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Design of Multiplexed link with zero Intersymbol Interference for Bandlimited Channels using Time-limited Orthogonal Signaling

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***Abstract*— This paper explores the design of time-limited input-output waveforms, the outputs being orthogonal, for band-limited channels, whose characteristics are known a-priori. Hence, the channel can support baseband multiplexed transmission without Intersymbol Interference. The problem has been worked out for two cases of channel modeling - the rational transfer function channel model and the distributed parameter channel model, using three methods of signal orthogonalization. For the rational function channel model, the signaling interval can be made arbitrarily small, thus permitting high signaling rates. For the distributed parameter channel model, the signaling rate works out to be a function of the line parameters and line length. At the receiving end, since the signals are orthogonal, a bank of matched filters separates the signals. The system performance has been evaluated in the presence of channel noise.**

***Index Terms*—Bandlimited channels, Bit Error Rate, Intersymbol Interference, Functions of compact support, Sufficiently differentiable functions, Time-limited orthogonal signaling.**

I. Introduction

Traditional multiplexing techniques like Frequency Division Multiplexing and Time Division Multiplexing fail for baseband transmission, when the transmission medium is band-limited, as certain portions of the signal spectrum will be cut-off resulting in Intersymbol Interference (ISI). A special case of synthesis of band-limited orthogonal signals for multichannel data transmission has been worked out in [1], [2]. In this approach, the channels operate on equally spaced center frequencies and transmit at the same data rate with signaling intervals synchronized. This limits the frequency spectrum and data rate of each channel. The transmitter-receiver complexity of OFDM/BFDM (orthogonal/biorthogonal frequency division multiplexing) systems is justified only for time-varying channels [3], [4].

Hence the design of time-limited orthogonal signals with the entire bandwidth of the transmission medium available to each baseband signal, is significant as high signaling rates can be achieved without ISI. Slated in practical terms, the methods outlined here permit the synthesis of a large class of practical orthogonal waveform sets that can be transmitted simultaneously through a channel with known characteristics. Since the received signals remain orthogonal, the receiver implementation involves only a bank of matched filters to separate the signals and maximize the SNR. The matched filter outputs can be sampled at the signaling rate to recover the transmitted data.

The rest of this paper is organized as follows. In Section II, we discuss the design methodologies for time-limited orthogonal outputs. In Section III, the above methods are applied to realize a multiplexed link with zero ISI for a Cat 5E UTP (Unshielded Twisted Pair) cable, modeled as a rational transfer function. Section IV gives the multiplexed link implementation for a lossy transmission line, with a distributed parameter channel model. Section V discusses the results. The orthogonality of the channel output waveforms and the correctness of the detected signals, in the presence of channel noise, at the corresponding matched filter outputs in terms of Bit Error Rate (BER) have been evaluated. Finally, conclusions are presented in Section VI.

II. The Principle

Our approach will be to cast the channel transfer function *H(s)* as the following ratio:

$$H(s) = \frac{N(s)}{D(s)} \tag{1}$$

with *N(s)* and *D(s)* of degrees *m* and *n* respectively, $n > m$

Let T be the signaling interval. Application of Paley-Wiener Theorem [5], makes possible, the design of finite energy pulses $e_{i1}(t), e_{i2}(t), \ldots\ldots e_{in}(t)$ with support in the interval (0, T), that, as input to the channel, yields as output, finite-energy pulses $e_{o1}(t), e_{o2}(t), \ldots\ldots e_{on}(t)$ with support in the interval (0, T). Let $\{e_k(t)\}$ be a finite-energy pulse-set with support in the interval (0, T). Any pulse $e_n(t)$ in the set satisfies the properties:

a) $e_n(t)$ is continuous and $D^{n-1} e_n(t)$ is continuous.

b) $e_n(0) = e_n(T) = \ldots = D^{n-1} e_n(0) = D^{n-1} e_n(T) = 0.$

c) $D^n e_n(t)$ is a finite-energy pulse.

The pulse $e_n(t)$ with these properties is said to be sufficiently differentiable for the given channel function.

With such a pulse-set $\{e_k(t)\}$, input-output pulse sets can be obtained as

$$\{e_{ik}(t)\} = \{D(D)\, e_k(t)\} \quad (2)$$

$$\{e_{ok}(t)\} = \{N(D)\, e_k(t)\} \quad (3)$$

where *D(D)* and *N(D)* denote polynomials of the differentiation operator *D*, equivalent to *D(s)* and *N(s)* respectively. To satisfy the requirement that the output pulse set $\{e_{ok}(t) = N(D)\, e_k(t)\}$ be orthogonal, three methodologies are used: Symmetric Pulses, Pulses with Common Zeros, The Gram-Schmidt Orthogonalization Process.

A. Symmmetric Pulses

If $\{e_k(t)\}$ is an orthogonal set of sufficiently differentiable, symmetric pulses, $\{N(D)e_k(t)\}$is an orthogonal set, provided the polynomial *N(D)* is composed entirely of even powers of *D*, or entirely of odd powers of *D*. This is because, if any pulse $e_n(t)$ in the set $\{e_k(t)\}$is even/odd symmetric, all its existing derivatives of even order will be even/odd symmetric and all its existing derivatives of odd order will be odd/even symmetric.

B. Pulses with common Zeros

With this technique, the outputs have a closer structural uniformity with common zeros in time, than that obtainable from a set of symmetric orthogonal pulses.

Divide the interval (0, T) into j subintervals, not necessarily equal. Let the first subinterval contain the support of a pulse $c_1N(D)q_1(t)$, where c_1 is a constant, $q_1(t)$ is an arbitrary pulse that is sufficiently differentiable, and $N(D)q_1(t)$ is normalized. Let the second subinterval contain the support of a pulse $c_2N(D)q_2(t)$, etc. One pulse with support in the interval (0, T) is,

$$e_{o1}(t) = N(D)e_1(t) = N(D)\sum_{i=1}^{j} c_i q_i(t)$$

A second pulse with support contained in the interval (0, T), and with waveform structure similar to $e_{o1}(t)$, is

$$e_{o2}(t) = N(D)e_2(t) = N(D)\sum_{i=1}^{j} d_i q_i(t)$$

where each d_i is a constant to be determined. If $e_{o1}(t)$ and $e_{o2}(t)$ are to be orthogonal, their scalar product must be zero.

$$(\, N(D)e_1(t),\ N(D)e_2(t)\ ) = 0.$$

or

$$c_1d_1 + c_2d_2 + \ldots\ldots\ldots + c_jd_j = 0.$$

Proceeding in this fashion, an orthogonal set of pulses $\{e_{ok}(t)\}$ can be derived. The members of $\{e_{ok}(t)\}$ have zero amplitudes at the same points of time.

C. Gram-Schmidt Orthogonalization Process

For a set of sufficiently differentiable pulses $\{q_k(t)\}$, each pulse with support in the interval (0, T), the set $\{N(D)q_k(t)\}$contains pulses that are possible outputs, but generally not orthogonal. The set $\{\gamma_k(t)\} = \{N(D)\, q_k(t)\}$can be transformed into an orthonormal set $\{e_{ok}(t)\}$ in a manner analogous to that for transforming a general set of vectors.

Let $e_{o1}(t)$ represent pulse $\gamma_1(t)$ normalized,

$$e_{o1}(t)=\frac{\gamma_1(t)}{\|\gamma_1(t)\|},\ where\ \|\gamma_1(t)\|^2 = \int_0^T \gamma_1^2(t)dt$$

Define $\tilde{\gamma}_2(t)$,

$$\tilde{\gamma}_2(t)=\gamma_2(t) - (\gamma_2(t), e_{o1}(t))\ e_{o1}(t)$$

$$\text{where } (\gamma_2(t), e_{o1}(t)) = \int_0^T \gamma_2(t)\ e_{o1}(t)dt$$

Pulses $e_{o1}(t)$ and $\tilde{\gamma}_2(t)$ are orthogonal since their scalar product is zero. Let $e_{o2}(t)$ represent pulse $\tilde{\gamma}_2(t)$ normalized,

$$e_{o2}(t) = \frac{\tilde{\gamma}_2(t)}{\|\tilde{\gamma}_2(t)\|},\ \text{where } \|\tilde{\gamma}_2(t)\|^2 = \int_0^T [\tilde{\gamma}_2(t)]^2 dt$$

Pulses $e_{o1}(t)$, $e_{o2}(t)$ are two members of an orthonormal set. Proceeding in this fashion, the set $\{\gamma_k(t)\}$ can be transformed into the orthonormal set $\{e_{ok}(t)\}$. The orthonormal set $\{e_{ok}(t)\}$ can be written from Theorem 2 in [5] as

$$\{e_{ok}(t)\} = \{N(D)e_k(t)\}, where\ \{e_k(t)\} = \left\{\frac{\tilde{q}_k(t)}{\|N(D)\tilde{q}_k(t)\|}\right\}$$

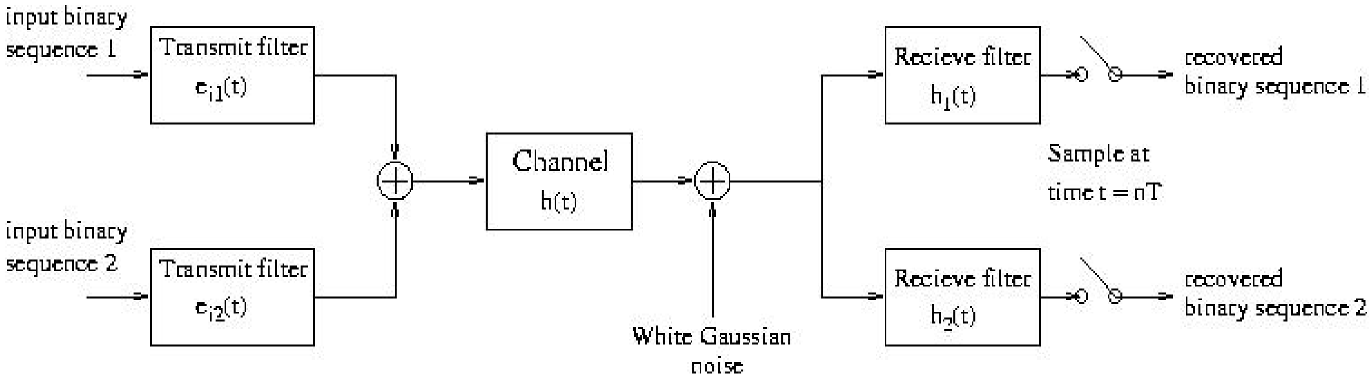
$$and\ \tilde{q}_k(t) = q_k(t) - \sum_{n=1}^{k-1} (\, N(D)q_k(t), N(D)e_n(t)\ )\ e_n(t)$$

D. Implementation Procedure

Input-output waveform pairs are worked out for a Cat 5E UTP cable and a lossy transmission line, by applying the above explained techniques. For both cases, multiplexed output is generated by multiplying independent data streams with the designed input wave shapes and adding them. At the receiving end, the multiplexed signal, mixed with noise, is applied as input to a bank of matched filters, and gets convolved with their corresponding impulse responses. The matched filter outputs are sampled at the signaling rate to recover the corresponding transmitted data. Thus, a multiplexed link with zero ISI can be established using time-limited orthogonal signaling as given by Fig. 1.

The recovered data is compared with the transmitted data and BER computed in the presence of channel noise. The orthogonality of the outputs is also checked by evaluating their dot-product. The SNR used for performance measure is given as:

$$SNR = 10\times\log_{10}\left(\frac{E_i}{N_o}\right) \quad (4)$$



where E_i is the signal energy in joules at the channel input and N_o is the noise power density in Watts/HZ.

Fig. 1. Multiplexed link with zero ISI using time-limited orthogonal signaling

III IMPLEMENTATION FOR CAT 5E UTP CABLE

A rational transfer function model, *H(D)*, is obtained for a Berk-Tek Cat 5E UTP cable, by fitting the attenuation characteristics given in the data sheet, using standard regression techniques, as given by (5). Input-output waveshapes pairs, with the outputs being orthogonal, are worked out for a period of *T = 10 nsec*. i.e., signaling rate of *100 Mbps*.

$$H(D) = \frac{\delta D^2 + \eta}{\alpha D^4 + \beta D^2 + \gamma} = \frac{N(D)}{D(D)}$$

$$\text{where } \alpha = 6.42\times10^{-4}$$
$$\beta = -48.2\times10^{12}$$
$$\gamma = 40303.02\times10^{24}$$
$$\delta = -19.45\times10^{12}$$
$$\eta = 32648.52\times10^{24} \quad (5)$$

Using the three methodologies explained in Section II, we derive two sufficiently differentiable and orthogonal functions $e_1(t)$ and $e_2(t)$ suitable for (5) and obtain corresponding input-output waveforms as:

$$e_{i1}(t) = (\alpha D^4 + \beta D^2 + \gamma)\, e_1(t)$$
$$e_{i2}(t) = (\alpha D^4 + \beta D^2 + \gamma)\, e_2(t)$$
$$e_{o1}(t) = (\delta D^2 + \eta)\, e_1(t)$$
$$e_{o2}(t) = (\delta D^2 + \eta)\, e_2(t) \quad (6)$$

The pulse width T may be made arbitrarily small, permitting the use of high signaling rates, as T is not dependent on the channel function.

A. Symmetric Pulses

This technique, as explained in Section II *A*, can be applied to the channel model given by (5) as *N(D)* is composed only of even powers of *D*. Two sufficiently differentiable and orthogonal functions for (5) are:

$$e_1(t) = (1-\cos(bt))^4 \left[u(t) - u\left(t - \frac{2\pi}{b}\right)\right]$$

$$e_2(t) = (1-\cos(2bt))^4 \left[u(t) - 2u\left(t - \frac{\pi}{b}\right) + u\left(t - \frac{2\pi}{b}\right)\right]$$

$$\text{where } T = \frac{2\pi}{b}$$

$e_1(t)$ and all its existing derivatives of even order are even symmetric and $e_2(t)$ and all its existing derivatives of even order are odd symmetric. From $e_1(t)$ and $e_2(t)$, input-output waveforms can be obtained using (6).

B. Pulses with Common Zeros

The interval (0, T) is divided into two equal subintervals, for convenience i.e. (0, T/2) and (T/2, T). Two pulses that are sufficiently differentiable for the channel function given by (5) are:

$$q_1(t) = (1-\cos(2bt))^4 \left[u(t) - u\left(t - \frac{\pi}{b}\right)\right]$$

$$q_2(t) = \left(1-\cos\left(2b\left(t - \frac{\pi}{b}\right)\right)\right)^4 \left[u\left(t - \frac{\pi}{b}\right) - u\left(t - \frac{2\pi}{b}\right)\right]$$

$$\text{where } T = \frac{2\pi}{b}$$

$q_1(t)$ has support in the interval (0, T/2) and $q_2(t)$ has support in the interval (T\2, T). Following the procedure explained in Section II *B*,

$$e_1(t) = q_1(t) + q_2(t)$$
$$e_2(t) = q_1(t) - q_2(t)$$

From $e_1(t)$ and $e_2(t)$, input-output waveforms can be obtained using (6).

C. Gram-Schmidt Orthogonalization Process

One pair of sufficiently differentiable pulses for the channel function given by (5) is

$$q_1(t) = (1-\cos(bt))^4 \left[u(t) - u\left(t - \frac{2\pi}{b}\right)\right]$$

$$q_2(t) = (1-\cos(2bt))^4\left[u(t)-u\left(t-\frac{2\pi}{b}\right)\right]$$
$$where\ T=\frac{2\pi}{b}$$

From $q_1(t)$ and $q_2(t)$, $e_1(t)$ and $e_2(t)$ are derived following the procedure explained in Section III *C*. Then, input-output waveforms are obtained using (6).

IV IMPLEMENTATION FOR TRANSMISSION LINE

In this case, since the channel transfer function *H(s)* is not a rational function, input-output waveshape pairs cannot be derived directly from (2) and (3). The design of a time-limited input-output pair involves the following steps [6], [7]:

1. Normalization of *H(s)* to *H'(s)*
2. Design of $\{e_i'(t), e_o'(t)\}$ corresponding to *H'(s)*
3. Transformation of $\{e_i'(t), e_o'(t)\}$ to $\{e_i(t), e_o(t)\}$

To get orthogonal outputs, the procedure followed is to design $p_1(t)$ and $p_2(t)$, by the techniques detailed in Section II, such that $e_{o1}(t)$ and $e_{o2}(t)$ are orthogonal. $p_1(t)$ and $p_2(t)$ are then transformed into $e_1(t)$ and $e_2(t)$ respectively, to get input-output waveform pairs.

We have implemented the multiplexed link for a lossy transmission line with line length of 100m and per unit length parameters as listed below:
$R = 0.1$ *ohms/m*, $L = 0.01*10^{-6}$ *Henry/m*,
$G = 0.01$ *mhos/m*, $C = 100*10^{-12}$ *Farad/m*
$Z_o = Z_l = 50\ \Omega$

For the parameters listed above, the input-output waveshape equations reduce to

$$e_i(t) = \frac{1}{a}e^{-bt}e_i'\left(\frac{t}{a}\right)$$
$$e_o(t) = \frac{1}{a}e^{-bt}e_o'\left(\frac{t}{a}\right) \qquad (7)$$
$$\text{where } a = 10^{-7}, b = 5.5\times10^{7}$$

$$e_i'(t) = (0.05D+0.225)[\tilde{h}_1(t)*e_1(t)+e_1(t)+e_1(t-2)] + (0.13D^2+1.125D+2.43)[\tilde{h}_2(t)*e_1(t)]$$
$$e_o'(t) = (0.05D+0.225)[e_1(t-1)] \qquad (8)$$

$$\tilde{h}_1(t) = \begin{cases} -\lambda(t^2-2t)^{-\frac{1}{2}}I_1\left[\lambda(t^2-2t)^{\frac{1}{2}}\right] & for\ 0\le t\le 2 \\ 0 & elsewhere \end{cases}$$
$$\tilde{h}_2(t) = \begin{cases} I_0\left[\lambda(t^2-2t)^{\frac{1}{2}}\right] & for\ 0\le t\le 2 \\ 0 & elsewhere \end{cases}$$

I_0 and I_1 are modified Bessel functions of zeroth and first order respectively.

The equation transforming $p_1(t)$ into $e_1(t)$ works out as:
$$e_1(t) = ae^{ab(t+1)}p_1(at+a) \qquad (9)$$
where $e_1(t)$ has a period $T'' = 2$ (as $\hat{h}_1(t)$ and $\hat{h}_2(t)$ are defined for $(0\le t\le 2)$). $\{e'_i(t), e'_o(t)\}$ is an input-output pair corresponding to *H'(s)* and time-limited to (0, 4). $e_i(t)$ and $e_o(t)$ have duration $T = 4a$. Note that unlike with the Cat 5E cable that has a rational transfer function model, the time-durations of the input and output pulses we have designed for the distributed parameter model cannot be chosen arbitrarily, as *T* is determined by *a*, which is a function of the line parameters and the line length.

Using (9), $p_1(t)$ and $p_2(t)$ can be transformed into $e_1(t)$ and $e_2(t)$ respectively. Since $e_1(t)$ and $e_2(t)$ are defined from *0-2*, $p_1(t)$ and $p_2(t)$ have periods from *a-3a* i.e. $T' = 2a$.

The technique of Symmetric pulses as explained in Section II *A*, cannot be implemented for this case, as (8), defining the output, is not composed entirely of even powers of *D* or odd powers of *D*. Orthogonal outputs are designed using the other two techniques i.e. Pulses with common zeros and the Gram-Schmidt orthogonalization process.

A. Pulses with Common Zeros

Sufficiently differentiable functions $q_1(t)$ and $q_2(t)$ corresponding to (7) and (8) with support in the intervals $(0, T'/2)$ and $(T'/2, T')$ respectively are:

$$q_1(t) = (1-\cos(2bt))^2\left[u(t)-u\left(t-\frac{\pi}{b}\right)\right]$$
$$q_2(t) = \left(1-\cos\left(2b\left(t-\frac{\pi}{b}\right)\right)\right)^2\left[u\left(t-\frac{\pi}{b}\right)-u\left(t-\frac{2\pi}{b}\right)\right]$$
$$where\ T'=\frac{2\pi}{b}$$

Following the procedure explained in Section II *B*,
$$p_1(t) = q_1(t)+q_2(t)$$
$$p_2(t) = q_1(t)-q_2(t)$$

Actually, $p_1(t)$ and $p_2(t)$ are to be defined from *a-3a*. For the $q_1(t)$ and $q_2(t)$ we have chosen, $p_1(t) = p_1(t-a)$ and $p_2(t) = p_2(t-a)$. Transforming $p_1(t)$ and $p_2(t)$ into $e_1(t)$ and $e_2(t)$ using (9), the pair of orthogonal outputs and the corresponding inputs are worked out using (7) and (8).

B. Gram-Schmidt Orthogonalization Process

Here also, we first design $p_1(t)$ and $p_2(t)$ for $T' = 2a$. One pair of sufficiently differentiable pulses for the transmission line specified by (7) and (8) are:

$$q_1(t) = (1-\cos(bt))^2\left[u(t)-u\left(t-\frac{2\pi}{b}\right)\right]$$
$$q_2(t) = (1-\cos(2bt))^2\left[u(t)-u\left(t-\frac{2\pi}{b}\right)\right] \quad where\ T'=\frac{2\pi}{b}$$

Since $p_1(t)$ and $p_2(t)$ to be defined from *a-3a*, $q_1(t)$ and $q_2(t)$ are translated to *a-3a*, giving,

$$q_1(t) = \left(1+\cos\left(\frac{\pi t}{a}\right)\right)^2 [u(t-a)-u(t-3a)]$$

$$q_2(t) = \left(1-\cos\left(\frac{2\pi t}{a}\right)\right)^2 [u(t-a)-u(t-3a)]$$

Following the procedure for orthonormalization explained in Section II *C*, $p_1(t)$ and $p_2(t)$ are derived from $q_1(t)$ and $q_2(t)$. Transforming $p_1(t)$ and $p_2(t)$ into $e_1(t)$ and $e_2(t)$ using (9), the pair of orthogonal outputs and the corresponding inputs are worked out using (7) and (8).

V RESULTS

The dot-product of the outputs, $e_{o1}(t)$ and $e_{o2}(t)$, taking random data, for both Cat 5E cable and transmission line, for all the techniques, gave zero, thus proving their orthogonality over the signaling interval. The BER results were also satisfactory, thus establishing the reliability of the multiplexed link even in the presence of channel noise.

A sample result, taking random data, for the method given in Section III *B* is given below:

Dot product of $e_{o1}(t)$ and $e_{o2}(t)$ = 4.392144e-016

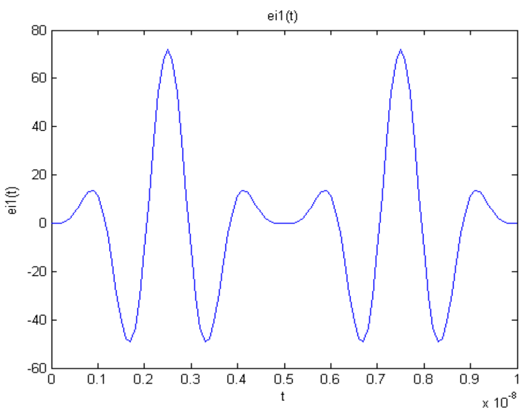
For channel without noise, BER1 = 0 and BER2 = 0.

For channel SNR = 50 dB, BER1 = 0 and BER2 = 0.

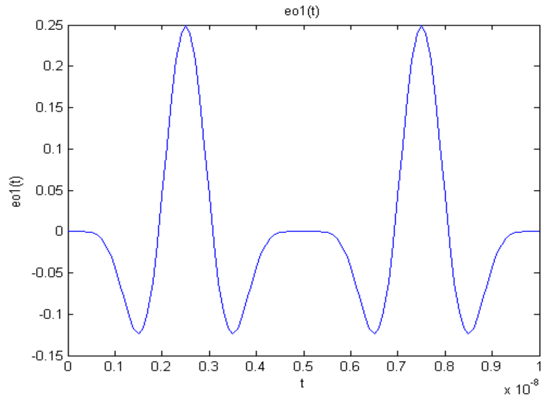
For channel SNR = 40 dB, BER1 = 0.0090 and BER2 = 0.0040

BER1 is the BER at the output of the Receive filter $h_1(t)$ and BER2 is the BER at the output of the Receive filter $h_2(t)$.

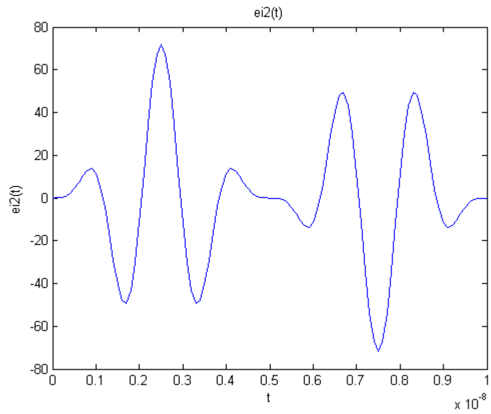
Fig 2 gives the corresponding input-output waveforms.



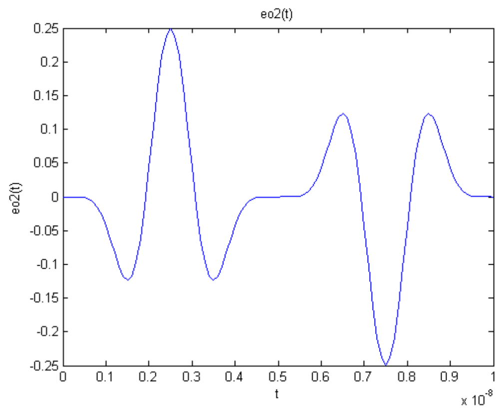
(a)



(b)



(c)



(d)

Fig. 2. Cat 5E cable input-output waveform pairs for the method: Pulses with common zeros
(a) Channel input 1, $e_{i1}(t)$ (b) Channel output 1, $e_{o1}(t)$
(c) Channel input 2, $e_{i2}(t)$ (d) Channel output 2, $e_{o2}(t)$

VI CONCLUSION

The results presented in Section V prove that a channel, whose characteristics are known a priori, can be used as an ISI free multiplexed link for baseband transmission. It is possible to synthesize a large class of time-limited input-output waveform sets, with the output pulses orthogonal and time-bound to the same value. They can be transmitted simultaneously along the channel length without ISI, increasing the channel capacity. A bank of matched filters at the receiving end can successfully separate out the signals even in the presence of channel noise. Such a multiplexed link is implemented for both the Cat 5E UTP and the lossy transmission line using different orthogonalization techniques. In all the cases, the corresponding matched filter at the receiver is able to successfully separate out the correct data stream, from the multiplexed signal corrupted by noise.

For the Cat 5E cable, where the channel is modeled as a rational transfer function, the time-duration of the input-output pulses are independent of the channel parameters, thus permitting arbitrarily high signaling rates. For the case of the lossy transmission line, the time duration of the input-output pulses is found to be dependent on channel parameters and line length. As the line length increases, the signaling rate for zero ISI operation decreases.

In the solutions worked out, the only restraints imposed on the input-output pulses, in addition to them being required to be time-bound and output pulses to be orthogonal, are that both be real-valued finite-energy pulses. By proper choices of sufficiently differentiable sets $\{e_k(t)\}$, many sets of input-output waveshapes satisfying the above criteria can be obtained. Additional criteria such as, energy considerations of transmitting and receiving systems and detection reliability in the presence of noise, could be taken into consideration to choose an optimal set of orthogonal outputs.

Slated in practical terms, the application of the principles derived in this paper, makes possible the synthesis of a large class of orthogonal waveform sets, which can be used to transmit numerous data streams simultaneously along a known bandlimited channel without ISI. Since the entire bandwidth of the transmission medium is available to each data stream, high signaling rates can be achieved without ISI.

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