ is negative definite. Hence, it implies that P^{-1} exists. Define

$$Q \triangleq P^{-1} = \begin{bmatrix} Q_{11} & 0 \\ Q_{21} & Q_{22} \end{bmatrix},$$
$$\tilde{Q} \triangleq \mathrm{diag}[Q,\ Q_{11},\ I,\ I].$$

Then we calculate

$$\tilde{Q}^T \Phi_{ij} \tilde{Q} = \begin{bmatrix} Q^T \Phi_{ij} Q & \Xi_{12ij} & \Xi_{13i} & \Xi_{14ij} \\ * & \Xi_{22} & 0 & \Xi_{24ij} \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -I \end{bmatrix}$$

where we let $L_j = K_j Q_{11},\ R = (\mu Q_{11})^{-1}$ and

$$\begin{aligned} Q^T \Phi_{11ij} Q = & \begin{bmatrix} 0 & I \\ A_i + B_{2i}K_j & -I \end{bmatrix} Q \\ & + Q^T \begin{bmatrix} 0 & I \\ A_i + B_{2i}K_j & -I \end{bmatrix}^T \\ & + Q^T \begin{bmatrix} 0 & 0 \\ 0 & h_M(\mu Q_{11})^{-1} \end{bmatrix} Q \\ = & \ \Xi_{11ij} + Q^T \begin{bmatrix} 0 & 0 \\ 0 & h_M(\mu Q_{11})^{-1} \end{bmatrix} Q. \end{aligned}$$

By using Schur complement formula, we obtain the conditions (14). If conditions (14) hold, state feedback control gain matrices K_i are obviously given by (15).

C. Robust H_∞ Control

We extend the result in the previous section to the robust H_∞ disturbance attenuation.

Theorem 3: The sampled-data controller (4) makes the uncertain fuzzy system (1) achieve the H_∞ disturbance attenuation γ if there exist $Q_{11} > 0,\ Q_{21},\ Q_{22},\ L_i,\ \varepsilon_{ij} > 0,\ i,j = 1,\cdots,r$ such that for some $\mu > 0$

$$\begin{bmatrix} \Xi_{ij} + \varepsilon_{ij}\bar{H}_i\bar{H}_i^T & \bar{E}_{ij}^T \\ \bar{E}_{ij} & -\varepsilon_{ij} I \end{bmatrix} < 0,\ i,j = 1,\cdots,r \tag{16}$$

where $\Xi_{ij},\ i,j = 1,\cdots,r$ are given in Theorem 2, and

$$\begin{aligned} \bar{H}_i^T &= [0 \quad H_i^T \quad 0 \quad 0 \quad 0 \quad 0], \\ \bar{E}_{ij} &= [E_{1i}Q_{11} + E_{2i}L_j \quad 0 \quad -hE_{2i}L_j \ 0\ 0\ 0]. \end{aligned}$$

Proof: Substituting $A_i,\ B_{2i}$ by $A_i + H_iF_i(t)E_{1i},\ B_{2i} + H_iF_i(t)E_{2i}$ in Ξ_{ij}, respectively, we have

$$\Xi_{ij} + \bar{H}_iF_i(t)\bar{E}_{ij} + \bar{E}_{ij}^T F_i^T(t)\bar{H}_i^T < 0, i,j = 1,\cdots,r.$$

It follows from Lemma 2.2 that the above LMIs hold if and only if there exist $\varepsilon_{ij} > 0,\ i,j = 1,\cdots,r$ such that

$$\Xi_{ij} + \varepsilon_{ij}\bar{H}_i\bar{H}_i^T + \frac{1}{\varepsilon_{ij}}\bar{E}_{ij}^T\bar{E}_{ij} < 0, \quad i,j = 1,\cdots,r.$$

By Schur complement formula, we have (16).

Conditions (16) are feasible LMIs and we can use a mathematical software like Matlab to easily solve them for solutions.

IV. Numerical Example

Consider the nominal system

$$\begin{aligned} \dot{x}(t) = & \sum_{i=i}^{2} \lambda_i(x(t))\{A_i x(t) + B_{1i}w(t) + B_{2i}u(t)\}, \\ z(t) = & \sum_{i=i}^{2} \lambda_i(x(t)) C_i x(t) \end{aligned} \tag{17}$$

where

$$\begin{aligned} & A_1 = \begin{bmatrix} 1 & 0.5 \\ -0.1 & -1 \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & 0.5 \\ 0.1 & -1 \end{bmatrix}, \\ & B_{11} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}, \\ & B_{21} = \begin{bmatrix} 0.7 \\ -1 \end{bmatrix},\ B_{22} = \begin{bmatrix} 1.3 \\ -1 \end{bmatrix}, \\ & C_1 = [0.2 \quad 0],\ C_2 = [0.5 \quad 0], \\ & \lambda_1(x(t)) = \frac{2}{1 + e^{x_1(t)}},\ \lambda_2(x(t)) = 1 - \lambda_1(x(t)). \end{aligned} \tag{18}$$

We first consider the H_∞ sampled-data control for the above nominal fuzzy system. Given $\gamma = 1$, Theorem 2 shows the H_∞ sampled-data state feedback controller for the above system exists for the sampling interval $h_M \leq 0.3757$. When $h_M = 0.3757$, we obtain the H_∞ sampled-data state feedback controller

$$u(t) = \sum_{i=1}^{2} \lambda_i(x(t_k))K_i x(t_k) \quad (19)$$

with $K_1 = [-2.4753 \ \ -0.6194]$, $K_2 = [-2.4620 \ \ -0.6159]$ by Theorem 2 with $\mu = 0.5080$. For $h_M = 0.3$, Theorem 2 gives the minimum $\gamma = 0.0492$. In this case, control gain matrices $K_i, \ i = 1, 2$ in (19) are given by $K_1 = [-2.7758 \ \ -0.6363]$, $K_2 = [-2.7754 \ \ -0.6362]$ with $\mu = 0.5120$ in Theorem 2.

Next, we consider the H_∞ control by the continuous-time state feedback control. The continuous-time controller

$$u(t) = \sum_{i=1}^{2} \lambda_i(x(t))K_i x(t) \quad (20)$$

with $K_1 = [-5.5068 \ \ -0.5708]$, $K_2 = [-5.5067 \ \ -0.5707]$ can be found to achieve the H_∞ disturbance attenuation $\gamma = 1$ for the system (17) by the method in [19]. Theorem 1 guarantees that the sampled-data controller of the form (19) with the same gain matrices also achieves the H_∞ disturbance attenuation $\gamma = 1$ for (17) with $h_M \leq 0.1515$. This verifies the result of Corollary 1.

Moreover, we consider the robust H_∞ disturbance attenuation for an uncertain fuzzy system

$$\begin{aligned} \dot{x}(t) = & \sum_{i=i}^{2} \lambda_i(x(t))\{(A_i + H_i F_i(t) E_{1i})x(t) \\ & + B_{1i} w(t) + (B_{2i} + H_i F_i(t) E_{2i})u(t)\}, \\ z(t) = & \sum_{i=i}^{2} \lambda_i(x(t)) C_i x(t) \end{aligned}$$

with matrices (18) and

$$H_1 = H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$E_{11} = E_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \ E_{21} = E_{22} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}.$$

For $\gamma = 1$, Theorem 3 gives the maximum upper bound $h_M = 0.2333$ for which the closed-loop system achieves the H_∞ disturbance attenuation $\gamma = 1$. In this case, control gain matrices $K_i, \ i = 1, 2$ in the controller (19) are given by $K_1 = [-3.1029 \ \ -0.7645]$, $K_2 = [-3.1027 \ \ -0.7644]$ with $\mu = 0.6711$ in Theorem 3.

V. CONCLUSIONS

We have considered the robust H_∞ sampled-data control for uncertain fuzzy systems. Our method was based on a control input delay approach. We first showed the closed-loop system with the sampled-data state feedback control became the state delayed system. Then we gave sufficient conditions for the closed-loop system to achieve the H_∞ disturbance attenuation, and we proposed a design method of the H_∞ sampled-data state feedback controller for uncertain fuzzy systems. We also showed that if the continuous-time state feedback controller that makes the system achieve the H_∞ disturbance attenuation, then so does the sampled-data state feedback controller with the same control gains for small sampling intervals.

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Jun Yoneyama was born in Hokkaido, Japan. He received the B.Eng. and M.Eng. degrees in Electrical Engineering from Waseda University, Tokyo, Japan. He received the Ph.D degree in Electrical Engineering from University of California, Los Angeles, USA in 1996.

From 1996 to 2000, he was a reseach associate in Electrical and Electronic Engineering in Shizuoka University, Hamamatsu, Japan. Since 2000, he has been an Associate Professor in Depertment of Electrical Engineering and Electronics in Aoyama Gakuin University, Tokyo, Japan.

Prof. Yoneyama is a member of the IEEE, the Society of Instrument and Control Engineers(SICE), the Institute of Electrical Engineers, Japan(IEEJ), Japan Society for Fuzzy Theory and Intelligent Informatics(SOFT), and the Robotics Society of Japan(RSJ). His current research interest includes fuzzy systems, time-delay system, robust control, stochastic control, sampled-data control, and their applications.