

Interacting Multiple Model Particle-type Filtering Approaches to Ground Target Tracking

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Abstract—Ground maneuvering target tracking is a class of nonlinear and/or no-Gaussian filtering problem. A new interacting multiple model unscented particle filter (IMMUPF) is presented to deal with the problem. A bank of unscented particle filters is used in the interacting multiple model (IMM) framework for updating the state of moving target. To validate the algorithm, two groups of multiple model filters: IMM-type filters and particle-type multiple model filters, are compared for their capability in dealing with ground maneuvering target tracking problem. Simulation shows that particle-type filters outperform IMM-type filters in the estimate accuracy and the IMMUPF method relatively has much better performance than the IMMPF method.

Index Terms—particle filter, unscented particle filter (UPF), interacting multiple model (IMM), ground target tracking

I. INTRODUCTION

Ground target tracking has become the focus of increased investigation with the increased demands in recent years of modern warfare [1]-[4]. Complex ground target tracking, with non-maneuvering, low maneuvering and high maneuvering targets simultaneously, belongs to the class of nonlinear and/or non-Gaussian filtering problems. A single model cannot sufficiently model the complex variables. One popular technique for tracking maneuvering targets is the multiple model (MM) approach, especially the interacting multiple model (IMM) approach [5], [6]. The MM approach has proven to be an appropriate method for handling such nonlinear filtering problems. The IMM estimator relatively performs much better than other MM methods [5].

It is well known that the Kalman filter is the optimal estimator to linear and Gaussian systems. The extended Kalman filter (EKF) is the most popular approach to implement recursive nonlinear estimations. However, it may suffer from larger errors if the system has strong nonlinearity [7]. Recently, particle filter (PF) has been introduced. Particle filtering methods, using a large number of random samples to directly approximate the probability density function of a state distribution, may deal with any nonlinearity in the dynamics and

measurements. Furthermore, the assumption that the noises are Gaussian can be neglected. Gordon proposed the first working particle filter or bootstrap filter [8]. A sequential importance re-sampling (SIR) scheme is used to curtail the degeneracy of data. It simply takes the prior distribution as the proposal distribution in the calculation of the importance weights. If there is little overlap between the prior and the likelihood, this approximation will introduce large errors because it fails to use the latest available measurement. To overcome the drawback, some filtering methods may efficiently generate the accurate mean and covariance of the proposal distribution. An unscented particle filter (UPF) [9], [10], which uses an unscented Kalman filter (UKF) [11], [12] to generate the proposal distribution, is used for the updating stage of the sequential importance sampling (SIS). The UPF method has two advantages: first, it makes efficient use of the latest available information; second, it has heavier tails. To approximate the true mean and covariance of Gaussian random variable, the EKF method can only achieve the first-order level. The unscented filter, in contrast, accurately captures up to the third-order level (Taylor series expansions) for any nonlinearity [13]. Particle filtering approaches for Markovian switching systems have been proposed in [14]-[16]. A major drawback of these methods is that there is no control over the number of particles in a mode. In [17], a multiple model particle filter is proposed. However, there is no interaction between the modes. To address this problem, Boers and Driessen proposed an interacting multiple model particle filter (IMMPF) which uses a regularized particle filter for the filtering step [18].

Combining an unscented particle filter with the IMM approach, an interacting multiple model unscented particle filtering (IMMUPF) method which uses an UKF to generate the proposal distribution is presented. Similar to the standard IMM filters, the IMMUPF consists of four steps: interaction/mixing, filtering, mode probability update, and state combination. To validate the IMMUPF algorithm, two groups of nonlinear IMM estimators: the standard IMM filters and particle-type IMM filters are compared in their abilities to handle a typical ground

moving target tracking problem.

The organization of this paper is as follows. System dynamic and measurement models are described in Section II. In Section III several nonlinear filtering methods based on a single model frame are described. In Section IV the basic structure of the IMM estimator is introduced and the IMMUPF method is then proposed. To validate the IMMUPF approach, a typical ground target tracking example is given in Section V.

II. SYSTEM DYNAMIC AND MEASUREMENT MODELS

A. Dynamic Model

The target state vector is defined: $\mathbf{x} = [x, y, v_x, v_y, a_x, a_y]^T$, where position vector $\mathbf{s} = [x, y]^T$, velocity vector $\mathbf{v} = [v_x, v_y]^T$, and acceleration vector $\mathbf{a} = [a_x, a_y]^T$. The system noise vector is given by $\mathbf{w} = [w_x, w_y]^T$. Note that w_x and w_y correspond to zero-mean white noise “accelerations” along X and Y axes respectively.

Non-Maneuvering Model (M_1): A constant velocity (CV) model is modeled by

$$f(\mathbf{x}_k, t_k, M_1) = \begin{bmatrix} 1 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}_k, \quad (1)$$

$$g(\mathbf{x}_k, t_k, M_1) = \begin{bmatrix} \frac{1}{2}(\Delta t)^2 & 0 & \Delta t & 0 & 0 & 0 \\ 0 & \frac{1}{2}(\Delta t)^2 & 0 & \Delta t & 0 & 0 \end{bmatrix}^T \mathbf{w}_k, \quad (2)$$

where Δt is the sampling time.

Maneuvering Model: A constant acceleration (CA) is given as follows:

$$f(\mathbf{x}_k, t_k, M_{2,3}) = \begin{bmatrix} 1 & 0 & \Delta t & 0 & \frac{1}{2}(\Delta t)^2 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & \frac{1}{2}(\Delta t)^2 \\ 0 & 0 & 1 & 0 & \Delta t & 0 \\ 0 & 0 & 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_k, \quad (3)$$

$$g(\mathbf{x}_k, t_k, M_{2,3}) = \begin{bmatrix} \frac{1}{2}(\Delta t)^2 & 0 & \Delta t & 0 & 1 & 0 \\ 0 & \frac{1}{2}(\Delta t)^2 & 0 & \Delta t & 0 & 1 \end{bmatrix}^T \mathbf{w}_k. \quad (4)$$

For the above system, maneuvers can be modeled by positive and negative accelerations with the same process noise. Therefore, we may obtain two maneuvering models: positive CA model, denoted by M_2 and negative CA model, denoted by M_3 . The general dynamic discrete system for the multiple models is defined in (1) and (2), where $M \in \{1, 2, 3\}$, $M=1$, $M=2$ and $M=3$ correspond to M_1 (CV), M_2 (positive CA) and M_3 (negative CA) respectively [19].

To model abrupt acceleration changes of maneuvering targets, a heavy-tailed non-Gaussian distribution

$$w(k) \sim C(0, Q_C),$$

$$Q_C = \begin{bmatrix} q_x^2 & 0 \\ 0 & q_y^2 \end{bmatrix} \quad (5)$$

is introduced to the process noise, where $C(0, Q_C)$ is a Cauchy distribution and denotes a heavy-tailed distribution with central position 0 and covariance Q_C .

B. Measurement Model

Measurements in a polar format taken by a ground moving target indicator (GMTI) radar in discrete time including range and bearing, is given by

$$z(k) = \begin{bmatrix} r_k \\ \phi_k \end{bmatrix} = h(\mathbf{x}_k) + \mathbf{v}_k$$

$$= \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \text{atan}\left(\frac{y_k}{x_k}\right) \end{bmatrix} + \begin{bmatrix} v_{r_k} \\ v_{\phi_k} \end{bmatrix}, \quad (6)$$

where \mathbf{v}_k is a zero-mean Gaussian noise vector with variance $R_k = \text{diag}(\sigma_r^2, \sigma_\phi^2)$, and σ_r and σ_ϕ are standard deviations for the range and bearing respectively. Although the platform is moving, for simplicity, the GMTI radar is assumed at the original point. The sampling time is set to be $T=\Delta t=t_k=1s$.

C. A General System Model

A general dynamic system for multiple models in discrete time is rendered by

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, t_k, M_k) + g(\mathbf{x}_k, t_k, M_k) w(\mathbf{x}_k, M_k), \quad (7)$$

$$\mathbf{z}_k = h(\mathbf{x}_k, t_k, M_k) + \mathbf{v}(\mathbf{x}_k, M_k), \quad (8)$$

where

- $f(\cdot)$ and $h(\cdot)$ are the parameterized state transition and measurement functions¹,
- \mathbf{x}_k and \mathbf{z}_k are the dynamical state and measurement of the system in mode M_k ,
- $M_k \in M \subset \mathbb{N}$ is the modal state in effect in the interval $(k-1, k]$ of the system, and the system itself is a Markov chain,
- w, v are the process noise and measurement noise with means \bar{w} and \bar{v} and covariances Q_k and R_k respectively,

¹In the context, $f(\cdot)$, $h(\cdot)$ and $g(\cdot)$ are the abbreviations of $f(\mathbf{x}_k, t_k, M_k)$, $h(\mathbf{x}_k, t_k, M_k)$ and $g(\mathbf{x}_k, t_k, M_k)$ respectively.

$g(\cdot)$ is the control input,
 t_k is the sampling time, in this context a constant variable.

The model transition probability is modeled as a Markov chain with

$$\pi_{ij} = P_{ij} = \text{Prob}\{M_k = j | M_{k-1} = i\}, \forall i, j \in M \quad (9)$$

$$M = \{1, \dots, r\},$$

where r is the number of possible models. In continuous time, M_k , under assumption, is in effect within the semi-closed interval $(t_{k-1}, t_k]$.

III. NONLINEAR FILTERING METHODS FOR GROUND TARGET TRACKING

In this section three typical nonlinear estimators are described, including the EKF, UKF and UPF, to be used for tracking ground maneuvering target. These nonlinear filters determine in an approximate way the mean and covariance of the probability of a target state conditioned to the measurements. A single model equation in discrete time for numeric study is given by

$$x_{k+1} = f(x_k, w_k), \quad (10)$$

$$z_k = h(x_k, v_k). \quad (11)$$

A. Extended Kalman filter

If the system is nonlinear, a local linearization of the equations may be used to describe the nonlinearity. This approximation is applied to compute the estimate $\hat{x}_{k-1|k-1}$ and propagation $\hat{x}_{k|k-1}$. The extended Kalman filter (EKF) [4], [20] is based on the Taylor series expansion of the nonlinear functions $f(\cdot)$ and $h(\cdot)$ with only linear terms being used. The state propagation and update are as follows:

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}), \quad (12)$$

$$P_{k|k-1} = F_{k-1} P_{k-1|k-1} F_{k-1}^T + Q_{k-1}, \quad (13)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - h(\hat{x}_{k|k-1})), \quad (14)$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}, \quad (15)$$

where P is the state covariance², and the Kalman filter gain K_k is determined by

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \quad (16)$$

and F_{k-1} and H_k are the Jacobians of the system equation and measurement equation. The EKF only uses the first-order terms of the Taylor series expansion of the nonlinear state equations. If the system is strong nonlinear or a local linearization assumption breaks down, it may introduce large estimation errors due to filter

²In the context, P (capital) is the state covariance, P (capital and italic) is a symbol of the probability, and p (small letter and italic) is a symbol of the probability density.

divergence.

B. Unscented Kalman filter

The unscented transform has been used in an extended Kalman filter frame lately [11], [12]. Same as the EKF, the unscented Kalman filter (UKF) is a recursive minimum mean square error (MMSE) filter. However, the UKF does not approximate the nonlinear state and measurement equations. It uses the true nonlinear model of state and/or measurement equation but approximates the probability density function (PDF) of the state vector. The density is represented by a set of deterministically chosen sample points (or sigma points) which completely captures the true mean and covariance of the Gaussian density. Therefore, the UKF can capture the posterior mean and covariance accurately to the third order for any nonlinearity.

Assume n -dimension state vector x_{k-1} with mean $\bar{x}_{k-1|k-1}$ and covariance $P_{k-1|k-1}$ be approximated by $2n+1$ weighted samples or sigma points. Then one cycle of the UKF is as follows:

(1) Calculate sigma points:

$$\begin{cases} \chi_{k-1|k-1}^0 = \bar{x}_{k-1|k-1} & i = 0 \\ \chi_{k-1|k-1}^i = \bar{x}_{k-1|k-1} + \left(\gamma \sqrt{P_{k-1|k-1}}\right)_i & i = 1, \dots, n \\ \chi_{k-1|k-1}^{i+n} = \bar{x}_{k-1|k-1} - \left(\gamma \sqrt{P_{k-1|k-1}}\right)_i & i = 1, \dots, n, \end{cases} \quad (17)$$

where $\gamma = \sqrt{n + \kappa}$, the corresponding weight of samples is given by

$$\begin{cases} w_0 = \kappa / (n + \kappa) \\ w_i = 1 / (2(n + \kappa)) \quad i = 1, 2, \dots, 2n, \end{cases} \quad (18)$$

where κ is a scaling factor and $(\sqrt{(n+\kappa)P_{k|k-1}})_i$ is the i th row or column of the matrix square root of $(n+\kappa)P_{k|k-1}$ and w_i is the normalized weight which is associated with the i th point such that $\sum_{i=0}^{2n} w_i = 1$. Numerically efficient and stable methods such as the Cholesky decomposition [21] are needed for the matrix square root.

(2) Propagation (time update): The sigma points are propagated and the estimated mean and covariance of the state are computed as follows:

$$\chi_{k|k-1}^i = f(\chi_{k-1|k-1}^i), \quad (19)$$

$$\hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i \chi_{k|k-1}^i, \quad (20)$$

$$P_{k|k-1} = Q_{k-1} + \sum_{i=0}^{2n} w_i [\chi_{k-1|k-1}^i - \hat{x}_{k|k-1}] [\chi_{k-1|k-1}^i - \hat{x}_{k|k-1}]^T. \quad (21)$$

(3) Measurement update: Using $h(\cdot)$ to calculate the measurement sigma points $\xi_{k|k-1}^i$ and update the mean and covariance by

$$\xi_{k|k-1}^i = h(\chi_{k|k-1}^i), \quad (22)$$

$$\hat{z}_{k|k-1} = \sum_{i=0}^{2n} w_i \xi_{k|k-1}^i. \quad (23)$$

(4) Calculate filter gain:

$$P_z = \sum_{i=0}^{2n} w_i (\xi_{k|k-1}^i - \hat{z}_{k|k-1})(\xi_{k|k-1}^i - \hat{z}_{k|k-1})^T, \quad (24)$$

$$P_{xz} = \sum_{i=0}^{2n} w_i (\chi_{k|k-1}^i - \hat{x}_{k|k-1})(\xi_{k|k-1}^i - z_{k|k-1})^T, \quad (25)$$

$$K_k = P_{xz} P_z^{-1}. \quad (26)$$

(5) Output:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - \hat{z}_{k|k-1}), \quad (27)$$

$$P_{k|k} = P_{k|k-1} - K_k P_z K_k^T. \quad (28)$$

C. Unscented particle filter

It has been proven that the EKF and UKF can deal well with nonlinear filtering problems. However, they always approximate $p(x_k|Z_{1:k})$ to be Gaussian. If the true density is non-Gaussian, they can never address the problems well. In such cases, particle filters and its varieties, such as an unscented particle filter (UPF), may yield better performances in comparison to that of an EKF or UKF. To improve the estimate accuracy, the UKF can be used to generate the true mean and covariance of the proposal distribution. Starting from $k=1$, one cycle of the unscented particle filter can be described in brief as follows.

(1) Initialization

Assume $\{x_0^i\}_{i=1}^N$ be a set of particles sampled from the prior $p(x_0)$ at $k=0$ and set

$$\begin{aligned} \hat{x}_0^i &= E[x_0^i], \\ P_0^i &= E[(x_0^i - \hat{x}_0^i)(x_0^i - \hat{x}_0^i)^T]. \end{aligned} \quad (29)$$

(2) Importance sampling

(i) Update each particle with the UKF to obtain mean $\bar{x}_{k|k}^i$ and covariance $P_{k|k}^i$.

(ii) Sample $\hat{x}_{k|k}^i$ can be drawn from $q(x_k^i | x_{0:k-1}^i, Z_{1:k}) = N(\bar{x}_{k|k}^i, P_{k|k}^i)$, Set $\tilde{x}_{0:k}^i = \{x_{0:k-1}^i, \hat{x}_{k|k}^i\}$ and $\tilde{P}_{0:k}^i = \{P_{0:(k-1)}^i, P_{k|k}^i\}$.

(iii) The importance weight can be evaluated by

$$w_k^i \propto \frac{p(z_k | \hat{x}_{k|k}^i) p(x_k^i | x_{k-1}^i)}{q(\hat{x}_{k|k}^i | x_{0:k-1}^i, Z_{1:k})}, i = 1, \dots, N, \quad (30)$$

where $Z_{1:k}$ denotes the all measurements from $t=1$ to $t=k$, N is the number of samples. The normalized weight is given by

$$\tilde{w}_k^i = \frac{w_k^i}{\sum_{j=1}^N w_k^j}. \quad (31)$$

(3) Selection or re-sampling

To obtain N random samples $\{\hat{x}_{0:k}^i, P_{0:k}^i\}_{i=1}^N$, it is necessary to resample N times from $\{\tilde{x}_{0:k}^i, \tilde{P}_{0:k}^i\}_{i=1}^N$ with the

importance weights. The procedure multiplies the high-weight particles and eliminates the low-weight particles in the state space to match the changes of the PDF over the state transition. Then, the weights of all reserved particles are set to be: $w_k^i = \tilde{w}_k^i = 1/N$.

Note that the above resampling scheme reduces the degeneracy phenomenon at the cost of increasing the computational load and losing the particle diversity. Other efficient resampling schemes, such as regularized particle filter (RPF) [14] and MCMC (Markov chain Monte Carlo) step [22], may be applied to compensate the drawbacks.

(4) Output

Via the output of a set of samples used to approximate the posterior distribution given by

$$p(x_{0:k} | Z_{1:k}) \approx \hat{p}(x_{0:k} | Z_{1:k}) = \frac{1}{N} \sum_{i=1}^N \delta_{(x_{0:k}^i)}(dx_{0:k}), \quad (32)$$

The state is eventually updated by means of the particles as follows:

$$\hat{x}_{k|k} \approx \frac{1}{N} \sum_{i=1}^N \hat{x}_{k|k}^i. \quad (33)$$

IV. TWO GROUPS OF IMM FILTERING METHODS FOR GROUND TARGET TRACKING

In many ground target tracking systems, it is difficult to use a single model to match the dynamics of nonlinear systems. Generally, multiple model techniques, especially the interacting multiple model method, are applied to deal with such complexity filtering problems. A group of standard IMM filters based on Gaussian approximation are simply described at first. Another group of particle-type filters based on sequential Monte Carlo methods, especially the IMMUPF algorithm, are developed subsequently.

A. The Standard IMM Filters

The standard IMM estimator is a suboptimal hybrid filter that has been proven to be one of the most cost-effective hybrid state estimation schemes [23]. The main advantage of this method is its ability to estimate the state of a dynamic system with several behavior modes which can transit from one to another. The general structure of the IMM consists of a bank of filters for the state cooperating with a filter for the parameters. One cycle of the IMM estimator consists of the following:

(1) Calculation of the mixing probabilities

$$\mu_{k-1|k-1}^{ij} = p_{ij} \mu_{k-1}^i / \bar{c}_j \quad i, j = 1, \dots, r, \quad (34)$$

where the normalizing constants are

$$\bar{c}_j = \sum_{i=1}^r p_{ij} \mu_{k-1}^i \quad j = 1, \dots, r. \quad (35)$$

(2) Mixing

Starting with the estimate $\hat{x}_{k-1|k-1}^i$, the mixed initial condition for the filter matched to the mode M_k^j is computed by

$$\hat{x}_{k-1|k-1}^{0j} = \sum_{i=1}^r \hat{x}_{k-1|k-1}^i \mu_{k-1|k-1}^{ij} \quad j = 1, \dots, r. \quad (36)$$

The corresponding covariance is

$$\begin{aligned} P_{k-1|k-1}^{0j} &= \sum_{i=1}^r \mu_{k-1|k-1}^{ij} \{P_{k-1|k-1}^i \\ &+ [\hat{x}_{k-1|k-1}^i - \hat{x}_{k-1|k-1}^{0j}][\bullet]^T\} \quad j = 1, \dots, r. \end{aligned} \quad (37)$$

(3) Mode-matched filtering

Using (36), (37) and z_k , the estimate and covariance of the filter matched to the mode M_k^j are yielded. The likelihood functions are

$$\begin{aligned} \Lambda_k^j &= p[z_k | M_k^j, Z_{1:k-1}] \\ &= p[z_k | M_k^j, \hat{x}_{k-1|k-1}^{0j}, P_{k-1|k-1}^{0j}] \quad (38) \\ j &= 1, \dots, r. \end{aligned}$$

(4) Mode probability update

$$\mu_k^j = \Lambda_k^j \bar{c}_j / c \quad j = 1, \dots, r, \quad (39)$$

where c is the normalization constant, given by

$$c = \sum_{j=1}^r \Lambda_k^j \bar{c}_j. \quad (40)$$

(5) Estimate and covariance combination

The combined estimate and covariance are computed according to the mixture equations:

$$\hat{x}_{k|k} = \sum_{j=1}^r \hat{x}_{k|k}^j \mu_k^j, \quad (41)$$

$$P_{k|k} = \sum_{j=1}^r \mu_k^j \{P_{k|k}^j + [\hat{x}_{k|k}^j - \hat{x}_{k|k}][\bullet]^T\}. \quad (42)$$

In comparison to other MM estimators such as the generalized pseudo Bayes (GPB) method, only the step “interaction/mixing” is typical for the IMM estimator. A basic structure of the IMM is shown in Fig. 1.

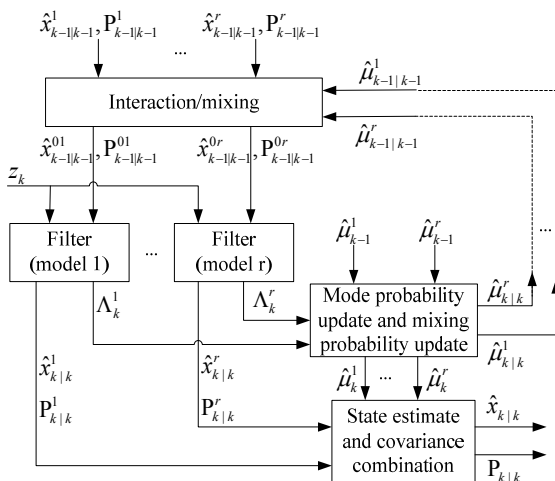


Figure 1. A basic structure of the standard IMM estimators.

Two standard IMM estimators, IMMEKF and IMMUKF, are derived when the nonlinear filtering

approaches such as the EKF and the UKF are used in the “filter” modules. A detailed implementation of the standard IMM filters refers to [5], [6].

B. Interacting Multiple Model Unscented Particle Filter

Integrating a bank of unscented particle filters with the IMM method, an interacting multiple model unscented particle (IMMUPF) is derived. Starting from $k-1$, one cycle of the IMMUPF algorithm can be described in detail as follows.

(1) Interaction/mixing: Based on the Markov jump model, the model likelihoods and the posterior probability densities for different modes at $k-1$, the initial densities $\hat{p}^{0j}(x_{k-1}^{0j} | Z_{1:k-1})$ are computed as Gaussian sum probability densities. The state estimate and covariance for each filter are initialized as

$$\hat{x}_{k-1|k-1}^{0j} = \sum_{i=1}^r \hat{x}_{k-1|k-1}^i \mu_{k-1|k-1}^{ij}, \quad (43)$$

$$P_{k-1|k-1}^{0j} = \sum_{i=1}^r \mu_{k-1|k-1}^{ij} \{P_{k-1|k-1}^i + [\hat{x}_{k-1|k-1}^i - \hat{x}_{k-1|k-1}^{0j}][\bullet]^T\}, \quad (44)$$

where $\hat{x}_{k-1|k-1}^{0j}$ and $P_{k-1|k-1}^{0j}$ are the mixed initial condition for mode-matched filter j at time $k-1$. Note that, in this context, i and j denote the corresponding model index respectively; n is the total number of sampling particles. Then the Gaussian mixing probabilities are computed via the equations:

$$\mu_{k-1|k-1}^{ij} = \frac{1}{c_j} \pi_{ij} \mu_{k-1}^i, \quad (45)$$

$$c_j = \sum_{i=1}^r \pi_{ij} \mu_{k-1}^i, \quad (46)$$

where c_j is a normalization factor.

(2) State update/importance sampling: The sample set $\{\hat{x}_{k-1|k-1}^{j,n}, P_{k-1|k-1}^{j,n}\}_{j=1, \dots, r}^{n=1, \dots, N}$ is drawn from the state estimate $\hat{x}_{k-1|k-1}^{0j}$ with the probability densities $\hat{p}_{k-1|k-1}^{0j}$. Then, propagate and update the sample set using the UPF to obtain the posterior samples $\{\hat{x}_{k|k}^{j,n}, \tilde{w}_k^{j,n}, P_{k|k}^{j,n}\}_{j=1, \dots, r}^{n=1, \dots, N}$ at time k , where $\tilde{w}_k^{j,n}$ is the normalized importance weight. At the same time, the mean of propagate output \bar{h}_k^j , innovations r_k^j , residual covariance \hat{S}_k^j and likelihood functions $\Lambda_k^{j,n}$ can be calculated respectively as follows:

$$\bar{h}_k^j = \sum_{n=1}^N h(\hat{x}_k^{j,n}, k, j), \quad (47)$$

$$r_k^j = z_k - h(\hat{x}_k^{j,n}, k, j), \quad (48)$$

$$\hat{S}_k^j = \sum_{n=1}^N [h(\hat{x}_k^{j,n}, k, j) - \bar{h}_k^j][\bullet]^T, \quad (49)$$

$$\Lambda_k^{j,n} = N(r_k^{j,n}; 0; \hat{S}_k^j). \quad (50)$$

(3) Residual resampling: Resample the sample set by evaluating the importance weights of all particles for each model to ensure the particles are distinct for an accurate posterior. The procedure, propagating the particles with

³Symbol $[\bullet]$ is same as the content on its left $[\bullet]$.

higher-weights and suppressing the particles with lower-weights, is to generate a new sample set with identical weights $\{\hat{x}_{k|k}^{j,n}, \tilde{w}_k^{j,n}=1/N\}_{j=1,\dots,r}^{n=1,\dots,N}$. A residual resampling scheme is adopted for its smaller Monte Carlo variance and favourable computation time [20].

(4) *Update of the mode probabilities:*

$$\mu_k^j = \frac{1}{c} \Lambda_k^{j,n} c_j, \quad (51)$$

$$c = \sum_{j=1}^r \Lambda_k^{j,n} c_j. \quad (52)$$

(5) *Estimate and covariance combination (output):*

Taking into account the mode probabilities, a combined state estimate can be obtained by averaging over the step 3 samples as follows:

$$x_{k|k}^n = \sum_{j=1}^r \hat{x}_{k|k}^{j,n} \mu_k^{j,n}, \quad (53)$$

$$\hat{x}_{k|k} = \frac{1}{N} \sum_{n=1}^N x_{k|k}^n. \quad (54)$$

The corresponding covariance is updated by the following equations:

$$P_{k|k}^n = \sum_{j=1}^r \mu_k^{j,n} \{P_{k|k}^{j,n} + [\hat{x}_{k|k}^{j,n} - \hat{x}_{k|k}] [\cdot]^T\}, \quad (55)$$

$$P_{k|k} = \frac{1}{N} \sum_{n=1}^N P_{k|k}^n. \quad (56)$$

V. SIMULATION AND ANALYSIS

To compare the performances of the IMMEKF, IMMUKF, IMPPF [18] and IMMUPF, a ground target tracking scenario is studied in this section. The four filters are then evaluated with the same sampling period of $\Delta t=t_k=1s$. The tracking performance is chosen as the root mean square errors (RMSEs).

A. Scenario and Simulation Conditions

From the location of $x=0km$ and $y=1km$, the target starts to make a positive CA movement for 54s with an initial velocity, initial acceleration and heading of $0m/s$, $0.5m/s^2$ and 30° . Then it makes an almost CV motion for 162s. At last, the target makes a negative CA movement for about 54s with an acceleration of $-0.5m/s^2$ until it stops.

Remark 1: For an almost linear movement with an angle, it may be simplified the state dimensions through a coordinate rotation. But here in order to generalize the issue, we do not simplify this operation. Moreover, the measurements in a polar format may be converted to those in a Cartesian format through a coordinate conversion.

The mode transition matrix is given by

$$\pi_{ij} = P_{ij} = \begin{bmatrix} 0.96 & 0.02 & 0.02 \\ 0.02 & 0.96 & 0.02 \\ 0.02 & 0.02 & 0.96 \end{bmatrix}. \quad (57)$$

The initial mode probabilities are set as $u_0^1=u_0^2=u_0^3=0.5$. The standard deviations are taken to be: $Q_{ax}^1=Q_{ay}^1=0.1 m^2/s^4$, $Q_{ax}^2=Q_{ay}^2=2 m^2/s^4$ and $Q_{ax}^3=Q_{ay}^3=2 m^2/s^4$ for the process noise in the M_1 , M_2 and M_3 respectively; $\sigma_r=2 m$ and $\sigma_\phi=0.001^\circ$ for the measurement noise. For the IMMPPF and IMMUPF approaches, the total particle number is set as $N=600$ with 200 particles for each model.

B. Simulation

One hundred Monte Carlo simulations with different measurement noises to the same trajectory have been done. The RMSEs plots of each state vector for the four filters are shown in Fig. 2-7. A comparison of RMSEs over the whole period and the maneuvering phases and computational complexity for the four filters are shown in Table 1.

C Performance Analysis

(1) *The estimate accuracy or RMSEs:* It can be seen from Fig. 2-7 and Table 1 that the four filters perform almost the same in the non-maneuvering phase (between $k=54s$ and $k=216s$); but in the maneuvering phase (before $k=54s$ and after $k=216s$), the IMM particle filters outperform the standard IMM filters in reducing the state estimate errors. The reason is that the particle-type multiple model filters use the mixture of multiple Gaussians while the standard IMM filters use only single Gaussian mixing. Furthermore, the IMMUPF algorithm obtains more accurate estimates than the IMPPF algorithm because an UKF method is used to update the true mean and covariance of the proposal distribution.

(2) *The computational complexity:* It is an undeniable fact that the particle-type filters are computationally expensive. However, certain parallel mechanism in implementing the IMM-type filtering approaches (including the particle-type filters and IMM-type filters) can reduce the computational burden. Table 1 shows a comparison of total computational loads according to the CPU time (system configurations: Celeron (R) CPU 2.0 GHz, memory 512 MB). It can be seen from Table 1 that the standard IMM filters have better performance than the particle-type filters in the execution time.

VI. CONCLUSION

An interacting multiple model unscented particle filter which uses an unscented Kalman filter to generate the true mean and covariance of the proposal distribution has been presented. Besides the theoretical analysis, simulation shows that the IMM particle filters obtain decent estimate accuracy whereas the standard IMM filters behave well in the computational complexity. In a word, the standard IMM filters, IMMEKF and IMMUKF, are recommended to track maneuvering targets with weak nonlinearity for computational resource saving; to track maneuvering targets with strong nonlinearity, especially with non-Gaussianity, the particle-type filters, IMPPF and IMMUPF, are considered if the parallel processing capacity is sufficient. Which estimator should be selected depends on the user's emphases.

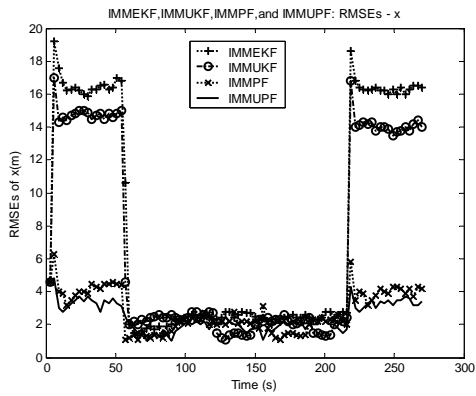


Figure 2. A comparison of the four filters: RMSEs-x.

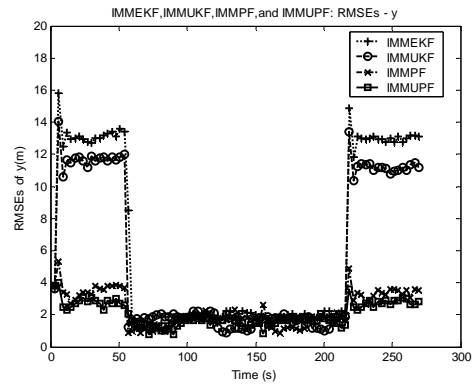


Figure 3. A comparison of the four filters: RMSEs-y.

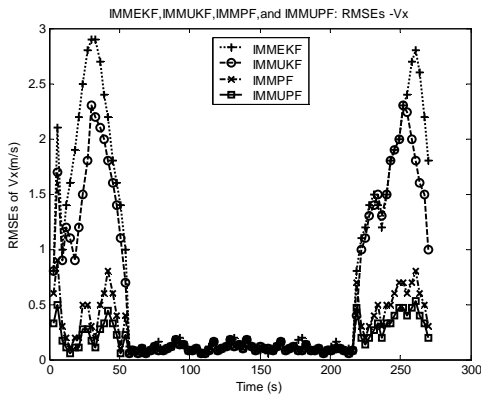


Figure 4. A comparison of the four filters: RMSEs-v_x.

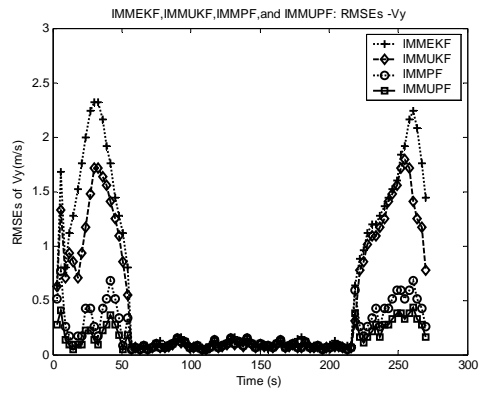


Figure 5. A comparison of the four filters: RMSEs-v_y.

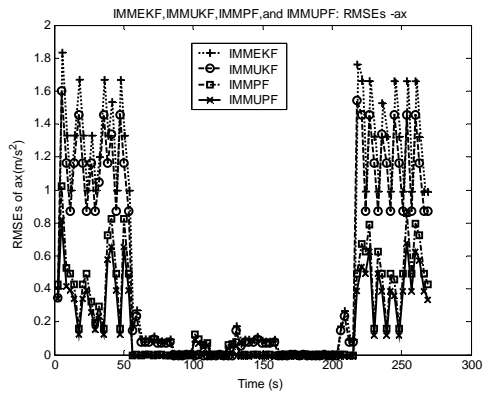


Figure 6. A comparison of the four filters: RMSEs-a_x.

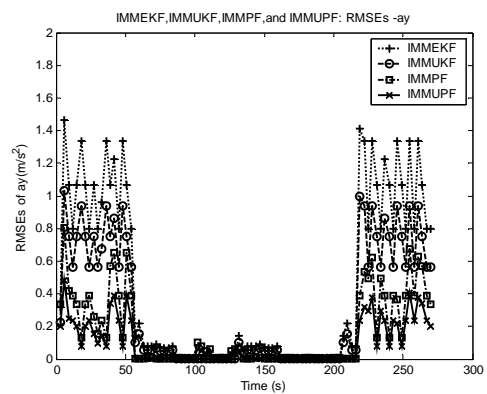


Figure 7. A comparison of the four filters: RMSEs-a_y.

TABLE 1
A COMPARISON OF RMSEs AND COMPUTATIONAL COMPLEXITY FOR THE FOUR FILTERS (100 RUNS, 3 MODELS)

Filters	RMSEs (the whole period ($k \in [1, 270]$)/the maneuvering phases($k \in [1, 54] \cup [216, 270]$))						Total time(s)
	x (m)	y (m)	v _x (m/s)	v _y (m/s)	a _x (m/s ²)	a _y (m/s ²)	
IMMEKF	7.964/16.20	6.346/12.96	0.8216/1.883	0.6635/1.506	0.5575/1.304	0.4442/1.043	8.335
IMMUKF	6.970/14.22	5.524/11.376	0.6596/1.498	0.5176/1.168	0.4906/1.147	0.3127/0.9176	15.234
IMMPF	2.791/4.156	2.372/3.533	0.2447/0.4611	0.208/0.392	0.2127/0.504	0.1651/0.393	1.5823×10^3
IMMUPF	2.271/3.419	1.960/2.905	0.174/0.2846	0.1427/0.233	0.1702/0.403	0.099/0.314	2.0965×10^3

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