

Fuzzy Reliability of a Network Using Fuzzy Sum of Disjoint Product Technique

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Abstract— In this study, we proposed a method to find the fuzzy reliability of the network using fuzzy sum-of-disjoint product technique. We use the fuzzy graph theory techniques for addressing the reliability of a network with known minimal paths. We have also presented an efficient algorithm to find the reliability of the network.

Index Terms— Network, Fuzzy Graph, Fuzzy Markov Model, Sum of Disjoint Product, Triangular Fuzzy Number, Fuzzy Reliability.

I. INTRODUCTION

Lofti A. Zadeh introduced fuzzy sets in 1965 to represent/manipulate data and information possessing non-statistical uncertainties [10]. Fuzzy systems are suitable for uncertain or approximate reasoning, especially for the system with a mathematical model that is difficult to derive.

Complex networks are used to interconnect multiple processing centers in distributed systems for several applications, such as telecommunications, the airline industry, banking and internet etc., Distributed processing offers several advantages such as increased performance, improved reliability of applications and decreased cost through resource sharing. In designing such networks, an important consideration is often network reliability [5]. Sum of Disjoint Product (SDP) is one of the most convenient techniques discussed in the literature for the network reliability evaluation.

The reliability engineering is one of the important engineering tasks in design and development of a technical system. It is well known that the conventional reliability analysis, using the probabilities, has been found to be inadequate to handle uncertainty of failure data and modeling. Simonovic [6] States that the two major sources of uncertainty are randomness and lack of knowledge. To overcome this problem, the concept of fuzzy approach has been used in the evaluation of the reliability of a system [7] presented a fuzzy set approach for fault tree and reliability analysis.

Fuzzy reliability has been studied by many authors. The theory of fuzzy reliability was proposed and developed by several authors, Cai, Wen and Zhang [3, 4] (1991, 1993); Cai (1996); Chen, Mon (1993); Hammer (2001); El-Hawary (2000), Onisawa, Kacprzyk (1995); Utkin, Gurov (1995). The recent collection of papers by Onisawa and Kacprzyk (1995), gave many different approach for fuzzy reliability. In many cases, the notion of representing component reliability indices (such as

failure rate or mean repair time) by crisp numbers must be challenged.

The rest of this paper is organized as follows: Section 2 we have given the basic definitions which are necessary to this paper. In section 3, we define a fuzzy sum of disjoint product. An algorithm to find the fuzzy reliability between two vertices in a given network using fuzzy sum disjoint product technique has been given in section 4. In section 5 we give an algorithm to find the fuzzy reliability of a network and give a numerical example in section 6. We end up with the conclusion in section 7.

II. NETWORK RELIABILITY AND SUM OF DISJOINT PRODUCT

A network is a directed graph or digraph is a pair $G = (V, A)$ where a set V , whose elements are called vertices or nodes, a set A of ordered pairs of vertices, called arcs, directed edges or arrows in which each edges associated with a non-negative real number called weight of the edge.

A fuzzy graph $G(V, E)$ [1] consists of crisp set of vertices V , a crisp set of weighted edges E in which each edge e_i is associated with a triangular fuzzy number as a fuzzy weight $\mu(e_i)$.

Network reliability refers to the reliability of the overall network to provide communication in the event of failure of a component or components in the network.

In sum disjoint product (SDP) the addition law of probabilities is the underlying justification. If two or more events have no element in common, the probability that at least one of them will occur is the sum of the probabilities of the separate sets. For example, with two events A and B , let \bar{A} denote the complement of A . Then we have:

$$P\{A \text{ or } B\} = P\{A\} + P\{\bar{A} \text{ and } B\}$$

Similarly with three events A, B, C , then we have,

$$P\{A \text{ or } B \text{ or } C\} = P\{A\} + P\{\bar{A} \text{ and } B\} + P\{\bar{A} \text{ and } \bar{B} \text{ and } C\}$$

III. FUZZY SUM OF DISJOINT PRODUCT

In this paper, we are using fuzzy possibility [9] as a triangular fuzzy number [2] to address the problem of finding the fuzzy reliability of a network.

In fuzzy sum of disjoint product (FSDP) the addition law of possibilities is the underlying justification. If two or more events have no element in common, the

possibility that at least one of them will occur is the sum of the possibilities of the separate sets. We define the *fuzzy sum of disjoint product* as follows:

With two events A and B , let \bar{A} denote the complement of A . Then we have:

$$\tilde{P}\{A \text{ or } B\} = \max\{\tilde{P}\{A\}, \min\{\tilde{P}\{\bar{A}\}, \tilde{P}\{B\}\}\}$$

Similarly with three events A, B, C , then we have,

$$\tilde{P}\{A \text{ or } B \text{ or } C\} = \max\{\tilde{P}\{A\}, \min\{\tilde{P}\{\bar{A}\}, \tilde{P}\{B\}, \min\{\tilde{P}\{\bar{A}\}, \tilde{P}\{\bar{B}\}, \tilde{P}\{C\}\}\}\}$$

In a similar way, we can extend this fuzzy sum disjoint product to n - number of events.

IV. AN ALGORITHM TO FIND THE FUZZY RELIABILITY BETWEEN TWO VERTICES IN A GIVEN NETWORK USING FUZZY SUM OF DISJOINT PRODUCT TECHNIQUE

Fuzzy reliability is defined as the failure free operation of the system at any given time. The fuzzy reliability of a Fuzzy Markov Model depends on the transition from one state to another state [8].

A *fuzzy Markov model* is a *Markov model* [8] consists of a set of states $\{s_1, s_2, s_3, \dots, s_m\}$ together with the *transition possibilities* m_{ij} , $i, j = 1, 2, 3, \dots, m$, where m is the total number of states in the *FMM* and m_{ij} is the *transition possibility* from state i to state j as a *triangular fuzzy number*.

A *Fuzzy Markov Model (FMM)* can be viewed as a *fuzzy graph* in which the states are considered as the set of vertices, the transition link between the vertices as the edges and the *link possibility (transition possibility)* as the *fuzzy edge possibility* $\mu(e_i)$.

For a given pair of vertices (states) v_s and v_t in a given network, we can find the fuzzy reliability between them. Using these fuzzy reliability's, we are finding the fuzzy reliability of the network, we give an algorithm to find the *fuzzy reliability* between every pair of vertices is given below:

Consider a fuzzy graph $G(V, E)$ where V is a crisp set of vertices(states), E is a crisp set of edges in which each edge e_i is associated with fuzzy weight $\mu(e_i)$ as a triangular fuzzy number.

Aim: To find the *fuzzy reliability* between every pair(s, t) of vertices in a given network.

Input: A connected graph $G(V, E)$ with vertex set V , and edge set E in which each edge associated with a fuzzy weight as a triangular fuzzy number, a source node s , a sink node t and all minimum paths that connect the source node s and a sink node t , $A_1, A_2, A_3, \dots, A_m$, where $|A_i| \leq |A_j|$, $i \leq j$ and $i, j = 1, 2, 3, \dots, m$.

Output: *fuzzy reliability* between the pair of vertices as a *triangular fuzzy number*.

Algorithm:

$$\text{Step 1: } \tilde{R}_1 = \min_{e_i \in A_1} \{\mu(e_i)\}$$

Step 2: Do the following steps $k = 2, 3 \dots m$

$$\tilde{R}_k = \min_{e_i \in A_k} \{\mu(e_i), 1 - \min_{e_l \in A_j \cap A_k} \{\mu(e_l)\}_{j=1,2,3,\dots,k-1}\}$$

Step 3: Therefore fuzzy reliability between the pair of vertices (s, t) is

$$\tilde{R}_{s,t} = \max\{\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_m\}$$

In the above algorithm we have found the *fuzzy reliability* between any two vertices v_s and v_t in a given network which is used to find the *fuzzy reliability* of the network. In the next section, we give an algorithm to find the *fuzzy reliability* of the network

V. FUZZY RELIABILITY OF A NETWORK

By using the above algorithm, we can find the fuzzy reliability between every pair of vertices in a given network (system). Using these fuzzy reliability's, we are finding the fuzzy reliability of the network as follows:

- For every pair of vertices, we get the fuzzy reliability using the above algorithm.
- Fuzzy reliability of the given network is

$$\tilde{R} = \min_{s,t \in V(G)} \{\tilde{R}_{s,t}\}$$

- Note that fuzzy unreliability of the given network is $\tilde{R}_U = 1 - \tilde{R}$.

This method of finding fuzzy reliability of a network (system) using *fuzzy sum disjoint product (FSDP)* technique is illustrated in the following example.

VI. NUMERICAL EXAMPLE

The website of any organization or institution can be considered as a Fuzzy Markov Model (FMM) consisting of various web pages (states) and the link between the web pages (edges). In this case, the reliability of the website is the accessibility of any page from the existing page. Hence every pair of web pages has to be considered as a source and sink for finding the reliability. To illustrate the above technique we have considered the website www.ssn.edu.in. In this, we have taken the department which consists of four web pages Information Technology (IT), Electrical and Electronics Engineering (EEE), Electronics and Communication Engineering (ECE) and Chemical Engineering (CHEM). The following graph is the model of the component (departments) of the website www.ssn.edu.in under study. The number on the each link represents the fuzzy possibility of the end nodes which was calculated based on the data, collected from the access log files and error log files.

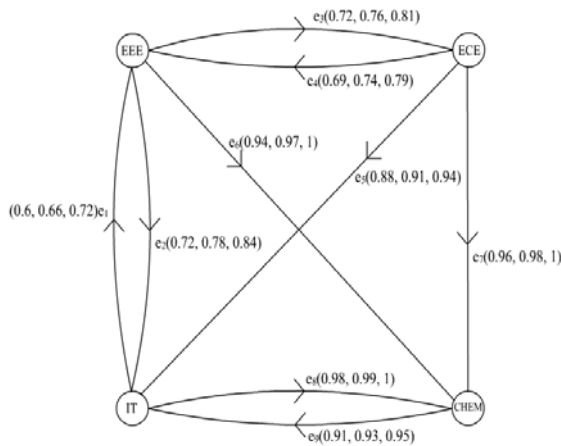


Figure 1: Network with Fuzzy Weights

We treat the above FMM as a network (fuzzy graph) with fuzzy weight. We are finding the fuzzy reliability between every pair of vertices using the algorithm given in section V, which is represented in the following table.

Table 1: Calculation Table

Serial No.	Source(v_s)	Sink (v_t)	Fuzzy Reliability($\tilde{R}_{s,t}$)
1.	EEE	ECE	(0.72,0.76, 0.81)
2.	EEE	CHEM	(0.94, 0.97, 1)
3.	EEE	IT	(0.72, 0.78, 0.84)
4.	ECE	EEE	(0.69, 0.74, 0.79)
5.	ECE	CHEM	(0.96, 0.98, 1)
6.	ECE	IT	(0.88, 0.91, 0.94)
7.	CHEM	IT	(0.91, 0.93, 0.95)
8.	CHEM	EEE	(0.6, 0.66, 0.72)
9.	CHEM	ECE	(0.6, 0.66, 0.72)
10.	IT	EEE	(0.6, 0.66, 0.72)
11.	IT	ECE	(0.6, 0.66, 0.72)
12.	IT	CHEM	(0.98, 0.99,1)

Using the algorithm in section IV, we can find Fuzzy reliability of the considered network is = (0.6, 0.66, 0.72).

CONCLUSION

We have presented an algorithm to analyze the fuzzy reliability of a network using fuzzy sum disjoint product technique where the fuzzy reliability and the possibility of the links are represented as a triangular fuzzy number. In this paper we have considered Fuzzy Markov Model as a network (fuzzy graph) with fuzzy weight as a triangular fuzzy number. We have found the fuzzy reliability between every pair of vertices using fuzzy sum disjoint product technique. Using these fuzzy reliability's, we are finding the fuzzy reliability of the network. Since the edge possibilities are represented as a triangular fuzzy number, the fuzzy reliability of a network is also a triangular fuzzy number. And this procedure is illustrated by a numerical example.

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