Adaptive Cerebellar Model Articulation Controller for a Class of Nonlinear Systems with Only Output Measurement

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***Abstract*—Based on the cerebellar model articulation approach, an adaptive output-feedback control for a class of nonlinear systems. A control law of the closed-loop system is derived from an adaptive cerebellar model articulation control approach, and the adjustable parameters of the adaptive cerebellar model articulation controller are updated online by an adaptive law. The stability of the closed-loop system is verified by strictly positive real Lyapunov theory, under the constraint that only the system output measurement is available for feedback. In addition, the system output is guaranteed to asymptotically track a given bounded reference signal. Eventually, the performance of the proposed approach is illustrated by two simulated examples of the nonlinear systems.**

***Index Terms*—adaptive control, cerebellar model articulation control, nonlinear systems, output feedback control.**

I. INTRODUCTION

Cerebellar model articulation system was first proposed and applied to manipulator control [1]. The cerebellar model articulation systems have been successfully applied to many closed-loop controls of the complex nonlinear systems because they possess the characteristics of rapid learning convergence, good generalization capability and easy hardware implementation [2]. Since the network output is a linear combination by the activated weights, this results in that the design of the control law is easier than some nonlinear neural networks. Therefore, many researchers have studied the learning properties and applications of the cerebellar model articulation system in recent years [3-12].

By utilizing the universal approximation feature of neural networks [13], many adaptive control methods [11-12] have been proposed to achieve the control goal for a class of nonlinear systems, and some adaptive control schemes [14-18] also have been proposed to acquire better performance for uncertain nonlinear systems under the constraint that only system output is available for measurement. In addition, the cerebellar model articulation system is also a perfect universal function approximator, but a traditional control of the cerebellar model articulation system uses the rectangle function as the receptive field function such that it generates the same output for different training samples [19-20]. In [20], a state feedback control of the cerebellar model articulation system with a Gaussian receptive field basis function has been developed. The rectangle function was replaced by the Gaussian function in an attempt to obtain the differential information. However, in most real systems, the state feedback control does not always hold because system states are not always available. Thus, the objective of this paper is to develop an adaptive cerebellar model articulation controller for a class of nonlinear systems with only the system output measurement. The adaptive and control laws are derived from strictly positive real (SPR) Lyapunov approach, and the stability of the closed-loop system can be guaranteed. In addition, the simulation results demonstrate the effectiveness of the proposed method.

This paper is organized as follows. Problem formulation is described in Section II. The design method of the proposed adaptive cerebellar model articulation controller is presented in Section III. Simulation results

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are provided to verify the effectiveness of the proposed method Section IV. Conclusions are given in Section V.

II. PROBLEM FORMULATION

Consider the nth order nonlinear dynamic system of the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= F(x_1,\ldots,x_n,u)+d \\ y &= x_1 \end{aligned} \quad (1)$$

where $\mathbf{x} = [x_1, x_2, ..., x_n]^T = [x, \dot{x}, ..., x^{(n-1)}]^T \in R^n$ be the state vector, $d(t)$ is the external bounded disturbance, and $u(t) \in R$ and $y \in R$ are the control input and output of the system, respectively, and $F(\cdot)$ is an unknown continuous function. It is assumed that $0 < \partial F/\partial u < \infty$ and only the system output y is measurable. The control objective is to design an adaptive cerebellar model articulation controller such that the system output y can track a given bounded reference signal r, and guarantee that all signals involved are bounded.

First, by using Taylor series expansion of the nonlinear system (1) around $u = 0$, equation (1) can be rewritten as a state-space model

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}(f(\mathbf{x}) + g(\mathbf{x})u + d + d_h) \\ y &= \mathbf{C}^T\mathbf{x} \end{aligned} \quad (2)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is a vector of states, d_h stands for higher order term, $g(\mathbf{x}) = \partial F/\partial u\,|_{u=0}$, and $f(\mathbf{x}) = F(\mathbf{x},u)\,|_{u=0}$. Note that $f(\cdot)$ is an unknown continuous function, and $g(\cdot)$ is an unknown but strictly positive function. Let $z = r - y$, where the desired bounded reference signal $\mathbf{r} = [r, \dot{r}, ..., r^{(n-1)}]$ and the output tracking error vector $\mathbf{z} = [z, \dot{z}, ..., z^{(n-1)}]^T = [z_1, z_2, ..., z_n]^T$. Then, assume an ideal control law is

$$\bar{u} = \frac{1}{g(\mathbf{x})}\left[-f(\mathbf{x}) - d - d_h + y_d^{(n)} + \mathbf{k}_c^T \mathbf{z} \right] \quad (3)$$

where $\mathbf{k}_c = [k_c^n, k_c^{n-1}, \cdots, k_c^1]^T$ is the feedback gain vector and determined such that the characteristic polynomial of $\mathbf{A} - \mathbf{B}\mathbf{k}_c^T$ is Hurwitz. Inserting (3) in (2) results in the following equation.

$$z^{(n)} + k_c^1 z^{(n-1)} + \cdots + k_c^n z = 0 \quad (4)$$

Namely, the system output y asymptotically tracks a given bounded reference signal r for any initial bounded states. However, since only the system output y is available for measurement, and the functions $f(\mathbf{x})$ and $g(\mathbf{x})$, and the external bounded disturbance are unknown, the ideal control law (3) can not be achieved. Thus, our goal is to design a cerebellar model articulation system with only the output state information to approximate this ideal control.

III. THE DESIGN METHOD OF THE PROPOSED ADAPTIVE CEREBELLAR MODEL ARTICULATION CONTROLLER

Suppose a control input u is

$$u = u_{ac} + u_c \quad (5)$$

where u_{ac} is the output of the cerebellar model articulation system, and u_c is the compensated controller which is employed to compensate the modeling error. Then, from (2), (3) and (5), we have

$$\begin{aligned} \dot{\mathbf{z}} &= \mathbf{A}\mathbf{z} - \mathbf{B}\mathbf{k}_c^T\hat{\mathbf{z}} + \mathbf{B}\left[g(\mathbf{x})\bar{u} - g(\mathbf{x})u_{ac} - g(\mathbf{x})u_c - d - d_h \right] \\ z_1 &= \mathbf{C}^T\mathbf{z} \end{aligned} \quad (6)$$

where $\hat{\mathbf{z}} = \mathbf{r} - \hat{\mathbf{x}}$ and $\hat{\mathbf{x}}$ are the estimates of $\mathbf{z}$ and $\mathbf{x}$, respectively. Thus, the tracking problem has been converted into the regulation problem of designing a state observer for estimating the state vector $\mathbf{z}$ in an attempt to regulate z_1 to zero.

A. Design of the cerebellar model articulation system

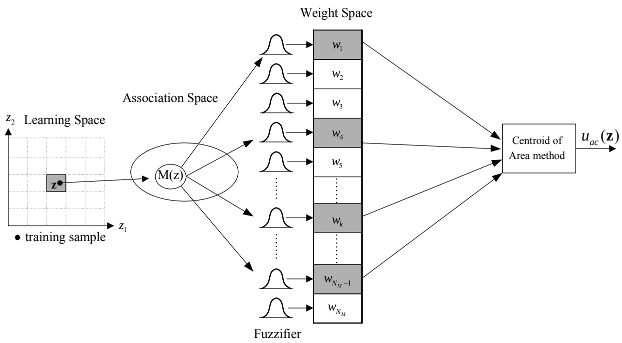

Fig. 1. Architecture of the cerebellar model articulation system

First, the architecture of the cerebellar model articulation system shown in Fig. 1 [21] includes learning space, association space, receptive-field, weight space and output. In learning space, it is quantized into several discontinuous states. The cerebellar model articulation system acquires an input vector from learning space, and finds the location of the input vector in learning space. According to the location of the input vector, a discontinuous state can be obtained, and then the cerebellar model articulation system maps the input vector to association space. Through association space, some receptive-fields covering this discontinuous state are activated. In the receptive-fields, Gaussian basis functions are chosen as receptive-field functions. By using centroid of area, the output can be expressed as

$$u_{ac} = \frac{\sum_{j=1}^{N_M} w_j \cdot P_j(\mathbf{z})}{\sum_{j=1}^{N_M} P_j(\mathbf{z})} \quad (7)$$

where N_M is the number of the receptive-fields, and $P_j(\mathbf{e})$ is the receptive-field function defined as

$$P_j(\mathbf{z}) = \prod_{i=1}^{n} \exp\left(-\left(\frac{z_i - m_{ij}}{\sigma_{ij}} \right)^2 \right) \tag{8}$$

where m_{ij} and σ_{ij} denote the mean values and variances, respectively. $w_1, w_2, ..., w_{N_M}$ are weighting parameters corresponding to the receptive-fields. Equation (7) can be rewritten as

$$u_{ac} = \boldsymbol{\theta}^T \cdot \psi(\mathbf{z}) \tag{9}$$

where $\boldsymbol{\theta} = [w_1, w_2, ..., w_{N_M}]^T$, and

$$\psi(\mathbf{z}) = \frac{P_j(\mathbf{z})}{\sum_{j=1}^{N_M} P_j(\mathbf{z})}$$

Then, since the state vector $\mathbf{z}$ in (6) can not be obtained, we design an observer that estimates the state vector $\mathbf{z}$. In addition, in order to achieve the control objective, the cerebellar model articulation system is used to approximate the ideal control $\bar{u}$ in (3), and the adaptive law is developed to adjust the parameters of the cerebellar model articulation system. Therefore, u_{ac} can expressed as

$$u_{ac}(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) = \hat{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) \tag{10}$$

B. Design of the the adaptive cerebellar model articulation controller

First, consider the following observer estimating the state vector $\mathbf{z}$ in (6)

$$\begin{aligned} \dot{\hat{\mathbf{z}}} &= \mathbf{A}\hat{\mathbf{z}} - \mathbf{B}\mathbf{k}_c^T \hat{\mathbf{z}} + \mathbf{B}(g\phi - g u_c) + \mathbf{k}_o(z_1 - \hat{z}_1) \\ \hat{z}_1 &= \mathbf{C}^T \hat{\mathbf{z}} \end{aligned} \tag{11}$$

where $\mathbf{k}_o = \left[k_o^n, k_o^{n-1}, \cdots, k_o^1 \right]^T$ is the observer gain vector and selected such that the characteristic polynomial of $\mathbf{A} - \mathbf{k}_o \mathbf{C}^T$ is strictly Hurwitz because $(\mathbf{C}, \mathbf{A})$ is observable. The purpose of the control term ϕ is to compensate the the modeling error. Define the observation errors as $\tilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$ and $\tilde{z}_1 = z_1 - \hat{z}_1$. Subtracting (11) from (6), we have

$$\begin{aligned} \dot{\tilde{\mathbf{z}}} &= (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T)\tilde{\mathbf{z}} + \mathbf{B}\left[g\bar{u} - g u_{ac}(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) - g\phi - d - d_h \right] \\ \tilde{z}_1 &= \mathbf{C}^T \tilde{\mathbf{z}} \end{aligned} \tag{12}$$

Then, the output error dynamics of (12) can be represented as

$$\tilde{z}_1 = W(s)\left[g u^* - g u_{FCMAC}(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) - g\phi - d - d_h \right] \tag{13}$$

where s is the Laplace variable, and $W(s) = \mathbf{C}^T (s\mathbf{I} - (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T))^{-1}\mathbf{B}$ is the transfer function of (12).

Next, define the ideal parameter vector as $\bar{\boldsymbol{\theta}} = \arg\min_{\hat{\boldsymbol{\theta}} \in M_{\hat{\boldsymbol{\theta}}}} \left[\sup_{\mathbf{z} \in U_{\mathbf{z}}, \hat{\mathbf{z}} \in U_{\hat{\mathbf{z}}}} \left| \bar{u} - u(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) \right| \right]$ [21,22], and an approximation error as $v = g\bar{u} - g u_{ac}(\hat{\mathbf{z}} | \bar{\boldsymbol{\theta}})$. In doing so, equation (12) can be rewritten as

$$\begin{aligned} \dot{\tilde{\mathbf{z}}} &= (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T)\tilde{\mathbf{z}} + \mathbf{B}\left[g u_{ac}(\hat{\mathbf{z}} | \bar{\boldsymbol{\theta}}) - g u_{ac}(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) - g\phi + v - d - d_h \right] \\ \tilde{z}_1 &= \mathbf{C}^T \tilde{\mathbf{z}} \end{aligned} \tag{14}$$

From (10), equation (14) can be rewritten as

$$\begin{aligned} \dot{\tilde{\mathbf{z}}} &= (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T)\tilde{\mathbf{z}} + \mathbf{B}\left[g\tilde{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) - g\phi + v - d - d_h \right] \\ \tilde{z}_1 &= \mathbf{C}^T \tilde{\mathbf{z}} \end{aligned} \tag{15}$$

where $\tilde{\boldsymbol{\theta}} = \bar{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}$. Because only the output $\tilde{z}_1$ in (15) is available for measurement, SPR Lyapunov approach is utilized to analyze the stability of (15) and the adaptive law for $\hat{\boldsymbol{\theta}}$ is obtained. Equation (15) can be rewritten as

$$\tilde{z}_1 = W(s)L(s)\left[\tilde{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) - \phi_f + v_f \right] \tag{16}$$

where $W(s) = \mathbf{C}^T (s\mathbf{I} - (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T))^{-1}\mathbf{B}$ is a known stable transfer function, $\phi_f = L^{-1}(s)[g\phi]$, $v_f = L^{-1}(s)v_d$, $v_d = v - d - d_h + g\tilde{\boldsymbol{\theta}}^T \xi(\hat{\mathbf{z}}) - L(s)\tilde{\boldsymbol{\theta}}^T \zeta(\hat{\mathbf{z}})$, and $\zeta(\hat{\mathbf{z}}) = L^{-1}(s)[\xi(\hat{\mathbf{z}})]$. $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper SPR transfer function. Then, the state space realization of (16) can be expressed as

$$\begin{aligned} \dot{\tilde{\mathbf{z}}} &= \boldsymbol{\Lambda}_c \tilde{\mathbf{z}} + \mathbf{B}_c\left[\tilde{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) - \phi_f + v_f \right] \\ \tilde{z}_1 &= \mathbf{C}_c^T \tilde{\mathbf{z}} \end{aligned} \tag{17}$$

where $\boldsymbol{\Lambda}_c = (\mathbf{A} - \mathbf{k}_o \mathbf{C}^T) \in \Re^{n \times n}, \mathbf{B}_c \in \Re^n$ and $\mathbf{C}_c^T = [1 \quad 0 \quad \cdots \quad 0] \in \Re^n$. To achieve the purpose of the stability of the adaptive cerebellar model articulation controller, the adaptive laws are chosen as

$$\dot{\hat{\boldsymbol{\theta}}} = \begin{cases} \gamma \tilde{z}_1 \psi(\hat{\mathbf{z}}) & if \ \|\hat{\boldsymbol{\theta}}\| < m_{\hat{\boldsymbol{\theta}}} \ or \ (\|\hat{\boldsymbol{\theta}}\| = m_{\hat{\boldsymbol{\theta}}} \ and \ \tilde{z}_1 \hat{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) \geq 0), \\ \Pr(\gamma \tilde{z}_1 \psi(\hat{\mathbf{z}})) & if \ \|\hat{\boldsymbol{\theta}}\| = m_{\hat{\boldsymbol{\theta}}} \ and \ \tilde{z}_1 \hat{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}}) < 0, \end{cases} \tag{18}$$

where the projection operator [24] is given as

$$\Pr(\gamma \tilde{z}_1 \psi(\hat{\mathbf{z}})) = \gamma \tilde{z}_1 \psi(\hat{\mathbf{z}}) - \gamma \frac{\tilde{z}_1 \hat{\boldsymbol{\theta}}^T \psi(\hat{\mathbf{z}})}{\|\hat{\boldsymbol{\theta}}\|^2} \hat{\boldsymbol{\theta}}$$

Base on the above discussions, the following theorems can be obtained.

Theorem: Consider the nonlinear dynamic system (1). It is assumed that Let $\hat{\boldsymbol{\theta}}$ be adjusted by the adaptive law (18), and let ϕ be given as

$$\phi = \begin{cases} \eta & if \ \tilde{z}_1 \geq 0 \\ -\eta & if \ \tilde{z}_1 < 0 \end{cases} \tag{19}$$

where $\eta \geq \kappa / g_L$, g_L is a positive constant and lower bound of $g(\mathbf{x})$, and κ is assumed as a positive constant and upper bound of $|v_d|$. Let $u_c = \phi$. The state observer (10) becomes

$$\begin{aligned} \dot{\hat{\mathbf{z}}} &= (\mathbf{A} - \mathbf{B}\mathbf{k}_c^T)\hat{\mathbf{z}} + \mathbf{k}_o \hat{z}_1 \\ \hat{z}_1 &= \mathbf{C}^T \hat{\mathbf{e}} \end{aligned} \tag{20}$$

Suppose that the control law is

$$u = u_{ac}(\hat{\mathbf{z}} | \hat{\boldsymbol{\theta}}) + u_c \tag{21}$$

Then (a) $\tilde{z}_1(t)$ converges to zero as $t \to \infty$. (b) All signals of the closed loop systems are bounded, and $e_1(t)$ converges to zero as $t \to \infty$. □

Proof of (a): Consider the Lyapunov-like function candidate

$$V = \frac{1}{2} \tilde{\mathbf{z}}^T \mathbf{P} \tilde{\mathbf{z}} + \frac{1}{2\gamma} \tilde{\boldsymbol{\theta}}^T \tilde{\boldsymbol{\theta}} \tag{22}$$

where $\mathbf{P} = \mathbf{P}^T > 0$. Differentiating (22) with respect to time and inserting (17) in the above equation yield

$$\dot{V} = \frac{1}{2}\tilde{\mathbf{z}}^T\left(\mathbf{A}_c^T\mathbf{P} + \mathbf{P}\mathbf{A}_c\right)\tilde{\mathbf{z}} + \tilde{\mathbf{z}}^T\mathbf{P}\mathbf{B}_c\left[\tilde{\boldsymbol{\theta}}^T\psi - v_f + w_f\right] + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}} \quad (23)$$

Because $H(S)L(S)$ is SPR, there exists $\mathbf{P} = \mathbf{P}^T > 0$ such that

$$\mathbf{A}_c^T\mathbf{P} + \mathbf{P}\mathbf{A}_c = -\mathbf{Q}$$
$$\mathbf{P}\mathbf{B}_c = \mathbf{C}_c$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$. By using (22), (23) becomes

$$\dot{V} = -\frac{1}{2}\tilde{\mathbf{z}}^T\mathbf{Q}\tilde{\mathbf{z}} + \tilde{z}_1\left[\tilde{\boldsymbol{\theta}}^T\psi \ - v_f + w_f\right] + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}} \quad (24)$$

By using (19) and the fact $\lambda_{\min}(\mathbf{Q})\|\tilde{\mathbf{z}}\|^2 \ge \lambda_{\min}(\mathbf{Q})|\hat{z}_1|^2$, where $\lambda_{\min}(\mathbf{Q}) \ge 0$, we have

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{z}_1|^2 + \tilde{z}_1\left(\tilde{\boldsymbol{\theta}}^T\psi + \frac{1}{\gamma}\dot{\tilde{\boldsymbol{\theta}}}^T\tilde{\boldsymbol{\theta}}\right) \quad (25)$$

Inserting (18) in (25) and after some manipulation yields

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(\mathbf{Q})|\tilde{z}_1|^2 \quad (26)$$

Equation (22) and (29) only guarantee that $\tilde{z}_1(t) \in L_\infty$ and $\tilde{\mathbf{z}}(t) \in L_\infty$, but do not guarantee the convergence. Because all variables in the right-hand side of (17) are bounded, $\dot{\tilde{z}}_1(t)$ is bounded, i.e., $\dot{\tilde{z}}_1(t) \in L_\infty$. Integrating both side of (26) and after some manipulation yields

$$\int_0^\infty |\tilde{z}_1(t)|^2 dt \le \frac{V(0) - V(\infty)}{\frac{1}{2}\lambda_{\min}(\mathbf{Q})}. \quad (27)$$

Since the right side of (27) is bounded, so $\tilde{z}_1(t) \in L_2$. By using Barbalat's lemma [25], we have $\lim_{t\to\infty}|\tilde{z}_1(t)| = 0$.

Proof of (b): First, from (a), we have $\lim_{t\to\infty}|\tilde{z}_1(t)| = 0$ and $\tilde{\mathbf{z}}(t) \in L_\infty$. By using (11) and the fact $u_c = \phi$, we obtain

$$\dot{\hat{\mathbf{z}}} = \left(\mathbf{A} - \mathbf{B}\mathbf{K}_c^T\right)\hat{\mathbf{z}} + \mathbf{K}_o\mathbf{C}^T\tilde{\mathbf{z}}$$
$$\hat{z}_1 = \mathbf{C}^T\hat{\mathbf{z}} \quad (28)$$

Similarly, because $\mathbf{A} - \mathbf{B}\mathbf{K}_c^T$ is a Hurwitz matrix and $\tilde{\mathbf{z}}(t)$ is bounded, $\hat{\mathbf{z}}(t)$ is bounded. From $\tilde{\mathbf{z}} = \mathbf{z} - \hat{\mathbf{z}}$, it follows that $z_1, \mathbf{z} \in L_\infty$ and $z_1(t) \to 0$ as $t \to \infty$. From $\hat{\mathbf{z}}, \mathbf{z}, \mathbf{r} \in L_\infty$, it follows that $\mathbf{x}, \hat{\mathbf{x}} \in L_\infty$. The boundedness of $y(t)$ follows that of $z_1(t)$ and $r(t)$. This completes the proof.

To summarize, the overall scheme of the adaptive cerebellar model articulation control system shown in Fig. 2.

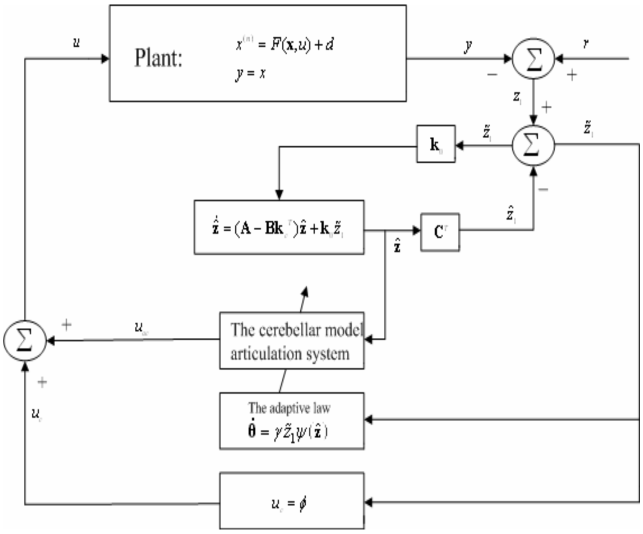

Fig. 2. Overall scheme of the adaptive cerebellar model articulation control system.

According to the above discussion, the design algorithm of the proposed method is as follows.

Design algorithm.

Step 1: Select the feedback and observer gain vectors $\mathbf{k}_c, \mathbf{k}_o$ such that the matrices $\mathbf{A} - \mathbf{B}\mathbf{k}_c^T$ and $\mathbf{A} - \mathbf{k}_o\mathbf{C}^T$ are Hurwitz matrices.

Step 2: Choose an appropriate value η in (19) and γ in (18). In order to remedy the control chattering, (19) can be modified as

$$\phi = \begin{cases} \eta & if\ \tilde{z}_1 \ge 0\ and\ |\tilde{z}_1| > \alpha \\ -\eta & if\ \tilde{z}_1 < 0\ and\ |\tilde{z}_1| > \alpha \\ \eta\,\tilde{z}_1/\alpha & if\ |\tilde{z}_1| < \alpha \end{cases} \quad (29)$$

where α is a positive constant.

Step 3: Solve the state observer in (20), where ϕ is given in (29).

Step 4: Determine the Gaussian basis function in (7) for $\hat{\mathbf{z}}(t)$. Then from (8) compute the basis vector ψ.

Step 5: Obtain the control law (21), and the adaptive law (18).

IV. THE SIMULATION EXAMPLES

The simulation results of the proposed design algorithm to illustrate that stability of the closed loop system is guaranteed, and all signals involved are bounded.

Example 1: Consider the Duffing forced oscillation system [23]

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = q_1x_2 + q_2x_1^3 + q_3\cos(wt) + u + d \quad (30)$$
$$y = x_1$$

Let $q_1 = -5$, $q_2 = -6$, $q_3 = 12$ $w = 1$. It is assumed that the external disturbance $d(t)$ is a square wave having an amplitude ± 1 with a period of 2π . The control objective is to control the state x_1 of the system to track the reference trajectory y_d , under the condition that only the system output y is available for measurement. The design parameters are selected as $\gamma = 0.5 \times 10^3$, $\alpha = 0.005$, $\eta = 20$, and $N_M = 64$. The feedback and observer gain vectors are given as $\mathbf{k}_c = [144 \quad 24]^T$ and $\mathbf{k}_o = [60 \quad 900]^T$, respectively. The filter $L^{-1}(s)$ is given as $L^{-1}(s) = 1/(s+2)$. The initial states are chosen to be $x_1(0) = x_2(0) = 1$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = 2$ and $\hat{\mathbf{z}}(0) = \mathbf{r}(0) - \hat{\mathbf{x}}(0)$.

Case 1: Simulation results are provided with a following model reference [26].

$$\begin{aligned} \dot{h}_{d1} &= h_{d2} \\ \dot{h}_{d2} &= -h_{d1} + 1.5(1 - h_{d1}^2) h_{d2} \\ r_j &= h_{dj}, \quad j = 1,2, \quad \mathbf{h_{d0}} = [1 \quad 1]^T \end{aligned} \tag{31}$$

Fig. 3 shows the system output can track the desired output well. Fig. 4 shows the trajectories of the states x_1 and $\hat{x}_1$. Fig. 5 shows the control input u .

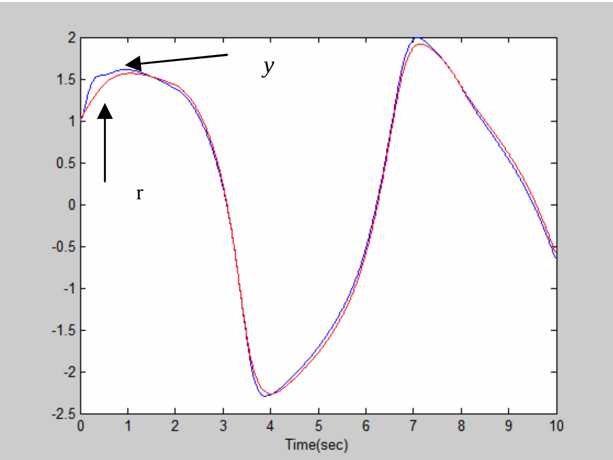

Fig. 3. Trajectories of y and r

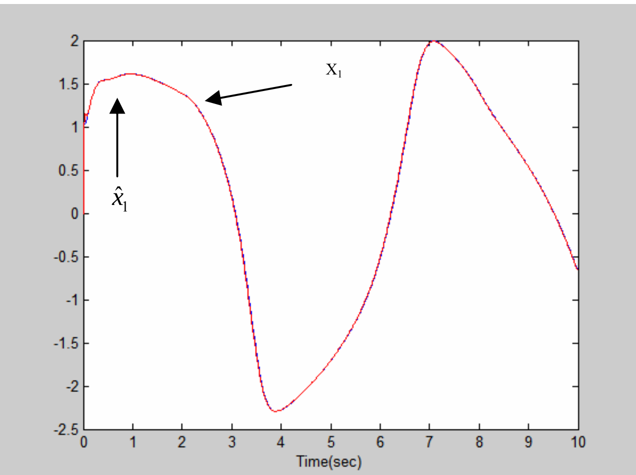

Fig. 4. Trajectories of x_1 and $\hat{x}_1$

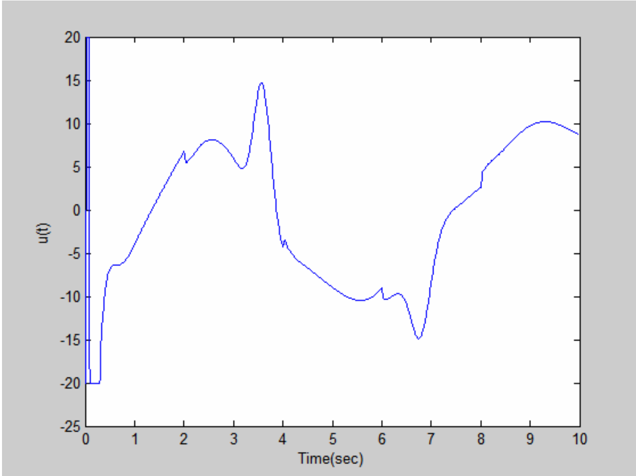

Fig. 5. The control input u

Case 2: Simulation results are provided with a model reference defined by

$$\begin{aligned} \dot{\mathbf{h}} &= \mathbf{M}\mathbf{h} + \mathbf{N}u_r \\ r &= \mathbf{J}\mathbf{h} \end{aligned} \tag{32}$$

where

$$\mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} 0 & 1 \\ -100 & -12 \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} 1 \\ 100 \end{bmatrix} \tag{33}$$

$$\mathbf{J} = [1 \quad 0], \quad \mathbf{h}_0 = [0 \quad 0].$$

Fig. 6 shows the system output can track the desired output well. Fig. 7 shows the trajectories of the states x_1 and $\hat{x}_1$. Fig. 8 shows the control input u .

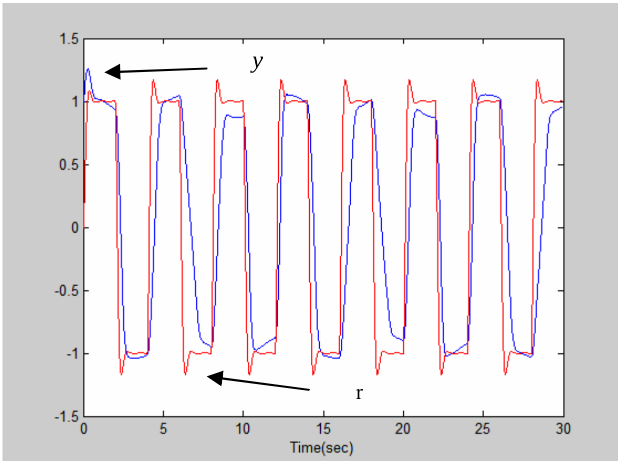

Fig. 6. Trajectories of y and r

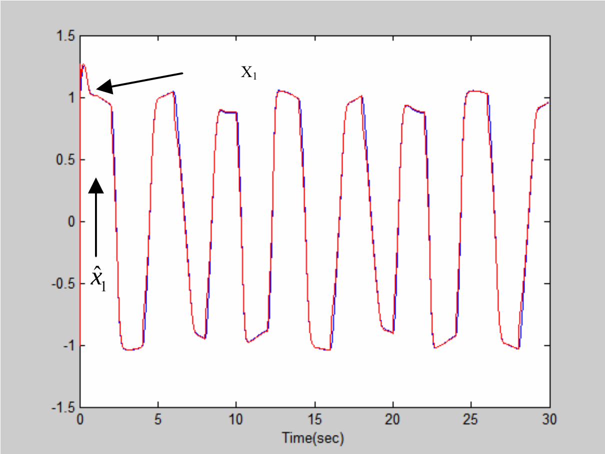

Fig. 7. Trajectories of x_1 and $\hat{x}_1$

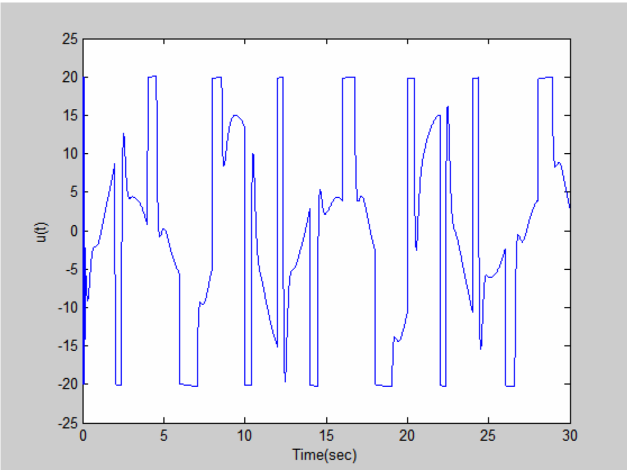

Fig. 8. The control input u

Example 2: Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 x_2 - 2x_1^2 + u + d \\ y &= x_1 \end{aligned} \tag{34}$$

The parameters are the same as those in example 1.The initial states are chosen to be $x_1(0) = x_2(0) = 2$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = 2$ and $\hat{\mathbf{z}}(0) = \mathbf{r}(0) - \hat{\mathbf{x}}(0)$.

Case 1: Simulation results are provided with a following model reference as the case 1 of example 1. Fig. 9 shows the system output can track the desired output well. Fig. 10 shows the trajectories of the states x_1 and $\hat{x}_1$.Fig. 11 shows the control input u.

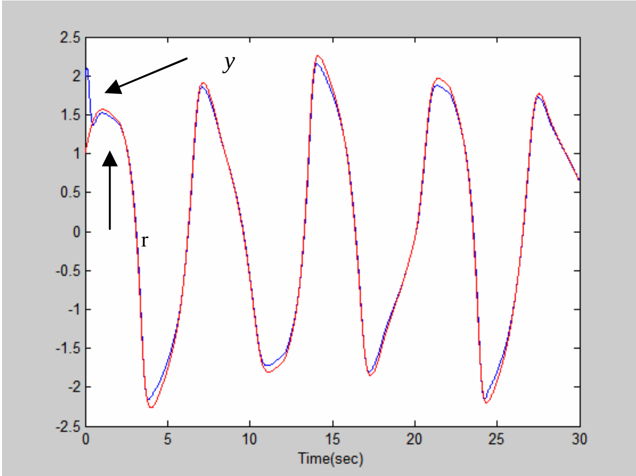

Fig. 9. Trajectories of y and r

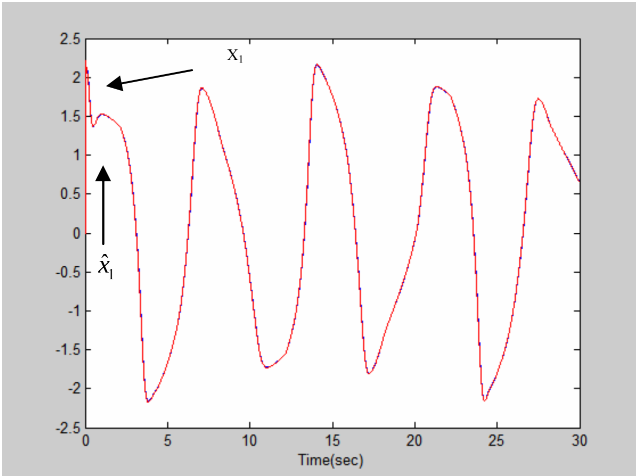

Fig.10. Trajectories of x_1 and $\hat{x}_1$

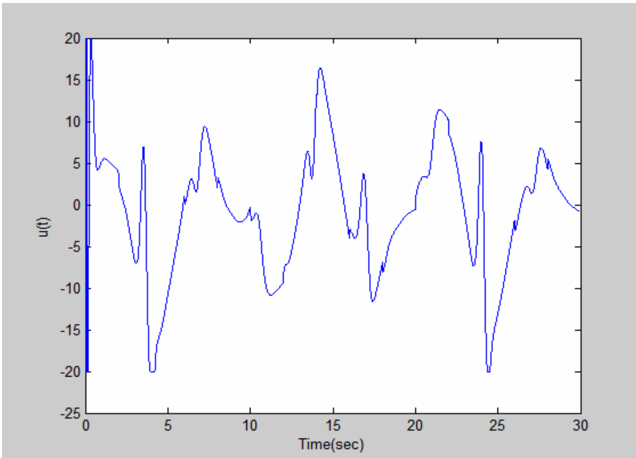

Fig. 11. The control input u

Case 2: Simulation results are provided with a model reference as the case 2 of example 1. Fig. 12 shows the system output can track the desired output well. Fig. 13 shows the trajectories of the states x_1 and $\hat{x}_1$.Fig. 14 shows the control input u.

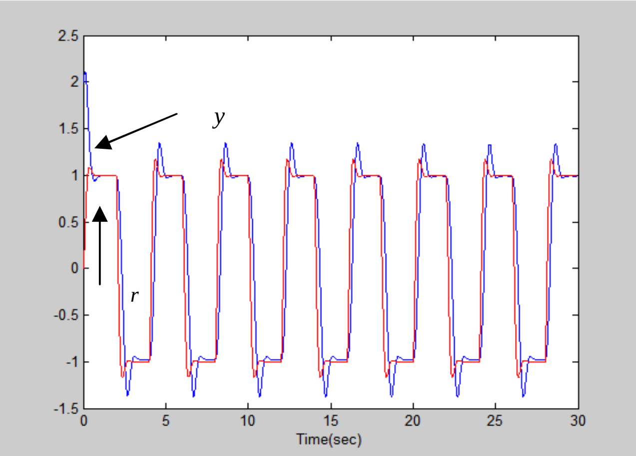

Fig. 12. Trajectories of y and r .

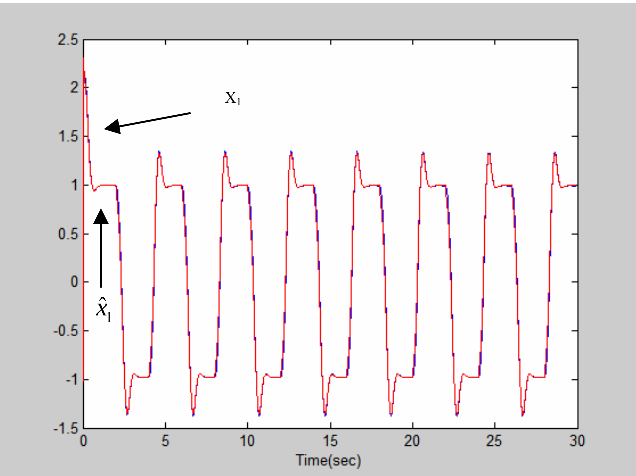

Fig.13. Trajectories of x_1 and $\hat{x}_1$

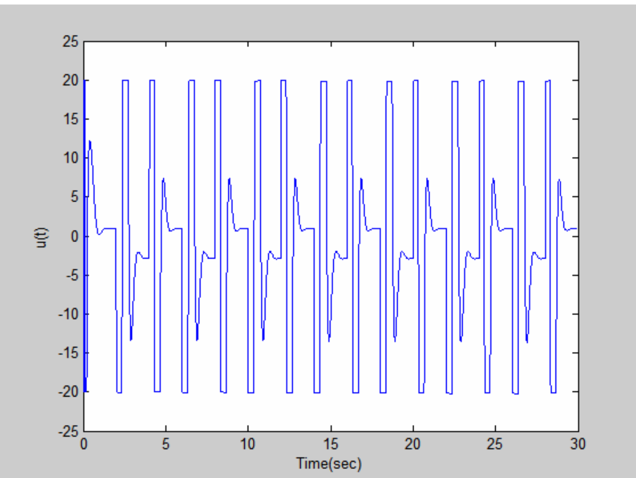

Fig. 14. The control input u

V. CONCLUSIONS

The adaptive cerebellar model articulation control system has been proposed for a class of nonlinear systems with only the system output measurement. The stability of the closed loop system can be guaranteed, and the output can asymptotically track a given bounded reference signal. The simulation results for two nonlinear systems demonstrate that the tracking performance can be achieved via the proposed control scheme.

IV. CONCLUSIONS

The adaptive cerebellar model articulation control system has been proposed for a class of nonlinear systems with only the system output measurement. The stability of the closed loop system can be guaranteed, and the output can asymptotically track a given bounded reference signal. The simulation results for two nonlinear systems demonstrate that the tracking performance can be achieved via the proposed control scheme.

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