

On Maximum-Likelihood Detection for Non-coherent FH/BFSK in Multitone Jamming

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Abstract— We derive the optimum maximum-likelihood (ML) non-coherent detector for FH/BFSK systems in the environment of multitone jamming and additive white Gaussian noise with no aid of jamming state indication and the resultant signal-plus-jammer tone, which, however, were required in K. C. Teh's previous work [1]. Because of the optimum ML detector's huge complexity, a novel complexity-reduced scheme which is called joint-suboptimum maximum-likelihood (JSML) detector is proposed. Simulation results show the superiority of the proposed optimum ML and JSML detectors over other existing diversity combining approaches. The performance analysis in terms of bit error probability (BEP) for the JSML detector with single diversity in $n = 1$ and $n = 2$ band multitone jamming (BMJ) is provided and further validated by simulation results. In moderate and high SNR conditions, though the proposed JSML detector performs slightly worse than the optimum ML detector, it does achieve much complexity reduction.

I. INTRODUCTION

Frequency hopping communication is nowadays a widely used technique to effectively combat various kinds of interference such as multipath fading, multiple access interference and interference from co-existence systems. In frequency hopping communications, multitone jamming remains a dominating factor to degrade the performance [5], [8]–[10].

In [1], [3], K. C. Teh *et al.* proposed maximum likelihood (ML) receivers for FFH/BFSK systems to combat the multitone jamming in additive white Gaussian noise (AWGN) and Rayleigh-fading channels, respectively. The side information required in [1] includes the indication of jamming state and resultant signal-plus-jammer tone amplitudes in each diversity reception (equ. (5) in [1]). Fig. 1 shows a case in which the desired symbol tone is hit by a jamming tone. The resultant signal-plus-jammer tone amplitude varies with the random phase difference, ϕ , between the desired symbol tone and the jamming tone. In the presence of noise or fading, it is usually not easy for the desired receiver to precisely get estimates of both ϕ and the indication of jamming state.

With no aid of *a priori* knowledge about the jamming state indication and resultant signal-plus-jammer tone amplitudes, the optimum ML detector for non-coherent FH/BFSK in multitone jamming and AWGN was derived in [11]. The optimum ML detector possesses huge complexity due to the absence of the above two kinds of jamming state information. First, without knowledge of the resultant signal-plus-jammer

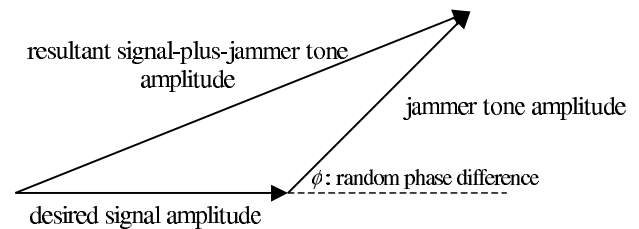


Fig. 1. Amplitudes of the desired signal, jammer tone, and resultant signal-plus-jammer tone

tone amplitudes, the optimum ML detector incorporates the integration of *modified Bessel* function with respect to the random phase difference, ϕ . Second, without the indication of jamming state, the optimum ML has to perform the composite hypothesis test where the decision metric is the sum of all possible jamming state hypotheses' metrics. The composite hypothesis test makes it more complicated to simplify the mathematical expression of the optimum ML detector.

In order to reduce the complexity, [11] further proposed a novel complexity-reduced scheme in moderate and high signal-to-noise-ratio (SNR) conditions. This scheme is called joint-suboptimum maximum-likelihood (JSML) detector. It performs the joint-suboptimum detection for every possible hypothesis of both the desired symbol tone and the jamming tone, to remove the jamming state composite parameter in the optimum ML detector. In this suboptimal detector, ϕ between the desired symbol tone and the jamming tone is no longer treated as a random variable, but an unknown deterministic parameter. The JSML detector removes the necessity of the integration with respect to ϕ , by performing the maximum-likelihood estimation for it.

In [11], simulation results in the environment of band multitone jamming (BMJ) [5] and AWGN showed the superiority of the proposed detectors over other existing diversity combining approaches. In insufficient jamming state information situations, the ML-I detector proposed in [1] performs worse than the proposed detectors. [11] also showed that the JSML detector is able to achieve much complexity reduction and it performs only slightly worse than the optimum ML detector in moderate and high SNR conditions.

This paper is an enhanced version of [11], where performance analysis in terms of bit error probability (BEP) calculation is presented for the JSML detector with single diversity, in $n = 1$ and $n = 2$ BMJs, respectively. The resulting analytical BEPs calculated with aid of MATLAB are validated by simulation results presented in [11].

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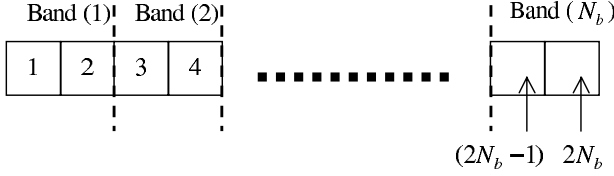


Fig. 2. The structure of N_b FH/BFSK sub-bands and corresponding N_t tones

In this paper, we re-state several major issues described in [11]. In VI, we present performance analysis of JSML detector with single diversity. In APPENDIX I, we provide a way to get the approximated ML estimate of a key parameter that is used to obtain the final mathematical form of the JSML detector.

II. SYSTEM MODEL

We consider non-coherent FH/BFSK systems and follow the system model in Chapter 2, part 2 in [5]. The entire spread spectrum bandwidth W_{ss} is equally partitioned into $N_t = W_{ss}/R_c = 2N_b$ tones, where R_c is the chip rate and N_b is the total number of bands. These tones are equally spaced and further partitioned into N_b non-overlapping sub-bands. Fig. 2 depicts the structure of N_b FH/BFSK sub-bands and corresponding N_t tones. In a one-hop-per symbol system, the hop rate equals R_c , so does the symbol rate, $R_s (= 1/T_s)$. Let J be the total jamming power, S be the power of the transmitted signal for the desired user, E_s be the desired symbol energy, N_J be the power spectrum density of the jamming noise with $N_J = J/W_{ss}$, and Q be the total number of jamming tones, each of which has power $J/Q = S/\alpha$.

Assume timing recovery is ideal between the desired user's receiver and transmitter. It is also assumed that one hopping interval of each jamming signal is synchronized to one symbol interval of the desired user. The received signal during the l -th symbol interval, $l = 0, 1, \dots$, is expressed by $R_l(t) = S_l(t) + J_l(t) + n(t)$, where $n(t)$ is the zero mean AWGN with two-sided power spectrum density $N_0/2$. $S_l(t) = \sqrt{2S}q(t - lT_s) \cos[2\pi(f_c(l) + f(l))t + \theta_s(l)]$ is the transmitted signal. $J_l(t)$ is the jamming signal expressed by $J_l(t) = \sum_{j=1}^Q \sqrt{2S/\alpha} q(t - lT_s) \cos[2\pi f_j(l)t + \theta_j(l)]$. $q(t)$ is the normalized rectangular pulse function with duration T_s . Assume $f_c(l) = 2h/T_s$, where h is a positive integer controlled by a hopping sequence. When $f(l)$ equals $-1/(2T_s) \equiv f_0$, it corresponds to symbol 0. When $f(l)$ equals $1/(2T_s) \equiv f_1$, it corresponds to symbol 1. $f_1(l), f_2(l), \dots, f_Q(l)$, are different. That is, in each hopping interval, the Q jamming signals randomly dwell at Q distinct tones. $f_j(l)$ equals a tone possibly used by the desired user.

In the following analysis, we consider the normalized received signal $r(t) \equiv R_m(t)/\sqrt{E_s}$. Let r_{ic} and r_{is} be respectively the in-phase and quadrature phase components of $r(t)$ at f_i , $i = 0, 1$, where $r_{ic} = \int_{mT_s}^{(m+1)T_s} r(t) \sqrt{\frac{2}{T_s}} \cos[2\pi(f_c(m) + f_i)t] dt$ and $r_{is} = \int_{mT_s}^{(m+1)T_s} r(t) \sqrt{\frac{2}{T_s}} \sin[2\pi(f_c(m) + f_i)t] dt$.

III. OPTIMUM ML DETECTION

Without loss of generality, we suppose the m -th symbol, b_m , is 0 (i.e. the symbol tone is f_0). In the absence of $n(t)$,

E_0 and E_1 are respectively the output energies at f_0 and at f_1 . These Q jamming tones are distinct. Therefore at most two jamming tones simultaneously dwell in a band (i.e. f_0 and f_1), which can be categorized to four hypotheses. We denote these four hypotheses by \mathbf{H}_{0k} , where the subscript 0 stands for the symbol 0 and $k = 1, 2, 3, 4$.

\mathbf{H}_{01} : Both tones are not jammed ($E_0(\mathbf{H}_{01}) = 1, E_1(\mathbf{H}_{01}) = 0$).

\mathbf{H}_{02} : Symbol tone is not jammed and the other tone is jammed ($E_0(\mathbf{H}_{02}) = 1, E_1(\mathbf{H}_{02}) = 1/\alpha$).

\mathbf{H}_{03} : Symbol tone is jammed and the other tone is not jammed ($E_0(\mathbf{H}_{03}) = 1 + 1/\alpha + 2 \cos(\phi)/\sqrt{\alpha}, E_1(\mathbf{H}_{03}) = 0$).

\mathbf{H}_{04} : Both tones are jammed ($E_0(\mathbf{H}_{04}) = 1 + 1/\alpha + 2 \cos(\phi)/\sqrt{\alpha}, E_1(\mathbf{H}_{04}) = 1/\alpha$).

We denote ϕ the phase difference between the symbol tone and the jamming tone. ϕ is uniformly distributed in $[0, 2\pi)$. The likelihood function of $b_m = 0$ is expressed by

$$p(\mathbf{r}_0, \mathbf{r}_1 | b_m = 0) = \sum_{k=1}^4 p(\mathbf{r}_0, \mathbf{r}_1 | \mathbf{H}_{0k}) p(\mathbf{H}_{0k}). \quad (1)$$

In (1), $\mathbf{r}_i = [r_{ic}, r_{is}]$, $i = 0, 1$, and $p(\mathbf{H}_{0k})$ is a priori probability of \mathbf{H}_{0k} . It varies with jamming strategy. The derivation of $p(\mathbf{H}_{0k})$ can be found in [5]. The conditional probability of $(\mathbf{r}_0, \mathbf{r}_1)$ given \mathbf{H}_{0k} is shown to be

$$p(\mathbf{r}_0, \mathbf{r}_1 | \mathbf{H}_{0k}) = E_{\theta_0, \theta_1, \phi} \left\{ \frac{1}{4\pi^2 \sigma^4} \exp\left\{ \frac{-1}{2\sigma^2} \int_{mT_s}^{(m+1)T_s} (r(t) - \sqrt{\frac{2E_0(\mathbf{H}_{0k})}{T_c}} \cos[2\pi(f_c(m) + f_0)t + \theta_0] - \sqrt{\frac{2E_1(\mathbf{H}_{0k})}{T_c}} \cos[2\pi(f_c(m) + f_1)t + \theta_1]) dt \right\} \right\}, \quad (2)$$

where $\sigma^2 = N_0/2$ and θ_i stands for the random phase difference between the desired transmitter and its receiver when detecting at f_i . In this non-coherent detection scheme, we assume θ_i , $i = 0, 1$, is uniformly distributed in $[0, 2\pi)$.

Based on [6], it can be shown that $p(\mathbf{r}_0, \mathbf{r}_1 | \mathbf{H}_{0k}) = p(\mathbf{r}_0 | \mathbf{H}_{0k}) p(\mathbf{r}_1 | \mathbf{H}_{0k})$, where

$$p(\mathbf{r}_0 | \mathbf{H}_{0k}) = E_{\theta_0, \phi} \left\{ \exp\left\{ \frac{-1}{2\sigma^2} [(r_{0c} - \sqrt{E_0(\mathbf{H}_{0k})} \cos \theta_0)^2 + (r_{0s} - \sqrt{E_0(\mathbf{H}_{0k})} \sin \theta_0)^2] \right\} / (2\pi\sigma^2) \right\}, \quad (3)$$

$$p(\mathbf{r}_1 | \mathbf{H}_{0k}) = E_{\theta_1, \phi} \left\{ \exp\left\{ \frac{-1}{2\sigma^2} [(r_{1c} - \sqrt{E_1(\mathbf{H}_{0k})} \cos \theta_1)^2 + (r_{1s} - \sqrt{E_1(\mathbf{H}_{0k})} \sin \theta_1)^2] \right\} / (2\pi\sigma^2) \right\}. \quad (4)$$

$p(\mathbf{r}_0, \mathbf{r}_1 | b_m = 1) = \sum_{k=1}^4 p(\mathbf{r}_0, \mathbf{r}_1 | \mathbf{H}_{1k}) p(\mathbf{H}_{1k})$ can be derived in a similar way. The optimum ML detector outputs \hat{b}_m the decision with the maximum $p(\mathbf{r}_0, \mathbf{r}_1 | b_m = \hat{b}_m)$.

Based on (3)(4), (1) can be simplified to yield the decision metric for $b_m = 0$ as shown in Fig. 3. This non-coherent optimum ML detector (square law energy detector) is involved in the integration of the modified Bessel function with respect to the random phase difference, ϕ , between the symbol tone and the jamming tone. Its implementation complexity is much high. In addition, $p(\mathbf{r}_0, \mathbf{r}_1 | b_m)$ shall be averaged with respect to \mathbf{H}_{ik} , which makes it more complicated to further derive a simpler form for this composite hypothesis test.

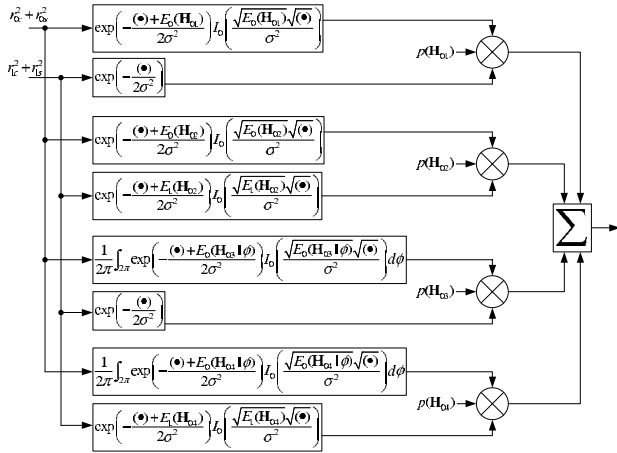


Fig. 3. Decision metric for $b_m = 0$ with single diversity in the optimum ML detector

IV. JOINT-SUBOPTIMUM ML DETECTION

Rather than detecting for b_m only, in this section, we consider the joint-optimum detection for \mathbf{H}_{ik} (i.e. the optimum detection for both the desired symbol and the jamming symbol). Combining b_m and the jamming symbol into a common hypothesis is to first remove the \mathbf{H}_{ik} parameter in the original optimum ML composite hypothesis test. We can then use properties of the natural logarithm and *modified Bessel* function's asymptotic approximations to yield the complexity-reduced detector.

Given $(\mathbf{r}_0, \mathbf{r}_1)$, the *a posteriori* probability of \mathbf{H}_{0k} , $p(\mathbf{H}_{0k}|\mathbf{r}_0, \mathbf{r}_1)$, is proportional to $p(\mathbf{r}_0, \mathbf{r}_1|\mathbf{H}_{0k})p(\mathbf{H}_{0k})$, which can further be simplified as $g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{0k})$ expressed as follows.

$$g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{01}) = p(\mathbf{H}_{01}) \exp\left(\frac{-1}{2\sigma^2}\right) I_0\left(\frac{\sqrt{r_{0c}^2 + r_{0s}^2}}{\sigma^2}\right), \quad (5)$$

$$g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{02}) = p(\mathbf{H}_{02}) \exp\left(\frac{-1}{2\sigma^2}\right) I_0\left(\frac{\sqrt{r_{0c}^2 + r_{0s}^2}}{\sigma^2}\right) \cdot \exp\left(\frac{-1}{2\sigma^2\alpha}\right) I_0\left(\frac{\sqrt{(r_{1c}^2 + r_{1s}^2)/\alpha}}{\sigma^2}\right), \quad (6)$$

$$g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{03}) = \frac{p(\mathbf{H}_{03})}{2\pi} \int_{2\pi} \left\{ \exp\left(\frac{1+1/\alpha+2\cos\phi/\sqrt{\alpha}}{-2\sigma^2}\right) I_0\left(\frac{\sqrt{1+1/\alpha+2\cos\phi/\sqrt{\alpha}}\sqrt{r_{0c}^2 + r_{0s}^2}}{\sigma^2}\right) \right\} d\phi, \quad (7)$$

$$g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{04}) = \frac{p(\mathbf{H}_{04})}{2\pi} \int_{2\pi} \left\{ \exp\left(\frac{1+1/\alpha+2\cos\phi/\sqrt{\alpha}}{-2\sigma^2}\right) I_0\left(\frac{\sqrt{1+1/\alpha+2\cos\phi/\sqrt{\alpha}}\sqrt{r_{0c}^2 + r_{0s}^2}}{\sigma^2}\right) \right\} d\phi \cdot \exp\left(\frac{-1}{2\sigma^2\alpha}\right) I_0\left(\frac{\sqrt{(r_{1c}^2 + r_{1s}^2)/\alpha}}{\sigma^2}\right), \quad (8)$$

where $I_0(\bullet)$ is the zero-th order *modified Bessel* function of the first kind.

Similarly, we have $g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{1k})$ by exchanging $r_{0c}^2 + r_{0s}^2$ and $r_{1c}^2 + r_{1s}^2$ in (5)-(8). $p(\mathbf{H}_{1k})$ equals $p(\mathbf{H}_{0k})$ due to the equally probable symbols 0 and 1.

In (7) and (8), ϕ is a random variable and should be averaged over 2π . However, in order to reduce the complexity, the

JSML detector treats ϕ as an unknown deterministic parameter to estimate, based on $(r_{0c}^2 + r_{0s}^2)$. It takes the approximate maximum-likelihood estimate of $(1 + 1/\alpha + 2\cos\phi/\sqrt{\alpha})$, which equivalently gives the estimate of ϕ , provided that α is already known. In moderate and high SNR conditions, it can be shown that the maximum-likelihood estimate [7] of $\sqrt{1 + 1/\alpha + 2\cos\phi/\sqrt{\alpha}}$ approximates $\sqrt{r_{0c}^2 + r_{0s}^2}$. In APPENDIX I, we present the derivation of the ML estimate. Thus,

$$(7) \approx \hat{g}(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{03}) = p(\mathbf{H}_{03}) \exp\left(\frac{r_{0c}^2 + r_{0s}^2}{-2\sigma^2}\right) I_0\left(\frac{r_{0c}^2 + r_{0s}^2}{\sigma^2}\right), \quad (9)$$

$$(8) \approx \hat{g}(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{04}) = p(\mathbf{H}_{04}) \exp\left(\frac{r_{0c}^2 + r_{0s}^2}{-2\sigma^2}\right) I_0\left(\frac{r_{0c}^2 + r_{0s}^2}{\sigma^2}\right) \cdot \exp\left(\frac{-1}{2\sigma^2\alpha}\right) I_0\left(\frac{\sqrt{(r_{1c}^2 + r_{1s}^2)/\alpha}}{\sigma^2}\right). \quad (10)$$

Furthermore, we utilize the asymptotic form (for large argument) of the zero-th order *modified Bessel* function: $I_0(x) \approx \exp(x)/\sqrt{2\pi x}$ [4]. After taking the natural logarithm of (5)(6)(9)(10) and scaling by σ^2 , we have

$$\sigma^2 \ln(g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{01})) \approx r_0 + A - \frac{\sigma^2}{2} \ln\left(\frac{2\pi}{\sigma^2}\right) - \frac{\sigma^2}{2} \ln(r_0) + \sigma^2 \ln(p(\mathbf{H}_{01})), \quad (11)$$

$$\sigma^2 \ln(g(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{02})) \approx r_0 + r_1 \sqrt{\frac{1}{\alpha}} + A + B - \frac{\sigma^2}{2} \ln\left(\frac{2\pi}{\sigma^2}\right) - \frac{\sigma^2}{2} \ln(r_0) - \frac{\sigma^2}{2} \ln(r_1) + \sigma^2 \ln(p(\mathbf{H}_{02})), \quad (12)$$

$$\sigma^2 \ln(\hat{g}(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{03})) \approx \frac{r_0^2}{2} - \frac{\sigma^2}{2} \ln\left(\frac{2\pi}{\sigma^2}\right) - \sigma^2 \ln(r_0) + \sigma^2 \ln(p(\mathbf{H}_{03})), \quad (13)$$

$$\sigma^2 \ln(\hat{g}(\mathbf{r}_0, \mathbf{r}_1, \mathbf{H}_{04})) \approx \frac{r_0^2}{2} + r_1 \sqrt{\frac{1}{\alpha}} + B - \frac{\sigma^2}{2} \ln\left(\frac{2\pi}{\sigma^2}\right) - \sigma^2 \ln(r_0) - \frac{\sigma^2}{2} \ln(r_1) + \sigma^2 \ln(p(\mathbf{H}_{04})), \quad (14)$$

where $r_i^2 = r_{ic}^2 + r_{is}^2$, $A = -1/2$, and $B = -1/(2\alpha) - \sigma^2 \ln(2\pi/(\sigma^2\sqrt{\alpha}))/2$. In moderate and high SNR conditions, we neglect the term with factor $\sigma^2 \ln(r_i)$. After removing the common term $C = -\sigma^2 \ln((2\pi)/\sigma^2)/2$ in (11)-(14), we have the following results.

$$(11) - C \approx r_0 + A + C_{01} \equiv L_{01}, \quad (15)$$

$$(12) - C \approx r_0 + r_1 \sqrt{1/\alpha} + A + B + C_{02} \equiv L_{02}, \quad (16)$$

$$(13) - C \approx r_0^2/2 + C_{03} \equiv L_{03}, \quad (17)$$

$$(14) - C \approx r_0^2/2 + r_1 \sqrt{1/\alpha} + B + C_{04} \equiv L_{04}, \quad (18)$$

where $C_{0k} = \sigma^2 \ln(p(\mathbf{H}_{0k}))$ and L_{0k} is the complexity-reduced decision metric for \mathbf{H}_{0k} . We can get L_{1k} by exchanging r_0 and r_1 in (15)-(18). Fig. 4 shows the system design for the joint-suboptimum ML detector. The JSML detector picks the maximum among L_{ik} and then outputs $\hat{b}_m = i$ as the decision. If we treat jamming signals as multiuser interference, the JSML detector equivalently performs the multiuser detection in the environment where the desired signal and other collocated, uncoordinated interfering signals coexist in the common spread spectrum band.

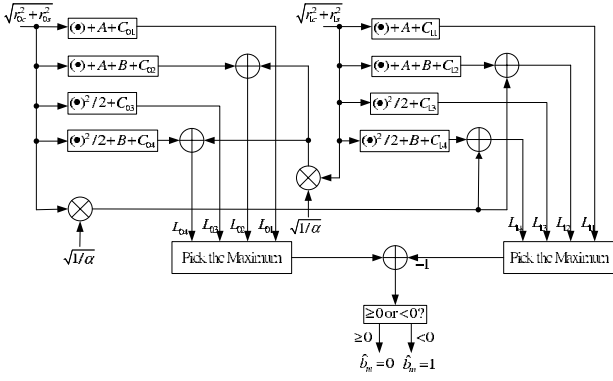


Fig. 4. Decision metrics for b_m with single diversity in the JSML detector

V. DIVERSITY COMBINING

In this section, we consider the scenario of one-hop-per-symbol with multiple diversities. We denote the diversity number by U . These U diversity receptions are assumed to be mutually independent. Based on (1), the decision metric of the optimum ML detector for symbol i is shown to be

$$\prod_{u=1}^U \left(\sum_{k=1}^4 p(\mathbf{r}_0^u, \mathbf{r}_1^u, |\mathbf{H}_{ik}) p(\mathbf{H}_{ik}) \right), i = 0, 1. \quad (19)$$

For JSML detector, it can be shown that the decision metric for symbol i is:

$$\sum_{u=1}^U \max(L_{i1}^{(u)}, L_{i2}^{(u)}, L_{i3}^{(u)}, L_{i4}^{(u)}), i = 0, 1. \quad (20)$$

In (19)-(20), \mathbf{r}_i^u is the normalized received vector of the u -th diversity at f_i . $L_{ik}^{(u)}$ is the corresponding complexity-reduced decision metric of \mathbf{H}_{ik} .

VI. PERFORMANCE ANALYSIS

In this section, we derive bit error probabilities (BEPs) of the JSML detector with the single diversity, in $n = 1$ BMJ and $n = 2$ BMJ [5]. Without loss of generality, we assume symbol 0 is transmitted.

A. BEP in $n = 1$ BMJ

In $n = 1$ BMJ, the hypothesis \mathbf{H}_{04} does not happen. The BEP is $p_b = \sum_{k=1}^3 (1 - p(c|\mathbf{H}_{0k})) p(\mathbf{H}_{0k})$, where $p(c|\mathbf{H}_{0k})$ is the conditional bit correct probability expressed by $p(c|\mathbf{H}_{0k}) = p(L_{01} \max|\mathbf{H}_{0k}) + p(L_{02} \max|\mathbf{H}_{0k}) + p(L_{03} \max|\mathbf{H}_{0k})$. It can be shown that

$$p(L_{01} \max|\mathbf{H}_{0k}) = p(L_{01} > L_{02}, L_{01} > L_{03}, L_{01} > L_{11}, L_{01} > L_{12}, L_{01} > L_{13}|\mathbf{H}_{0k}), \quad (21)$$

which is equivalent to

$$\begin{aligned} p(L_{01} \max|\mathbf{H}_{0k}) &= p(\sqrt{\alpha}(C_{01} - B - C_{02}) > r_1, \\ & r_0^2 - 2r_0 + 2(C_{03} - A - C_{01}) < 0, \\ & r_0 > r_1, \\ & r_0(1 - 1/\sqrt{\alpha}) + C_{01} - B - C_{02} > r_1, \\ & 2(r_0 - C_{03} + A + C_{01}) > r_1^2|\mathbf{H}_{0k}). \end{aligned} \quad (22)$$

Similarly, $p(L_{02} \max|\mathbf{H}_{0k})$ is shown to be

$$p(L_{02} \max|\mathbf{H}_{0k}) = p(L_{02} > L_{01}, L_{02} > L_{03}, L_{02} > L_{11}, L_{02} > L_{12}, L_{02} > L_{13}|\mathbf{H}_{0k}), \quad (23)$$

which implies that

$$\begin{aligned} p(L_{02} \max|\mathbf{H}_{0k}) &= p(\sqrt{\alpha}(C_{01} - B - C_{02}) < r_1, \\ & r_0^2 + 2(C_{01} - r_0 - A - r_1/\sqrt{\alpha} - B - C_{02}) < 0, \\ & r_0 > r_1(1 - 1/\sqrt{\alpha}) + C_{01} - B - C_{02}, \\ & r_0 < r_1, \\ & r_1^2 - 2(r_1/\sqrt{\alpha} - r_0 + C_{01} - A - B - C_{02}) < 0|\mathbf{H}_{0k}). \end{aligned} \quad (24)$$

And $p(L_{03} \max|\mathbf{H}_{0k})$ is expressed by

$$p(L_{03} \max|\mathbf{H}_{0k}) = p(L_{03} > L_{01}, L_{03} > L_{02}, L_{03} > L_{11}, L_{03} > L_{12}, L_{03} > L_{13}|\mathbf{H}_{0k}), \quad (25)$$

which indicates that

$$\begin{aligned} p(L_{03} \max|\mathbf{H}_{0k}) &= p(r_0^2 - 2r_0 + 2(C_{03} - A - C_{01}), \\ & r_0^2 + 2(C_{01} - r_0 - A - r_1/\sqrt{\alpha} - B - C_{02}) > 0, \\ & r_0^2 + 2(C_{03} - r_1 - A - C_{01}) > 0, \\ & r_0^2 + 2(C_{03} - r_1 - A - r_0/\sqrt{\alpha} - B - C_{02}) > 0, \\ & r_0 > r_1|\mathbf{H}_{0k}). \end{aligned} \quad (26)$$

By [6], r_i is Ricean-distributed random variable with parameter σ and noncentrality parameter $s_i = \sqrt{E_i(\mathbf{H}_{0k})}$. Therefore

$$p(L_{01} \max|\mathbf{H}_{0k}) =$$

$$\begin{cases} 0, & \text{if } 1 - 2(C_{03} - A - C_{01}) < 0, \\ \int_{x_0 \in Z_{01}} p_{R_0}(x_0) \cdot F_{R_1}(\min[\sqrt{\alpha}(C_{01} - B - C_{02}), \\ x_0, x_0(1 - 1/\sqrt{\alpha} + C_{01} - B - C_{02}), \\ \sqrt{2(x_0 - C_{03} + A + C_{01})}]) dx_0, & \text{else,} \end{cases}$$

where $p_{R_i}(x) = x/\sigma^2 \exp(-(x^2 + s_i^2)/(2\sigma^2)) I_0(xs_i/\sigma^2)$ is the probability density function of r_i , and $F_{R_i}(x) = \int_{y=0}^x p_{R_i}(y) dy$ is the corresponding cumulative density function. Z_{01} is a subset of x_0 such that each x_0 in Z_{01} satisfies the following conditions:

- 1) $\max(0, 1 - \sqrt{1 - 2(C_{03} - C - C_{01})}) < x_0$.
- 2) $x_0 < 1 + \sqrt{1 - 2(C_{03} - C - C_{01})}$.
- 3) $2(x_0 - C_{03} + A + C_{01}) \geq 0$.

In addition,

$$p(L_{02} \max|\mathbf{H}_{0k}) = \int_{x_0 \in Z_{02}} p_{R_0}(x_0) [F_{R_1}(a_1(x_0)) - F_{R_1}(a_2(x_0))] dx_0,$$

where

$$\begin{aligned} a_1(x_0) &= \min(x_0, 1 + \sqrt{a_3(x_0)}), \\ a_2(x_0) &= \max(1 - \sqrt{a_3(x_0)}, 1/2x_0^2 - 1/\sqrt{\alpha}x_0 + C_{03} - A \\ & - B - C_{02}, x_0(1 - 1/\sqrt{\alpha}) + C_{01} - B - C_{02}), \\ a_3(x_0) &= 1 - 2(C_{03} - A - B - C_{02} - x_0/\sqrt{\alpha}), \end{aligned}$$

and $Z_{02} \equiv \{x_0 | x_0 > 0, a_3(x_0) \geq 0, a_1(x_0) \geq a_2(x_0)\}$.

Also, $p(L_{03} \max | \mathbf{H}_{0k}) =$

$$\begin{cases} \int_0^\infty p_{R_0}(x_0) \cdot F_{R_1}(\min(x_0, a_4(x_0), a_5(x_0), a_6(x_0))) dx_0, & \text{if } D < 0, \\ \int_0^{1-\sqrt{D}} p_{R_0}(x_0) \cdot F_{R_1}(\min(x_0, a_4(x_0), a_5(x_0), a_6(x_0))) dx_0 + & \\ \int_{1+\sqrt{D}}^\infty p_{R_0}(x_0) \cdot F_{R_1}(\min(x_0, a_4(x_0), a_5(x_0), a_6(x_0))) dx_0, & D \geq 0, \end{cases}$$

where

$$\begin{aligned} a_4(x_0) &= \sqrt{\alpha}(1/2x_0^2 - x_0 + C_{03} - A - B - C_{02}), \\ a_5(x_0) &= 1/2x_0^2 + C_{03} - A - C_{01}, \\ a_6(x_0) &= 1 - 2x_0^2 - x_0/\sqrt{\alpha} + C_{03} - A - B - C_{02}, \end{aligned}$$

and $D = 1 - 2(C_{03} - A - C_{01})$.

B. BEP in $n = 2$ BMJ

In $n = 2$ BMJ, hypothesis \mathbf{H}_{02} and hypothesis \mathbf{H}_{03} do not happen. The BEP is written as $p_b = (1 - p(c|\mathbf{H}_{01}))p(\mathbf{H}_{01}) + (1 - p(c|\mathbf{H}_{04}))p(\mathbf{H}_{04})$, where $p(c|\mathbf{H}_{0k})$ is the conditional bit correct probability expressed by $p(c|\mathbf{H}_{0k}) = p(L_{01} \max | \mathbf{H}_{0k}) + p(L_{04} \max | \mathbf{H}_{0k})$. It can be shown that

$$p(L_{01} \max | \mathbf{H}_{0k}) = p(L_{01} > L_{04}, L_{01} > L_{11}, L_{01} > L_{14} | \mathbf{H}_{0k}), \quad (27)$$

which is equivalent to $p(L_{01} \max | \mathbf{H}_{0k}) =$

$$\begin{aligned} p(\sqrt{\alpha}(r_0 + A + C_0 - 1/2r_0^2 - B - C_{04}) > r_1, \\ r_0 > r_1, \\ 2(r_0 + A + C_{01} - r_0/\sqrt{\alpha} - B - C_{04}) > r_1^2 | \mathbf{H}_{0k}). \end{aligned} \quad (28)$$

On the other hand, $p(L_{04} \max | \mathbf{H}_{0k}) =$

$$p(L_{04} > L_{01}, L_{04} > L_{11}, L_{04} > L_{14} | \mathbf{H}_{0k}), \quad (29)$$

which implies that $p(L_{02} \max | \mathbf{H}_{0k}) =$

$$\begin{aligned} p(\sqrt{\alpha}(r_0 + A + C_{01} - 1/2r_0^2 - B - C_{04}) < r_1, \\ r_1(1/\sqrt{\alpha} - 1) > A + C_{01} - B - C_{04} - 1/2r_0^2, \\ r_1^2 - 2(r_1/\sqrt{\alpha} - r_0/\sqrt{\alpha}) - r_0^2 < 0 | \mathbf{H}_{0k}). \end{aligned} \quad (30)$$

Again, r_i is Ricean-distributed random variable with parameter σ and noncentrality parameter $s_i = \sqrt{E_i(\mathbf{H}_{0k})}$ [6]. Therefore,

$$p(L_{01} \max | \mathbf{H}_{0k}) =$$

$$\int_{x_0 \in Z_{03}} p_{R_0}(x_0) \cdot F_{R_1}(\min[\sqrt{\alpha}(x_0 + A + C_{01} - 1/2x_0^2 - B - C_{04}), \sqrt{2(x_0 + A + C_{01} - x_0/\sqrt{\alpha} - B - C_{04})}]) dx_0,$$

and $Z_{03} \equiv \{x_0 | 2(x_0 + A + C_{01} - x_0/\sqrt{\alpha} - B - C_{03}) \geq 0\}$.

And,

$$p(L_{04} \max | \mathbf{H}_{0k}) = \int_{x_0 \in Z_{04}} p_{R_0}(x_0) [F_{R_1}(b_1(x_0)) - F_{R_1}(b_2(x_0))] dx_0,$$

where

$$\begin{aligned} b_1(x_0) &= \min(1/(1/\sqrt{\alpha} - 1)(A + C_{01} - B - C_{04} - 1/2x_0^2), \\ &1/\sqrt{\alpha} + \sqrt{1/\sqrt{\alpha} - 2(x_0/\sqrt{\alpha} - 1/2x_0^2)}), \\ b_2(x_0) &= \max(\sqrt{\alpha}(x_0 + A + C_{01} - 1/2x_0^2 - B - C_{04}), \\ &1/\sqrt{\alpha} - \sqrt{1/\sqrt{\alpha} - 2(x_0/\sqrt{\alpha} - 1/2x_0^2)}). \end{aligned}$$

We denote Z_{04} is a subset of x_0 such that each x_0 in Z_{04} satisfies the following conditions:

- 1) $b_1(x_0) \geq b_2(x_0)$.
- 2) $1/\sqrt{\alpha} - 2(x_0/\sqrt{\alpha} - 1/2x_0^2) \geq 0$.

VII. NUMERICAL RESULTS

We set $N_b = 100$ and take the worst case (WC) BMJ, in which parameters setting for Q and α can be found in Chapter 2, part 2, [5] (Here we set $\alpha = 0.95$ to make the jamming tone's energy slightly greater than the desired tone's energy in $n = 1$ BMJ). High SNR (30 dB, in Fig. 5) and moderate SNR (13.35 dB, in Fig. 6) are considered. In the single-diversity scenario, we compare simulation results of the conventional detector, JSML, and the analytical BEP of JSML derived in VI. In the multiple-diversity scenario ($U = 3$ in Fig. 5(a)6(a) and $U = 2$ in Fig. 5(b)6(b)), simulation comparisons are made for hard-decision majority-vote (HDMV) receiver [1] (in Fig. 5(a)6(a) only), self-normalized (SN) receiver [1], product-combining (PC) receiver [2], optimum ML and JSML. Fig. 5(a)6(a) and Fig. 5(b)6(b) present results in the WC $n = 1$ BMJ and $n = 2$ BMJ, respectively. For single-diversity, simulation results in Fig. 5 and Fig. 6 validate the analytical BEP of the JSML. For multiple-diversity, the JSML in Fig. 5(a) gains around 4 dB, compared to the SN receiver at 10^{-5} BEP level. At the same BEP level in Fig. 5(b), it gains around 10 dB over the SN receiver.

In Fig. 5(a)(b), we observe the performances of ML-I receiver structure proposed in [1]¹, when the jamming state indications and the resultant signal-plus-jammer tone amplitudes are not always perfectly acquired (that is, without perfect "jamming state information", which is denoted by JSI). In the $n = 1$ BMJ and AWGN, it is assumed that the symbol tone and the jamming tone cannot be straightforwardly distinguished when they are located at different tones in a jammed band. Therefore, in this special case, the q_{nl} defined in [1] is selected by flipping a coin. Meanwhile, the resultant signal-plus-jammer tone amplitudes here are estimated by the received amplitudes, since the phase difference between the desired signal tone and jamming is random. In the WC $n = 2$ BMJ, we assume the desired receiver has correct knowledge of q_{nl} while still utilizing the estimated resultant signal-plus-jammer tone amplitudes. Fig. 6(a)(b) show that the ML-I receiver performs worse than other combining approaches, due to its incomplete side information in the realistic environment.

We can hardly distinguish the performance curves of optimum ML and JSML in the high SNR conditions (in Fig. 5(a)(b)). In moderate SNR (in Fig. 6(a)(b)), however, JSML behaves slightly worse than optimum ML. The major reason is that in deriving the JSML, we utilize *modified Bessel* function's large argument asymptotic approximations, to acquire the maximum-likelihood estimation of the resultant signal-plus-jammer energy, and to simply the complexity the likelihood function. Therefore, there will be more performance

¹In this structure, we treat each hop per symbol in [1] as one diversity defined in this paper. Doing this leads to the same diversity scenario when the ML-I receiver are compared to the proposed detectors and other diversity combining techniques.

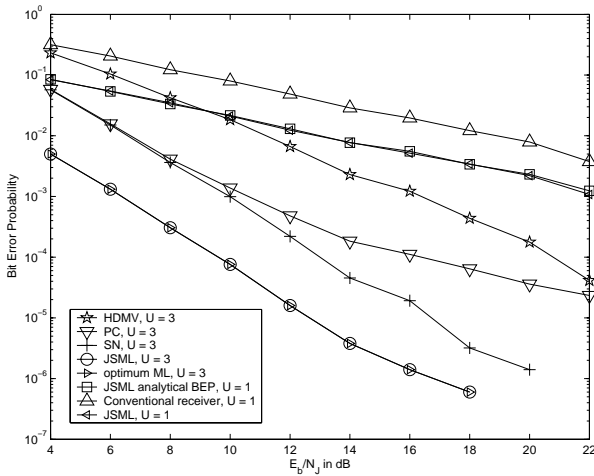
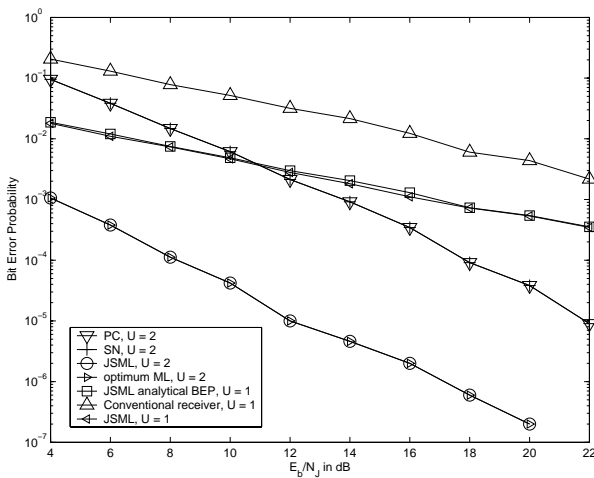
(a) Bit error probabilities in the WC $n = 1$ BMJ.(b) Bit error probabilities in the WC $n = 2$ BMJ.

Fig. 5. Performances of various combining techniques in SNR = 30 dB

gap in case the approximations could not apply appropriately in moderate, and even low SNR conditions. On the other hand, we present in Fig. 7 the elapsed time (in seconds) taken by optimum ML, JSML, SN and PC when detecting 10000 symbols². Based on Fig. 7, the optimum ML needs at least a hundredfold of the time taken by the JSML. This is because the optimum ML performs composite hypothesis test in which the random phase difference is averaged over 2π .

VIII. CONCLUSION

For non-coherent FH/BFSK systems in the presence of multitone jamming and AWGN, we re-state the optimum ML detector and a novel complexity-reduced joint-suboptimum ML (JSML) detector in high and moderate SNR conditions, proposed in [11]. They require amplitude information of the desired signal tone and the jamming tone only, with no aid of *a priori* knowledge about the indication of jamming state and the resultant signal-plus-jammer tone amplitudes as utilized in [1]. In the environment of band multitone jamming (BMJ) and

²This is done by MATLAB simulation in a laptop with OS: Microsoft Windows XP, Intel Pentium M processor: 1.4GHz, and RAM: 512 MB.

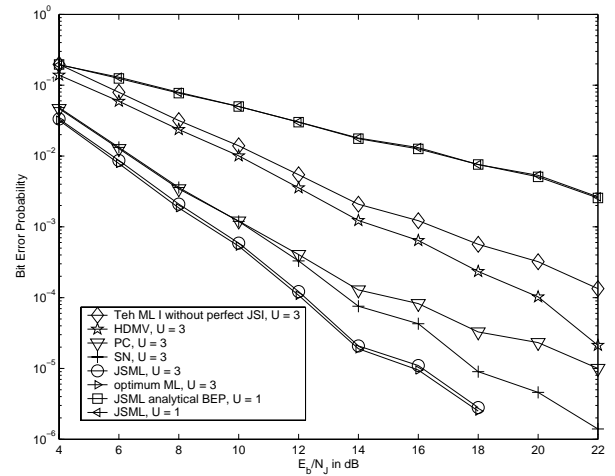
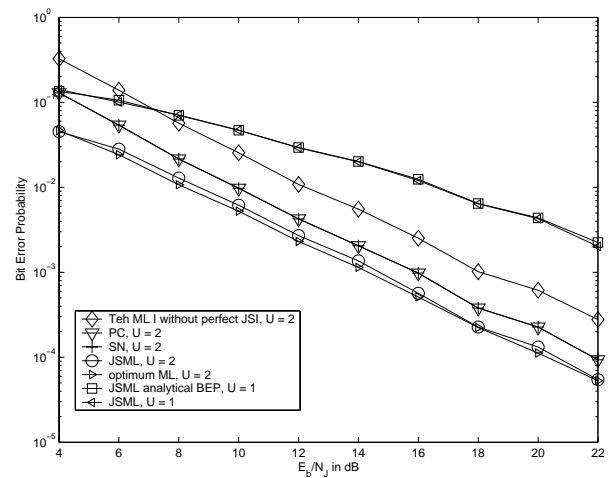
(a) Bit error probabilities in the WC $n = 1$ BMJ.(b) Bit error probabilities in the WC $n = 2$ BMJ.

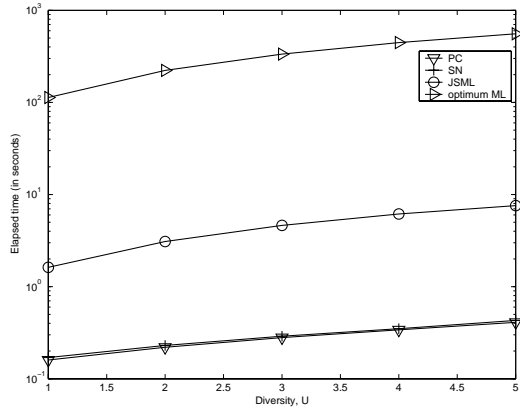
Fig. 6. Performances of various combining techniques in SNR = 13.35 dB

AWGN, the optimum ML detector and JSML detector perform better than existing diversity combining approaches such as SN, PC, HDMV and the conventional energy detector. In insufficient or inaccurate jamming state information situations, the ML-I detector (for moderate SNRs) proposed in [1] is shown by simulation results to lose its original superiority over SN, PC and HDMV. It also has performances inferior to the proposed detectors in [11]. In moderate and high SNR conditions, the JSML detector performs only slightly worse than the optimum ML detector while it does achieve much complexity reduction.

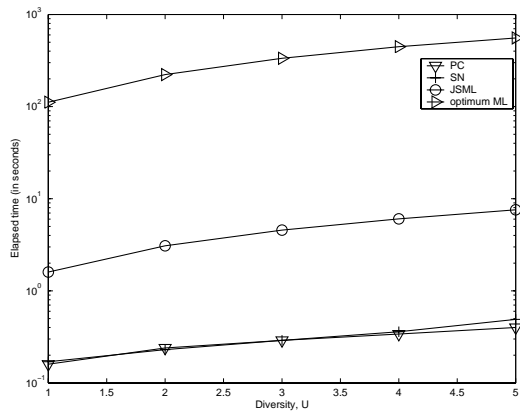
In this paper as an enhanced version of [11], analytical BEPs of JSML with single diversity are provided in detail, and validated by simulation results.

APPENDIX I

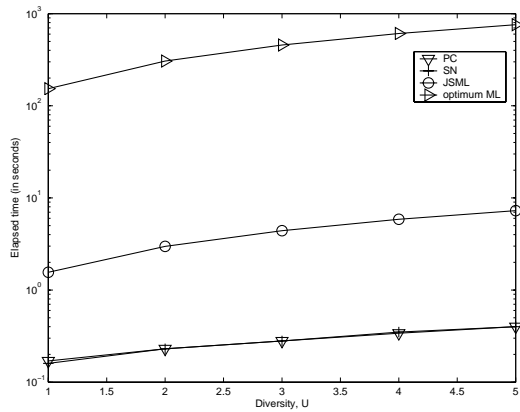
To find the ML estimate for $\sqrt{1 + 1/\alpha + 2 \cos \phi}/\sqrt{\alpha}$ is equivalent to find the ML estimate for the non-centrality parameter s [6], of a Rician-distributed random variable: $R = \sqrt{X_1^2 + X_2^2}$, based on the observation of $r_0 \equiv \sqrt{r_{0c}^2 + r_{0s}^2}$. X_1 and X_2 are independent Gaussian random variables with



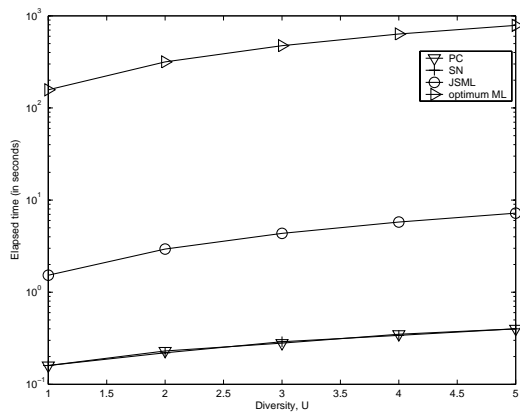
(a) WC $n = 1$ BMJ with SJR = 10 dB and SNR = 30 dB



(b) WC $n = 2$ BMJ with SJR = 10 dB and SNR = 30 dB



(c) WC $n = 1$ BMJ with SJR = 10 dB and SNR = 13.35 dB



(d) WC $n = 2$ BMJ with SJR = 10 dB and SNR = 13.35 dB

Fig. 7. Elapsed time taken by the optimum ML, JSML, SN and PC, to detect 10000 symbols

mean m_1 and m_2 , respectively, where $m_1^2 + m_2^2 = s^2$. X_1 and X_2 have common variance σ^2 . The probability density function of R is

$$f_R(r) = \frac{r}{\sigma^2} \exp(-(r^2 + s^2)/(2\sigma^2)) I_0\left(\frac{rs}{\sigma^2}\right).$$

Given r_0 , the ML estimate of s , \hat{s} , satisfies the following equation:

$$\partial \ln(f_R(r, \hat{s})) / \partial r |_{r=r_0} = 0.$$

In moderate and high SNR conditions, we have

$$\partial \left\{ \ln \left[\frac{r}{\sigma^2} \exp(-(r^2 + \hat{s}^2)/(2\sigma^2)) \frac{\exp(\frac{r\hat{s}}{\sigma^2})}{\sqrt{2\pi(\frac{r\hat{s}}{\sigma^2})}} \right] \right\} / \partial r |_{r=r_0} \approx 0,$$

which implies

$$\partial \left\{ \ln \left[\frac{r}{\sigma^2} \right] - \frac{(r^2 + \hat{s}^2)}{(2\sigma^2)} + \left(\frac{r\hat{s}}{\sigma^2} \right) - \frac{1}{2} \ln \left[2\pi \left(\frac{r\hat{s}}{\sigma^2} \right) \right] \right\} / \partial r |_{r=r_0} \approx 0.$$

Therefore,

$$\frac{1}{r_0} - \frac{r_0}{\sigma^2} + \frac{\hat{s}}{\sigma^2} - \frac{1}{2r_0} \approx 0.$$

The above approximation leads to $\hat{s} \approx r_0 - \sigma^2/(2r_0) \approx r_0 \equiv \sqrt{r_{0c}^2 + r_{0s}^2}$.

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